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Schopenhauer on space, time, causality and matter: a physical reexamination

Schopenhauer sobre espaço, tempo, causalidade e matéria: um reexame físico

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Abstract: According to Schopenhauer, Kant's arguments about the transcendental ideality of space and time have implications for matter through the concept of causality and the principle of sufficient reason. In this article, I examine to what extent this principle, together with space, time and causality can be considered a priori concepts in the light of classical and modern physics. The concepts of matter and field in present day physics, and their possible a priori fundaments, are revisited in a modern context.
Keywords: Space; Time; Matter; Modern physics

Resumo: Segundo Schopenhauer, os argumentos de Kant sobre a idealidade transcendental do espaço e do tempo têm implicações para a matéria por meio do conceito de causalidade e do princípio de razão suficiente. Neste artigo, examino até que ponto este pressuposto, juntamente com as noções de espaço, de tempo e de causalidade, podem ser considerados conceitos a priori à luz da física clássica e moderna. Os conceitos de matéria e de campo na física atual e seus possíveis fundamentos a priori são revisitados sob um contexto moderno.
Palavras-chave: Espaço; Tempo; Matéria; Física moderna

Kant argued that space and time are *a priori* forms of intuition (*Anschauungsformen*), and "transcendental aesthetics cannot contain more than these two elements" (B58)[1]. Objects affect our senses as phenomena in space and time, and the phenomena are produced by a *thing-in-itself* which is not directly accessible to the senses.

In *The World as Will and Representation* (W I), Schopenhauer accepted Kant's transcendental aesthetics, but reduced the categories of his transcendental logics to only one: causality. He argued that causality is another form of the perception of phenomena, though distinct from space and time, and that it is also given *a priori*, i.e., prior to all experience. He thus stood in opposition to empiricists such as Locke and Hume, who supposed that our concept of causality follows *a posteriori* from empirical experience.

In his famous doctoral thesis, *The Fourfold Root of The Principle of Sufficient Reason* (G), Schopenhauer argued that causality is indissolubly associated to the principle of sufficient reason. He identified four roots of this principle, two of them being related to the perception of matter which he stated to be "through and through causality".

Can the above principles be sustained in the light of modern science, particularly modern physics? Regarding these problems, I pointed out in previous works (Hacyan[2]) that the concepts of space and time, as known in the macroscopic world, do not apply to

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1 All references to KANT are to the Critique of Pure Reason.
2 HACYAN, S. *Física y metafísica del espacio y del tiempo* (2004). *On the trascendental ideality of space and time in modern physics* (2006).

the atomic world due the existence of non-local effects, and that this well-established fact is perfectly compatible with Kant´s thesis that space and time are forms of intuition, not to be found in the thing-in-itself producing a phenomenon, but in its perception.

The aim of the present article is to examine whether causality could be a concept *a priori*, as stated by Schopenhauer, and to analyze its relationship with the concepts of matter and mass in the light of classical and modern physics. In particular, I will consider the well-established notion of field in modern physics, since it has replaced the vague idea of substance as the substratum of matter in classical philosophy.

Sufficient reason and causality

According to Empiricism, we learn that a certain effect is due to a certain cause only by experience. Accordingly, causality should be a probabilistic concept, since we cannot know by experience that a certain phenomenon which has never occurred could occur someday. Schopenhauer, on the contrary, argued that causality is a form of understanding given *a priori*: the fact that we always associate a cause to an effect is an innate process and could not be otherwise, since it is the only way to give a meaning to all empirical experiences. By experience alone, we would perceive only a temporal succession of unconnected states.

Causality is closely related to the principle of sufficient reason, since any effect must have a cause, but this relation, according to Schopenhauer, is also given *a priori*. In his doctoral thesis (G), he identified the first root of the principle of sufficient reason as the causal association of sense stimuli to the phenomenon producing it. He gave as an example the physico-chemical stimuli produced by light on the retina and the transmission of this information to the brain through the optical nerve; this crude information must, however, be processed by a certain form of understanding in order to yield an image of reality. In the modern language of computers, we could say that the brain has a *software* that processes the received information and produces the images of the world, just as an actual computer transforms electrical currents in its circuits into images on a screen: in this case the stimuli are a myriad of tiny electronic currents. Anachronisms apart, Schopenhauer's point is that our mind is equipped with such an innate software: our brain is born with it and we learn to use it through experience.

