DOI: 10.5902/1983465965639

EVOLUTION OF THE DEGREE OF EFFICIENCY OF THE CRYPTOCURRENCY MARKET FROM 2014 TO 2020: AN ANALYSIS BASED ON ITS FRACTAL COMPONENTS

Evolução do grau de eficiência do mercado de moedas criptográficas de 2014 a 2020: uma análise baseada em seus componentes fractais

Submission: 17/12/2021
Accept: 19/03/2022

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ABSTRACT

Objective: This study aims to analyze the evolution of the cryptocurrency market efficiency based on fractal aspects of the historical price series of 15 cryptocurrencies and a benchmark developed for this market (CRIX).
Methodology: The proposed analyses start from the efficiency index proposed by Kristoufek and Vosvrda (2013), which captures long-and short-term memory biases as well as first-order autocorrelation. The database covers the period from 08/02/2014 to 12/31/2020. Using structural breakout analysis for time series, it was possible to divide the sample into five periods of analysis, and the efficiency index was calculated for each one.
Findings: It was identified the existence of oscillations between the efficiency indexes over the analyzed periods, verifying a greater inefficiency at times of market upswing. In addition, it can be seen that in general this market has been gaining efficiency over the years, although it has not yet reached the absence of inefficiency. This conclusion corroborates studies on the adaptation of market efficiency based on its investors and agents. Finally, one can characterize the current scenario as a speculative bubble, which, due to the presence of the herd effect, enables the existence of arbitrage.
Originality: The research in this area is still recent, as it is a new financial segment, so there are several doubts and gaps in the literature. In this sense, the adoption of a longitudinal approach to identify the evolution of efficiency of this market is not only interesting but it is also an approach little explored by the literature.

Keywords: Cryptocurrencies; Fractal Market Hypothesis; Adaptive Markets; Market Efficiency.

RESUMO

Objetivo: Este estudo visa analisar a evolução da eficiência do mercado criptoativos com base em aspectos fractais da série histórica de preços de 15 criptomoedas e um índice de referência desenvolvido para este mercado (CRIX).
Metodologia: As análises propostas partem do índice de eficiência proposto por Kristoufek e Vosvrda (2013), que captura os vieses de memória de longo e curto prazo, bem como a autocorrelação de primeira ordem. O banco de dados cobre o período de 02/08/2014 a 31/12/2020. Usando a análise de quebra estrutural para séries temporais, foi possível dividir a amostra em cinco períodos de análise, e o índice de eficiência foi calculado para cada um deles.
Resultados: Foi identificada a existência de oscilações entre os índices de eficiência ao longo dos períodos analisados, verificando uma maior ineficiência em momentos de ascensão do mercado. Além disso, pode-se observar que, em geral, este mercado vem ganhando eficiência ao longo dos anos, embora ainda não tenha alcançado a ausência de ineficiência. Esta conclusão corrobora os estudos sobre a adaptação da eficiência do mercado com base em seus investidores e agentes. Finalmente, pode-se caracterizar o cenário atual como uma bolha especulativa, o que, devido à presença do efeito de manada, permite a existência de arbitragem.
Originalidade: Pesquisas nesta área ainda são recentes, pois se trata de um novo segmento financeiro, portanto existem várias dúvidas e lacunas na literatura. Neste sentido, a adoção de uma abordagem longitudinal para identificar a evolução da eficiência deste mercado não só é interessante como também é uma abordagem pouco explorada pela literatura.

Palavras-chave: Criptomoedas; Hipóteses de Mercado Fractal; Mercados Adaptativos; Eficiência de Mercado.

1 INTRODUCTION

In the current context of growing innovations related to increasingly modern and fast technologies, several market segments have undergone changes to adapt to the demand for new products and services. The financial market is no exception. Financial innovations were driven by the advent of the Internet, a tool that facilitates interactions and exchanges of information between users at reduced costs (Kristoufek, 2013). Among the most relevant innovations in this segment in the last decade, the creation of cryptocurrencies stands out (Briére, Oosterlinck & Szafarz, 2013).