Let us illustrate the *a priori* search of sufficient reason with some examples taken from our perception of physical phenomena. Consider the obvious (and rather naïve) example of the fall of heavy bodies due to the action of gravity. If it were a purely empirical fact, it could occasionally happen that a body remained in levitation. Based on previous experiences, it would be extremely improbable but not fully impossible; after all, it is only a matter of probability. In an analogous example, it may be an empirical fact that I have never won at a lottery, but if I win someday, no law of nature would be violated. Of course, we accept that bodies fall by the action of gravity due to a universal law of nature. If, by any chance, it happened that a body did not fall, we would not be satisfied with the explanation that it is very improbable but not utterly impossible. Quite on the contrary, we would look for a reason for such a phenomenon; for instance, that the said body is lighter than air, that it is held by invisible strings, or levitated by magnetic fields, etc. Actually, that is the purpose of science: given a new and unexpected phenomenon, to look *always* for its cause. (Incidentally, Schopenhauer was aware that weight is knowable *a posteriori*, not *a priori* as Kant believed: see Ch. 4 of (W I).

A more general example of the principle of sufficient reason is given by Newton´s laws of dynamics. In the first law, Newton stated that a body remains in its state of rest or uniform motion if no forces act on it. However, the concept of force is specified only by the second law: a body changes its state of rest or motion (more precisely, its quantity of motion) if a force acts on it. But this is just an acknowledgment that the only way to define force is as the sufficient reason for the change of a state. Euler himself clearly made the point, in his treatise of mechanics, that the motion of a body is necessarily due to some reason, and that this reason is nothing else than what we call force: see particularly Proposition 7 in his *Mechanica*[3].

Another, more actual example is the phenomenon of superconductivity. In 1911, Kamerlingh Onnes made the surprising discovery that the electrical resistivity of mercury drops exactly to zero below a certain temperature. Afterwards, physicists spent several decades looking for an explanation of this unexpected phenomenon; they finally found its (sufficient) reason in the realm of quantum mechanics.

Yet another example is the actual situation in cosmology: the observational data fit quite well with the model of the Universe predicted by the theory of General Relativity, but only if an invisible "dark matter" is assumed to exist, together with an even more mysterious cosmic acceleration: the so-called "dark energy". At present, most cosmologists believe that the model is correct and look for direct evidence of dark matter (as a sufficient cause). Other researchers have pointed out that gravitation may not behave as believed at very large scales. In either case, a sufficient reason is sought, whether in the form of a new kind of matter or a departure from General Relativity, the currently accepted theory of gravity. In all the given examples, causality is closely related to the principle of sufficient reason: there is a reason for every phenomenon.

The perception and study of nature was mainly done through direct senses in Schopenhauer's times, as he discussed at length in 4R. It is mainly in the nineteenth century that scientists started to study the world with the massive support of sophisticated apparatuses intended to extricate its secrets from nature. Thus, for instance, Faraday performed all his crucial experiments with schemes and apparatuses designed to make visible the effects of the invisible mechanisms producing electric and magnetic forces. Gone was the direct perception of nature so celebrated by the poets (see Sect. 6 below)!

A century ago, Duhem[4] discussed this important point in detail: "An experiment in Physics is not simply the observation of a phenomenon; it is, in addition, the theoretical interpretation of this phenomenon". Indeed, a layperson visiting a physicist's laboratory would not deduce what it is all about but would only see a set of apparatuses and monitors displaying graphs and numbers. Physicists who study, say, subatomic particles do not see electrons, protons, quarks, Higgs bosons, etc. in a direct way: they can only analyze a huge quantity of data collected by appropriately designed detectors and investigate the processes that take place using a theoretical model. The data produced in modern experiments must be processed according to very complicated (real) software, something similar, in a sense, to what the brain does with the primary stimuli received from our sense organs; in other words, the brain uses a huge extension of its innate "software".

In summary, there are various steps in the process of understanding physical processes. In Kantian terms, there is a thing-in-itself that produces a phenomenon. This phenomenon is perceived as a direct stimulus in our sense organs and is further processed by the brain. Alternatively, a complex phenomenon can produce a set of data in an apparatus in the form of numbers or images, and these data in turn are processed according to a pre-established theoretical model. In the former case, the relation between cause and effect is provided by an internal innate "software" (*a priori*) in the brain; in the latter case, by an external real software in an artificial brain. In modern science, experimental results and

[3] EULER, L. *Mechanica.*

[4] DUHEM, P. *La théorie physique* (1914)

their interpretations are necessarily mediated by a theory providing the causal relations between crude data and the theoretical description of a physical phenomenon. If some unexpected result or inconsistency appears, it must have a sufficient reason: it may be something yet unknown within a known theory (such as dark matter) or an inadequacy of this theory (such as a possible modification of General Relativity).

Matter and causality

Matter is quantified as mass, and mass can be determined empirically, either directly as weight or indirectly by the motion of a body. Accordingly, Kant did not consider mass as a pure form of intuition *a priori*. On this point, he is quite explicit in stating that "the possibility of the synthesis of the predicate 'weight' with the concept of 'body'... rests upon experience" (B12).

Schopenhauer, however, dealing with the third root of the principle of sufficient reason, stated that matter is the "perceptibility of time and space, on the one hand, and causality that has become objective, on the other" (G §35). He further elaborated on this point in (W I); for instance, in Chap. 4: "Time and space, each for itself, can be mentally presented apart from matter, whereas matter cannot be so presented apart from time and space".

In Chap. 24, he stated, in more detail, that "the whole being of matter consists in *acting*. Only thus does it occupy space and last in time. It is through and through pure causality".

In short, Schopenhauer's conclusion was that causality manifests itself through matter as we perceive it. Accordingly, he included matter in parallel with space and time in his table of *prædicabilia a priori*, which he presented as "all the fundamental truths rooted in our *a priori* knowledge of perception" (W II Chap. IV of the Supp.). In his own words:

> Matter... is not *object* but *condition* of experience, just as are space and time. This is why, in the accompanying table of our pure fundamental knowledge *a priori, matter* has been able to take the place of *causality*, and, together with space and time, figures as the third thing which is purely formal, and therefore is inherent in our intellect.

Of course, some of the *a priori* truths in his table can be questioned by modern science, but in most cases their *a priori* nature is worth examining... even at the risk of falling into anachronisms!

Thus, for instance, the first *a priori* truth states: "there is only one time... -- only one space... -- only *one* Matter, and all different materials are different states of matter; as such it is called *Substance*". Of course, the concept of substance used by ancient philosophers may be nowadays completely outdated, but what Schopenhauer apparently had in mind is a substratum of *all* matter; see Section 4 below for a more detailed discussion.

The third *a priori* truth states; "time cannot be thought away... —space cannot be thought away... -- The annihilation of matter cannot be conceived, yet the annihilation of all its forms and qualities can". In a modern interpretation, we may say that matter *and energy* can be transformed into each other, but the total matter-energy cannot be annihilated (for instance, an electron and a positron produce two gamma rays, particles of pure energy). I will return to this point in section 4 below.

The fourth *a priori* truth is: "Matter exists, *i.e.*, acts in all the dimensions of space and throughout the whole length of time, and thus unites and thereby fills these two. In this consists the true nature of matter. It is therefore through and through causality." This seems to be obvious, but Schopenhauer's point is that it cannot be otherwise since the existence of what we perceive as matter is prior to experience.

The fifth *a priori* truth is that time, space and matter are infinitely divisible. Of course, we now know about the existence of quarks, electrons and other elementary particles, but in physical theories these are described as point particles having no structure and therefore admitting no further division[5]; if some structure were discovered in the future, more elementary constituents would be looked for.

The sixth *a priori* truth is that time, space and matter are homogeneous and form a continuum. This seems to contradict what we know about elementary particles, but the field of modern physics does form a continuum (see Section 4 below).

Finally, let us discuss at some length the eighteenth *a priori* truth:

> Time is not measurable directly through itself, but only indirectly through motion, which is in space and time simultaneously; thus, time is measured by the motion of the sun and of the clock.
> Space is measurable directly through itself, and indirectly through motion, which is in time and space simultaneously; thus, for example, an hour's walk, and the distance of the fixed stars expressed as so many light years.
> Matter as such (mass) is measurable, *i.e.*, determinable according to its quantity, only indirectly, thus only through the *magnitude of the motion*, which it receives and imparts by being repelled or attracted.