In the cryptocurrency market, the first and most widespread one is Bitcoin, developed by Nakamoto (2008) and launched in 2009. Since then, several other cryptocurrencies, the so- called altcoins, have been created. The rapid increase and dissemination on a global scale have drawn the attention of academia, in order to explain and understand the evolution of this new segment of the financial market. It can see an increasing concentration of institutional investors as participants in this market, mainly aiming to increase the earning potential of portfolios (Białkowski, 2020).

In the literature about the subject there were prevalent findings of relative inefficiency in this market (Kristoufek & Vosvrda, 2019; Tran & Leirvik, 2019; and Gurdgiev & O'Loughlin, 2020), caused by scenarios characterized by information asymmetries, decision-making with irrational effects, and differences in expectations among investors, which ultimately can lead results that go against the assumption of the law of one price, which in turn implies the possibility of arbitrage in the market (Fama, 1970).

Furthermore, the studies carried out under this viewpoint, end up finding that this inefficiency may be a consequence of the existence of inexperienced investors (outsiders), as well as the lack of maturity of the market as a whole, which is in line with the Adaptive Markets Hypothesis (AMH) of Lo (2004). Thus, as the market is composed of more rational investors (such as institutional investors), it converges to an efficient market proposed by Fama's (1970) Efficient Market Hypothesis (EMH).

Among the various ways of analyzing market efficiency, a set of metrics that have been proved to be relevant for the analysis are the tools of Econophysics (see Mandelbrot, 2005; Mosteanu & Faccia, 2021), a current field that aims to analyze complex socioeconomic systems, such as the financial market, from the perspective of models used by physics (Schinckus, 2011). According to Jovanovic and Schinckus (2013), Econophysics has the potential to complement the economic literature because, unlike the latter which is limited by paradigms, assumptions and theories, it starts from the use of real data and through physical modeling, conducts analysis of forecasts.

Previous studies that dealt with the efficiency of the cryptocurrency market used different analysis approaches, but most of they do not focus on the comparative analysis over the time. Thus, aiming to contribute to filling this gap, this paper seeks to analyze the market efficiency for these assets based on fractal aspects of the historical price series of 15 cryptocurrencies to build an efficiency index originally proposed by Kristoufek and Vosvrda (2013), which is based on studies of fractal dimensions to identify behavioral inefficiencies. Cording to the authors, the metrics used for the construction of the index capture long and short-term memory of temporal series, which can be associated with herd effect and anchor and availability bias. Such analysis becomes important due to changes in the financial market and its legislation about cryptocurrencies.

Thus, in addition to contributing to the theoretical framework, analyzing the efficiency of this market becomes important for the proper appreciation and safety of fund managers, as well as individual investors. The results of the analyzes allow to verify the adequacy of AMH to explain the behavior of efficiency in this market. In addition, it was possible to highlight that in bearish periods, the cryptocurrency market presented itself as being more efficient than in bullish. Finally, speculative movements were observed to a high degree, allowing to reinforce the high degree of risk in this market.

After this introduction, section 2 presents the theoretical background supporting the empirical research. Section 3 discusses the methodology used to analyze the efficiency of the cryptocurrency market, followed by the results found in section 4. Finally, section 5 presents the final considerations.

2 THEORETICAL BACKGROUND

2.1 Cryptocurrency Market

Lánský (2017) characterizes a cryptocurrency as a decentralized system independent of central authorities based on cryptographies, which not only maintains an overview of cryptocurrency units, but also defines circumstances for creating new currencies. Furthermore, this is a system that allows direct transactions between users, who at the same time act as servers in a system of peer-to-peer nodes, independently of monetary systems and bodies. Thus, no government or central authority can control the cryptocurrency demand and supply (Yermack, 2013). In addition, the system does not need an intermediary, this being one of its main revolutions, since it eliminates the double spread common in financial market transactions (Brito, Shadab & Castilho, 2014).

The first and best-known cryptocurrency is Bitcoin, developed by Satoshi Nakamoto, a pseudonym for a group of anonymous developers, in 2008 and the forerunner of the blockchain system for crypto-assets. After the system was adopted by the market, several new virtual currencies emerged. In general, altcoins were designed with the intention of improving the system offered by Bitcoin. The main changes proposed by them concern the speed of transactions, changes in the mining system and volume availability (Reed, 2017).