The point is that space, time and mass must be measured with respect to some preassigned standards. Time was measured in the past with the periodic motion of celestial bodies, and nowadays it is measured with the periodic vibrations of a cesium atom. Space is presently measured in terms of the distance covered by light in a (well defined) unit of time. As for mass, its quantification has been more problematic; in the eighth *a priori* truth Schopenhauer stated that "by reason of matter we weigh", but it has been considerably more difficult to find a standard of mass in terms of purely natural constants. It is only recently that this aim was achieved: the Sèvres standard is now replaced by an atomic standard fixing the value of the Planck constant.

3. Mass and motion

At this point, it is worth noticing that there are several definitions of mass in physics textbooks. It is usually defined as the "quantity of matter", although it is never specified how such a quantity can be measured. Moreover, there is a general confusion between mass and weight. We know that weight, though related to mass, is a manifestation of the gravitational force and vanishes in outer space. Obviously, this was unknown in antiquity, and there was even a confusion between size and weight since bigger bodies are usually heavier than smaller ones[6].

In the *Principia*, Newton defined mass as the "quantity of matter", which is "the measure of the same [matter], arising from its density and bulk conjointly". It appears, therefore, that density was for Newton a more primary concept than mass. As for Newton's second law, Euler argued that the primary concept should be force and not mass, since the mass of a body is measured from its motion produced by a force of prescribed magnitude. Accordingly, Euler postulated mass as the ratio of force to the acceleration it produces.

However, it is not obvious that force should be a primary concept. Jammer (1961) remarks that, due to the new positivistic attitude: "What once, in Newtonian physics, played a central role was now regarded as an obscure metaphysical notion that has to be banished from science". Is it then possible to measure mass without referring to force (or gravity)? Ernst Mach[7] conceived a scheme to deduce the relative masses of two bodies in

[5] "Superstrings" have some structure, but in an abstract mathematical space.
[6] JAMMER, M. *Concepts of mass.*
[7] MACH, E. *The science of mechanics.*

mutual interaction from their accelerations, but the method is far from being practical.

Summing up, while the concept of acceleration is given in terms of space and time, it is not clear that force or mass should be primary concepts. In any case, mass can be measured only through motion (or equilibrium) in space and time, as stated by Schopenhauer in his table of *prædicabilia*.

Substance, matter and field

Classical philosophers call "substance" the underlying and permanent element of the world. Kant thought that it is necessary to "presuppose its existence throughout all time" (B 228), since "the unity of experience would never be possible if we were willing to allow that new things, that is, new *substances*, could come into existence" (B 229).

Regarding mass, Kant gave the example of how a philosopher would determine the weight of smoke: "Subtract from the weight of the wood burnt the weight of the ashes which are left over, and you have the weight of the smoke". This, however, as Kant pointed out, is based on the presupposition that "matter (substance) does not vanish, but only suffers an alteration of form" (B 228). This should be known *a priori*, following from a *principle of permanence*, even though it can be experimentally confirmed or disconfirmed *a posteriori*. If disconfirmed, some sufficient reason for the discrepancy should be looked for without abandoning the general principle; otherwise, physics could not be an exact science!

As for Schopenhauer, he assumed that matter is "the true and admissible content of the conception of *substance*". Thus

> The eternity of matter follows from the fact that the law of causality refers only to the *states* of bodies... it is by no means related to the existence of *that which bears* these states and has been given the name of *substance... Substance is permanent...* (G § 20).

Hence the different manifestations of matter are *accidents* of the substance, or more precisely "a particular mode of action... *in concreto*" (G § 21). On this point, at least, Schopenhauer follows Locke, who divided qualities into primary and secondary, the latter being mutable.

Furthermore, Schopenhauer argued in his criticism of Kantian philosophy (appendix of W I) that "... the concept of *substance* was formed merely in order to be the vehicle for surreptitiously introducing the concept of the immaterial substance", namely soul. Substance is left only with matter if the concept of soul is eliminated.

Of course, physics is not concerned with the soul, and it is therefore irrelevant, from our present point of view, whether substance is identified with matter or vice versa. In any case, the conservation of mass-energy and the *field* are the fundamental concepts in modern physics, since they encompass both matter and its (immaterial!) interactions.

The fundamental concept of energy, together with its conservation law, appeared in physical theories in the middle of the nineteenth century. It is only then that physicists realized that there must be some conserved quantities besides matter in the physical world (in other words, satisfying Kant's principle of permanence). Since then, the concept of force was gradually substituted by the more abstract but mathematically well-defined concept of energy. Finally, when Einstein proved the equivalence between mass and energy, it became clear that what was conserved is mass *and* energy.

Though the conservation of mass and energy is at present perfectly well accepted, it is nevertheless an empirical principle of physics. As such, Kant would say that it lacks "strict universality and apodictic certainty". An historical curiosity may illustrate this point: When physicists discovered that energy was apparently not conserved in nuclear beta decays, no less an authority than Niels Bohr proposed the daring hypothesis that the conservation of energy could be a statistical principle that does not apply at the atomic level. The alternative proposed by Wolfgang Pauli was to keep this principle at the expense of postulating the existence of an unknown invisible particle that would carry the missing energy away. Thus, in a sense, Pauli was following the method of Kant's hypothetical philosopher: instead of smoke, he weighted an even more elusive object, which turned out to be the neutrino. At the time of this discussion, both alternatives seemed equally convincing as sufficient reasons, but experiments finally confirmed Pauli's hypothesis.

The concept of field, as the substratum of all particles *and* their interactions, was forged by physicists in the nineteenth century and is widely used nowadays. Obviously, it would be an anachronism to interpret it as the substance of classical philosophy, but the important point to be stressed is that the existence of something acting as the universal substratum of *all* material (or immaterial) phenomena must be a knowledge *a priori*, since it cannot be proved empirically.

The notion of field was introduced by Michael Faraday when he noticed that electrically or magnetically charged bodies interact through some invisible and immaterial "lines of force". Based on Faraday's empirical laws, James Clerk Maxwell elaborated a mathematical theory of the electromagnetic field that proved to be enormously successful. Quite generally, physicists nowadays define a field as a mathematical function of space and time, that is, a function that assigns a certain quantity or set of quantities to every point of space-time. The field, as something invisible but perfectly real, was fully accepted by physicists of the twentieth century. In this respect, Einstein noted that[8]:

> The concept of the material objects was gradually replaced as the fundamental concept of physics by that of the field. Under the influence of the ideas of Faraday and Maxwell the notion developed that the whole of physical reality could perhaps be represented as a field whose components depend on four space-time parameters.

Material objects interact between themselves in different ways and it is one of the most important purpose of physics to explain their interactions. As mentioned by Einstein, this was achieved in the nineteenth century for electromagnetism with the concept of field. Another force of nature, gravity, can also be described in terms of a field, as in general relativity (the gravitational field can be interpreted as a Riemannian space, but that is another matter).

With the advent of quantum mechanics, it was possible to describe atomic and nuclear processes with a mathematical theory of quantized fields. Quantum Field Theory (QFT) proved to be a most successful theory encompassing all electromagnetic and nuclear processes. The so-called Standard Model of elementary particles, which is based on QFT, has been confirmed in all possible ways and provides an accurate description of nature at the atomic and subatomic level… even though its success is rather surprising, and many physicists still look for a more profound theory that may sustain it.

QFT is a mathematical theory that describes various kinds of fields as functions of space and time, which are supposed to form a continuum[9]. Space and time, the two forms of intuition according to Kant, are therefore the primary and fundamental concepts of

[8] Preface to JAMMER, M. *Concepts of mass.*

[9] There have been some theoretical intents to quantize space and time, but without clear results.

QFT, and the field itself depend on certain physical parameters with units of length, time and mass-energy. In modern physics, it is the sufficient reason for the entire existence of matter and its interactions.

Moreover, according to the second *a priori* truth in the table of *prædicabilia*: "different matters are not so through substance but through accidents", and we can interpret "accidents" as particular manifestations of physical fields.