Studies in the area are still recent, mainly due to the market still being relatively new, in which, it is possible to identify the predominance of three main approaches: 1. determining the nature and characteristics of this market, comparing cryptocurrencies with financial assets, such as commodities, stocks and gold, reaching the conclusion that this asset modality could be considered as a new hybrid asset (Charfeddine, Benlagha & Maouchi, 2020; Nguyen, Nguyen, Nguyen and Nguyen, 2019;

Trimborn and Härdle, 2018); 2. analyzing the behavior of historical series and market efficiency of cryptocurrencies, whose main findings signal the high sensitivity of crypto-assets to macroeconomic shocks, the absence of correlations between them and traditional financial assets, and the identification of explosive behavior typical of speculative bubbles (Selmi, Tiwari & Hammoudeh, 2018; Cagli, 2019; Mnif, Jarboui & Mouakhar 2020; Gurdgiev & O'Loughlin 2020); 3. analyze the composition of investment portfolios, whether exclusively with cryptocurrencies or mixing these assets with the traditional financial markets ones. In that regard, Trimborn and Härdle (2018) explain the rationale for the creation of the Cryptocurrency Index (CRIX), used as a benchmark for the cryptocurrency market, a market index that updates quickly with every relevant change or event in the market. Based on the idea of diversification for risk reduction, CRIX calculates the optimal amounts of Bitcoin and/or altcoins to obtain the best possible risk-return ratio considering only assets in this segment.

2.2 Market Efficiency and Adaptive Markets

Fama (1970) proposed the EMH, presenting characteristics that must be observed so that a market can be considered efficient, which are: inexistence of transaction costs; homogeneous availability of information among all agents; homogeneous expectations of information about the prices of assets, among others. Thus, it is clear that in an efficient market, all information about an asset is available to the agents and it has already been reflected in prices, so that systematically arbitrage is impossible. Posteriorly, Fama (1991) states that given the reality of the market, there are indeed some imperfections in it, however, EMH can consider information asymmetry and transaction costs at reasonable levels to calibrate its models and analyzes.

However, as Galbraith (1994) argues, EMH cannot explain a number of events, such as speculative bubbles and crashes. Hence, while EMH postulates the rationality of agents, empirical data point to the existence of irrational reactions in the market, which has led researchers to use behavioral theory to describe market inefficiency and investors irrationality (Tversky & Kaneman, 1979).

In this context, the AMH emerged with the work of Lo (2004), which is based on the studies of Tversky and Kaneman (1979), considering behavioral effects as a key point to understand market inefficiency. Lo (2004) treats investors as individuals who make decisions imbued with behavioral biases, and investors with a greater share of these biases are considered non-adaptive. Consequently, they end up either making decisions with fewer behavioral effects and stay in the market, or they end up being naturally purged from it by a process similar to natural selection in biology (Lo, 2005).

From this evolutionist view, individuals act by impulse and bias, but they have the ability to learn from their mistakes and adapt, which ultimately drives the market to a higher level of efficiency (Urquhart & McGroarty, 2014). Thus, as stated by Lo (2004), market efficiency cannot be analyzed in a binary way, but rather as a time variable that depends on a series of contexts and macroeconomic and cultural variables.

2.2 Econophysics and the Fractal Markets Hypothesis

According to Mantegna and Kertész (2011), the approach between the field of finance and applied physics is not something recent, having as main reasons the need for modeling that incorporate correlations and dynamics of probabilities increasingly given the developments of financial systems. The studies of Mandelbrot (1963) and Mantegna (1991) according to Jovanovic and Schinckus (2013) are the base studies for the advancement of the area.

Unlike traditional economics modeling, which has as assumptions the rationality of agents and a wide range of theories, Econophysics develops its models based on empirical observations, in others words, first a model consistent with real data are developed to then identify the best theory to validate them (Schinchus, 2011). Moreover, Econophysics does not stick to using the Gaussian distribution as the basis for its models, resorting to stable Lévy processes for its modeling, which allows for greater statistical flexibility. Such a process uses an α-stable power law of the type $P(X > x) = x^{-\alpha}$, possessing as characteristics independent stationary increments called (càdlàge paths). Thus, the probability distribution of increments $X_t - X_s$ depends only on the length of the time interval $t - s$, so that the distribution of intervals of the same length will be i.i.d. (Mantegna, 1991).