Determinism and uncertainty

The concept of determinism is closely related to the principle of sufficient reason and causality. Let us consider it from the point of view of modern physics. To begin with, it must be pointed out that determinism and uncertainty are sometimes confused. By determinism it is understood that a given cause has always a unique effect… that could or could not be calculated in practice. Uncertainty is about the fact that the cause (or effect) cannot be known with absolute precision. Even though the laws of physics may be perfectly deterministic, the knowledge of the initial conditions may not be precise enough to calculate the future evolution of a physical system with absolute precision. As is well known in physics, to calculate the evolution of a mechanical system one must know both the dynamical equations that govern it *and* its initial condition. The uncertainty refers to the latter only.

In quantum mechanics, the evolution of an atomic system (in the non-relativistic limit) can be determined by the Schrödinger equation, which is fully deterministic. This equation enables the calculation of the so-called "wave function" describing *all* possible states of the system. According to the standard interpretation of quantum mechanics, it is the observation that forces the quantum system to manifest itself, with certain probability, in one of the possible states: this is known as the "collapse of the wave-function".

As for uncertainty, it is described in quantum mechanics by Heisenberg's famous principle that refers to the impossibility of measuring with absolute precision both the position and the momentum of a particle. This principle applies to the distance along a certain direction (say, the x axis) and the component of the momentum in that same direction. However, it does not apply to the component of the momentum in a perpendicular direction (say, along the y axis). In the language of quantum mechanics, a pair of two observables (such as position, velocity, energy, etc.) can be measured, in principle, with unlimited precision if the operators that represent them commute. In other words, one may not affect the outcome of the other observation when performing joint measurements. Despite the principle of uncertainty, quantum mechanics is the most precise theory ever elaborated, since it permits to calculate with amazing precision most processes in the atomic world. It all depends on what kind of observations are realized.

The real implication of the uncertainty principle in quantum mechanics is that not all measurements can be performed with absolute precision. In other words, not all measurements may be meaningful and not all the questions to nature can be answered without contradiction. However, this does not contradict the principle of sufficient reason, since this principle does not imply that a cause or an effect should be determined with absolute certainty.

In an essay on philosophical matters, Heisenberg[10] intended to refute Kant arguing that the principle of sufficient reason did not apply in quantum mechanics.

[10] HEISENBERG, W. *Physics and beyond.*

He gave as an example the decay of a radioactive nucleus: the exact time at which the nucleus decays emitting an electron cannot be calculated; so it is strictly probabilistic. However, Heisenberg was confusing determinism with sufficient reason. Moreover, he was considering the description of a physical phenomenon, but as Schopenhauer clearly stated (WWR, chap 53): "It is just the knowledge belonging to the principle of sufficient reason, with which we never reach the inner nature of things, but endlessly pursue phenomena only…".

It is true that it is not possible to predict the exact moment when the decay occurs, but that is not the point. The point is that physicists can look for the reason of such a process… and they have actually found one! It was discovered that the reason why a nucleus emits an electron is the process known as beta decay (the spontaneous transmutation of a neutron into a proton, an electron and an antineutrino), and, in turn, that the reason for this process is the existence of the *weak interactions* that occur in nuclear physics. And, again, the reason for the existence of such an interaction is the now well-established theory of electro-weak interactions. And the reason for the existence of such interactions may, perhaps, be found in the future in the context of a theory more fundamental than the presently known Standard Model of elementary particles. And so on.

On mathematics

Schopenhauer was not fond of mathematics. Without denying its practical use, he was convinced that mathematics could yield a quantitative description of the material world, but that it could never provide an understanding of its causal relations. "Where calculating begins, understanding ends" was his statement on this matter (G, §21; see also Chap. XIII of W I). His view may seem to be anachronic nowadays, but it must be realized that his dislike of mathematics was shared by many other intellectuals of his time who longed for a direct perception of nature, without the intermediacy of abstract concepts. Even Isaac Newton was criticized by his contemporary colleagues for having "only" described the motion of planets, without explaining the real cause of gravity. Goethe, a contemporary much-admired by Schopenhauer, was a strong critic of abstractions in the description of nature; they might be quite useful, he said, but "it does not occur to the architect to pass off his palaces as mountain sides and forests" (cited by Heisenberg[11]). Even among physicists, the case of Michael Faraday is noteworthy: his knowledge of mathematics was quite limited, and he deliberately avoided mathematical descriptions in the treatises he authored.