As for the α parameter, this would be a coefficient assumes values between 1 and 2. With $\alpha = 2$ the distribution becomes a normal distribution. In case of $\alpha = 1$ or $\alpha = 1.5$, for example, we have the Cauchy and Pareto distributions, respectively (Jovanovic & Schinckus, 2013). Therefore, one can see that Lévy stable processes are a generalization of the Gaussian distribution, which both corroborates the validation of modern financial modeling in certain scenarios and also allows the non-limitation to only one distribution to describe financial phenomena

Another fundamental points for understanding the stochastic processes analyzed by econophysicists concern self-similarity. A price equation as a function of time $p(t), t \in R_+$is considered as self-similar with index H if for all $c > 0, n > 0$ and $t_1, \dots t_n \in \mathrm{R}$, the vector $p(ct)_1, \dots p(ct)_n$ has the distribution equal to $c^H p(t_1), \dots c^H p(t_n)$. For the Brownian process, self-similarity occurs with $H = 0.5$. The stable Lévy process also has features of self-similarity, with independent increments and Parentian tails described by $\mathrm{P}\{|\mathrm{p}(\mathrm{t}+\Delta\mathrm{t}) - \mathrm{p}(\mathrm{t})| > \mathrm{x}\} \sim K_a \Delta \mathrm{t} x^{-\alpha}$, with $\mathrm{x} \to +\infty, \mathrm{a} = 1 / \mathrm{H} \in (0;2)$ and K_a a positive constant (Calvet & Fisher, 2013)

Among the tools used by econophysicists for market analysis, one that stands out is the use of fractal analysis. According to Kimura (2005), fractals are geometric objects that can be divided into infinitely many smaller parts maintaining similarities with the figure as a whole. According to Peters (1994), the theory of fractals is associated with a relation of global determinism in consonance with local randomness. Thus, one can consider new information as a random element that will affect the market in a deterministic way, that is, following a pattern. This idea guides the Fractal Market Hypothesis (FMH).

This hypothesis is linked with the chaos theory, in other words, the market assumes random movements in the short term but maintains a similar overall structure when expanding the horizon of analysis. In addition, for FMH, EMH would be a state of equilibrium and stability of the market both in the short and long term with respect to supply and demand. This is a possible condition, but not necessary for your models. (Peter, 1994; Peters, 1996). Starting from a state of market disequilibrium, AMH can be used to explain how this state moves to a stable version (EMH), thus being able to use elements from FMH to analyze the price structure regardless of whether it is in equilibrium or not. Thus, there is a validation of the use of FHM with EMH and AMH to explain the structure and efficiency of the market, respectively.

One of the first studies to verify this hypothesis was Elliot (1994), who identified a cycle pattern of five trend waves followed by three waves of corrections that were maintained when changing the time scale. This characteristic is called self-similarity, and is an element present in fractals. From this study, several others started to be conducted to identify levels of market efficiency based on FMH, one can mention the studies by Kristoufek and Vosvrda (2019), Selmi, Tiwari and Hammouudeh (2018), Caporale et. al. (2016) Kristoufek and Vosvrda (2013), Dubovikov, Starchenko and Dubovikov (2004) and Cajueiro and Tabak (2004).

Among the metrics used in these works, two stand out. The first one is Hurst exponent (H), a parameter that measures long-term self-similarity, that is, the persistence of long-term memory in prices. Its value ranges from 0 to 1. With $H = 0.5$, it is verified that the series is not correlated in the long run. For $H > 0.5$, the series is positively correlated in the long run and with $H < 0.5$, negatively correlated (Kristoufek & Vosvrda, 2013).

The second metric is the fractal dimension (D), a parameter that measures short-term memory effects in time series. In the financial market there is the presence of bear and bull movements. These changes have a local effect and no global effect. However, the fractal dimension is able to capture such an effect, so that at a neutral moment, $D = 1.5$, at moments of local persistence, $D < 1.5$, and at moments of local anti-persistence, $D > 1.5$ (Kristoufek and Vosvrda, 2013).

By the expected self-similarity process of a fractal figure, $H + D = 2$, that is, one can describe the fractal dimension as a function of long-term memory. However, this equality is only valid when there is a perfect reflection between long-term and short-term memory. This premise is not observed in financial time series, so that the short-term and long-term effects affect the time series heterogeneously (Kristoufek & Vosvrda, 2014).

3 METHODOLOGY

3.1 Sample and Data

The market index selected was the CRIX proposed and updated by Trimborn and Härdle (2016)[1]. The base consists of 2,327 observations, with index value calculated from 08/02/2014 to 12/31/2020. Fifteen cryptocurrencies were selected for the analyses, these being Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Bitcoin Cash (BCH), Bitshares (BTS), Litecoin (LTC), Cardano (ADA), Binance Coin (BNB), EOS (EOS), Neo (NEO), Dash (DASH), Stellar (XLM), Tronix (TRX), Tether (USDT) and Chainlink (LINK). The data was obtained via Yahoo Finance.

The choice of these crypto-assets was based in their liquidity, according to the work of Wei (2018), as well as their representativeness in the market and daily transaction volume. Adding up the participation of the 15 cryptocurrencies selected, it can be observed a total close to 90% of the existing market. The choice to start the analysis period on 08/02/2014 is due to this being the start of the CRIX calculation by Trimborn and Härdle (2016). As this would be the benchmark of this market, it was considered by the authors to analyze only the time series at times when this was already available, in order to compare the results of individual cryptocurrencies with their market portfolio.

The data collected considers asset prices in dollars, thus aiming to eliminate exchange rate effects from the sample. It is also noteworthy that daily closing data of cryptocurrencies and the index were used for the analyses.

3.2 Identification of the Analyzed Periods

To identify the evolution of the crypto-assets market, the analysis of Bitcoin, which has a correlation of over 97% with the index and represents over 64% of the crypto-assets market over the years. Thus, it can be seen that CRIX has theoretical (see Trimborn & Härdle, 2018) and empirical justifications for being the basis for identifying trend breaks. Table 1 summarizes the periods and their characteristics.

1 Available in: http://data.thecrix.de

Table 1 - Sample Period Segregation

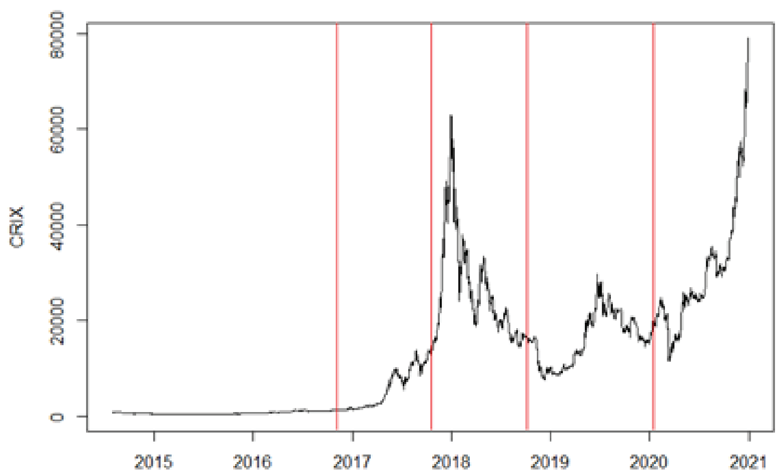
Period	Duration	N. Days	Feature
Period 1	08/02/2014 11/15/2016	821	Consolidation of the cryptocurrency market
Period 2	11/16/2016 10/27/2017	349	Entry of new investors, institutionalization of the legitimacy of crypto-activities, and great euphoria of new investors
Period 3	10/28/2017 10/11/2018	349	Beginning of the downturn in Bitcoin prices and end of the euphoria
Period 4	10/12/2018 01/16/2020	462	Launches of new altcoins, and increased international movement towards regulation
Period 5	01/17/2020 12/31/2020	347	Shrinkage due to pandemic COVID-19 and entry of institutional investors, reduction of American interest rates and new halving start a new period of euphoria

Source: Elaborated by the authors.

Thus, the CRIX time series was segmented into five periods based on test for structural breaks in time series using R's "strucchange" package. Based on the test results of the test, it was possible to identify breaking patterns and trends in this market. Figure 1 graphically illustrates each period from the theoretical CRIX quote.

Based on this Figure, one can verify the existence of oscillations between periods of investor euphoria and pessimism. This behavior is consistent with the behavior of emerging markets, especially the side support by the AMH. In addition, it was possible to verify similar behaviors to those highlighted in the other crypto-activities, especially in Bitcoin, Ethereum and Cardano.