Nowadays, we are used to the enormous success of mathematics in describing physical phenomena. Nevertheless, Schopenhauer was right in a certain sense; a mathematical description yields only numbers, but not a real understanding. Quantum mechanics is an excellent example of it: it is based entirely on abstract concepts, such as wave functions, operators, probability amplitudes, spin, etc., together with rigorous mathematical formulas that describe their evolution. But though it has proved to be the most precise description of physical phenomena, a description in terms of known concepts, such as particles and mechanical actions, has proved to be impossible. Any attempt to "explain" quantum mechanics in terms of familiar concepts leads inevitably to paradoxical conclusions.

The fact that mathematics is so effective may well be one of the greatest mysteries of modern physics. It has not been explained and such an effectiveness is, indeed, quite

[11] HEISENBERG, W. *Across the frontiers.*

unreasonable, as Eugene Wigner[12] has well pointed out. In any case, we can agree with Schopenhauer that where understanding ends, calculating begins… provided that we are aware that calculating may go unexpectedly far!

Conclusions

All physical theories and all measurements, in any system of units, involve space, time, and mass-energy. Accordingly, the irreducible elements of any system of units in physics are the standards of length, time and mass, the three *a priori* forms of understanding according to Schopenhauer, as he argued at length in his writings. It was in the twentieth century that a closer relation between space and time became manifest with Einstein's theory of relativity, and a further connection with mass was revealed when Planck discovered the quantization of energy.

As argued above, causality is a fundamental concept of physics that must not be confused with determinism. In quantum mechanics, the intervention of an observer causes the reduction of the wave-function to a particular state, which is the observed one, even though it cannot be predicted. Nevertheless, there is a sufficient reason for the "collapse of the wave-function" or any other physical phenomenon. Quite generally, every effect, whether in the atomic or the macroscopic world, must have a sufficient reason, and the basic principle of science is to investigate and discover it.

Finally, the main argument of the present article is that it is with *a priori* concepts that we may study and describe the world. Anachronisms apart, we have seen that the search for the substance of classical philosophy evolved in modern physics to the concept of field, interpreted as the ultimate underlying substance of the world. The field is described in terms of space and time, and it is interpreted as the sufficient reason for the existence of subatomic particles, that is, matter and its interactions.

References

KANT, I. *Critique of Pure Reason*. Trad. N. Kemp Smith, London: Macmillan, 1929.

SCHOPENHAUER, Arthur. *The World as Will and Representation* (W I and II). Trad. E. F. Payne, 1966, Dover.

SCHOPENHAUER, Arthur. *On the Fourfold Root of the Principle of Sufficient Reason* (G). Trad. E. F. J. Payne. 1974, Open Court.

HACYAN, Shahen. *Física y metafísica del espacio y el tiempo: la filosofía en el laboratorio,* Mexico: Fondo de Cultura Económica, 2004.

HACYAN, Shahen. On the Transcendental Ideality of Space and Time in Modern Physics, *Kant-Studien,* 97, 2006, pp. 382-395.

EULER, Leonard. *Mechanica,* 1736. Trad. I. Bruce. http://www.17centurymaths.com/contents/mechanica1.html

DUHEM, Pierre. *La théorie physique*, 1914. Fac-similé Paris: Vrin, 1997. Chap. IV

JAMMER, Max. *Concepts of Mass in Classical and Modern Physics*, Harvard University Press, 1961.

MACH, Ernst. *The Science of Mechanics. A Critical and Historical Exposition of its Principles,* 1893. Trad. Thomas J. McCormick. Chap. II, Sect. V.

EINSTEIN, Albert, foreword to Jammer, M. *Concepts of Space*, 1954. Harvard University Press.

[12] WIGNER, E. *The unreasonable effectiveness oh mathematics.*

HEISENBERG, Werner. Goethe's View on Nature and Science, in *Across the Frontiers,* 1990. Trad. Peter Heath. Ox Bow Press. Chap. X.

HEISENBERG, Werner. Kantian philosophy, in *Physics and beyond*, (*Der Teil und das Ganze,* 1969). Trad. 1971.

WIGNER, Eugene. The Unreasonable Effectiveness of Mathematics in the Natural Sciences, *Comm. Pure and Applied Math*, 13, 1960, pp. 1-14.