Figure 1 - Period Division by CRIX Quotations



Source: Elaborated by the authors.

3.3 Efficiency Index

Once the segregation of periods was done, the presence of market efficiency proposed by Fama (1970) was analyzed following the fractal analysis model proposed by Kristoufek and Vosvrda (2013). The authors define their indicator according to Equation [1], in which Mi is the i-th estimated efficiency indicator, Mi* is the expected value for Mi in an efficient market, and Ri is the range of Mi. Thus, EI assumes values from 0 to close to $EI = (^n/_2)^{1/2}$ with n equal to the number of metrics taken into account to construct EI. In this way, the index captures the average of the absolute deviations of indicators that capture effects of market inefficiency. Thus, EI = 0 indicates a fully efficient market and $EI = (^n/_2)^{1/2}$ the market is totally inefficient.

$$EI = \sum_{i=1}^{n} \frac{|M_i - M_i^*|}{R_i} \quad [1]$$

On the justification of the choice of this index, three main reasons can be cited. Firstly, its methodology enables a diversity of factors and dimensions to be included in the same index in a comparable manner, since the metrics are divided by their range. Furthermore, the calculation of the index itself is simple to perform and is quite intuitive, being basically a sum of the divergences between the actual values of metrics and what is expected in a purely efficient market. Finally, it can be adapted to include other metrics, which facilitates comparisons between results based on EI adaptations.

Following the original work of Kristoufek and Vovsrda (2013), eight metrics were selected for the composition of the EI. Firstly, three metrics were selected to calculate the Hurst exponent: detrended fluctuation analysis (H_{DFA}), detrended moving average (DMA) and height-height correlation analysis (HHCA).

The H_{DFA} is based on the variance of the series purged of its trend. To do so, the series is decomposed into subseries of size "s" and then the local average $X_{t,s}$ is estimated to build the trend-free series Y_t by subtracting the element X_t from its local average. Finally, the fluctuation $F^2_{DFA}(s)$ is defined as the mean of the squares of the mean errors between $X_{t,s}$ and X_t for each of the sub-period sizes, so that $F^2_{DFA}(s) \propto s^{2H}$, where H ins the Hurst exponent (Peng et al., 1993). As per Kristoufek and Vosvrda (2013), in this work $s_{min} = 5$ and $s_{max} = T/5$, were used, being T, the total size of the analyzed time series.

The H_{DMA} in turn, according to Barabási, Szépfalusy e Vicsek (1991), is based on the moving average effect of the series. For each compliance subperiod λ, a central average $X_{t,\lambda}$ is constructed. Similar to the H_{DFA} methodology, a series $F^2_{DMA}(s) \propto \lambda^{2H}$ is constructed from the difference of the mean square errors of the differences of $X_{t,\lambda}$ and X_t. Furthermore, as per the original EI metric, $\lambda_{min} = 3$ and $\lambda_{max} = 21$, with increments of λ by two units.

Finally, the H_{HHCA}, also called generalized Hurst exponent, is based on the scale of the height correlation function of X_t with time resolution v, $t = v, 2v \dots v\lfloor T/v \rfloor$, in which ⌊ ⌋ indicates the smallest integral operator. From the second-order height correlation function, expressed by $K_2(\dagger) = \sum_{\dagger}^{\lfloor T/v \rfloor} |X_{t+\dagger} - X_t|^2 / \lfloor T/v \rfloor$, in which † is the distance from $v = \dagger_{min}, \dots, \dagger_{max}$ (Di Matteo, Aste & Dacorongna, 2003). From this equation, one can obtain the distribution of K2(†) ∝ † 2H. For this work, we have $\dagger_{min} = 1$ and $\dagger_{max}$ ranging between 5 and 20, obtaining the Hurst exponent of this metric as the average of the calculated H_{HHCA}, as per the original work of Kristoufek and Vosvrda (2013).

For the fractal dimension metrics, the following were selected: Periodogram (D_p), Wavelet (D_w), Genton (D_G) and Hall-Wood (D_{HW}). The use of the Periodogram technique to calculate D_P was originally proposed by Chan, Hall and Poskitt (1995). For a series that is stationary and follows the Gaussian process, $B(\omega) = 2\int_0^1 X_t \cos(\omega[2t-1]dt$, and $J(\omega) = B(\omega)^2$ being the semi-periodogram. With $n_s = 2m+1$observations of X_t with $t = i/2m$ and $i = 1, 2 \dots 2m$, an approximation for the Periodogram would be $B(\omega) = \frac{1}{m}\left[\frac{X_0 - X_t}{2} + \sum_{l=1}^{2m-1} X_{i/2m} \cos\left(\omega \frac{i-m}{m}\right)\right]$ and so $D = \frac{1}{5} + \frac{1}{2}\left[\sum_{l=1}^{L}(s_l - \bar{s})\log\left(J(\omega)\right)\right]\left[\sum_{l=1}^{L}(s_l - \bar{s})^2\right)]^{-1}$, where $s_l = log(l/n)$, $\bar{s} = \frac{1}{L}\sum_{l=1}^{L} s_l$ and *n* is the size of the time series and l is the size of the boxes used for segregating the series.

Subsequently, Serroukh, Walden and Percival (2000) developed the Wavelet methodology for the calculation of D_w, this being an adaptation to a weighted least squares estimator for a fractional differentiation process of long-term memory. According to the authors, this metric considers high frequencies that were not captured by D_P. Using the discrete Wavelet transform, it is possible to perform the decomposition of the series into $W_j, j = 1, 2, \dots J_0$, having $J_0 = \lfloor log_2(n) \rfloor$ .Thus, the j-th coefficient associates with the scale$\tau_j = 2^{j-1}$, and the Wavelet variance is given by $V_2(\tau_j) = \frac{1}{2n}\left\|W_j\right\|^2$. With a large τ_i, one has that $V_2(\tau_j) \propto \tau_j^{4-2D}$.

The D_G methodology is based on Genton's robust variogram estimator (Genton, 1998), which can be defined as $V_2(l/n) = \frac{1}{2(n-1)}\sum_{i=1}^{n}\left(\frac{x_i}{n} - \frac{x(i-l)l}{n}\right)^2$. Based on these calculations, it is possible to construct the D_G, which in turn is expressed by $D = \frac{\sum_{l=1}^{L}(s_l - \bar{s})\log(V_2(l/n))}{2\sum_{l=1}^{L}(s_l - \bar{s})^2}$, whereby, according to Davies and Hall (1999), by making *L=2* there is a reduction in the bias of the metric.

Finally, D_{HW} is based on the process of box-counting and using stepwise scaling of absolute deviations, originally proposed by Hall and Wood (1993). The absolute deviations of the series is calculated via $A(l/n) = \frac{1}{n}\sum_{i=1}^{\lfloor n/l \rfloor}\left|X_{il/n} - X_{(i-1)l/n}\right|$ and the estimator is measured via $D = 2 - \frac{\sum_{l=1}^{L}(s_l - \bar{s})\log(A(l/n))}{2\sum_{l=1}^{L}(s_l - \bar{s})^2}$, where, as in the previous case, with *L=2* there is a reduction in the bias of the metric

In the end, Kristoufek and Vosvrda (2013) propose to add the first-order autocorrelation coefficient ρ(1) in their index, starting from the idea that if prices followed a random walk, the autocorrelations of returns should be close to 0. Equation 2 summarizes the final model.

$EI = [(H_{DFA} - 0.5)^2 + (H_{DMA} - 0.5)^2 + (H_{HHCA} - 0.5)^2 + (D_P - 1.5)^2 + (D_W - 1.5)^2 + (D_G - 1.5)^2 + (D_{HW} - 1.5)^2 + (\rho(1)/2)^2]^{1/2}$ [2]

Thus, the present work helps to complement studies such as those of Urquhart (2016), who proposes that as the market matures, an improvement in the level of efficiency would occur and Selmi, Tiwari, and Hammoudeh (2018), who, in turn, claim that there is a cyclical wave of market efficiency based on the volume of rational and irrational investors. Moreover, the present paper aims to compare its results with past studies, such as those of Wei (2018) and Kristoufek and Vosvrda (2019). To perform the calculations of Hurst exponents, fractal dimensions and first-order correlations, the R and Matlab software were used, based on the script used by Cajueiro and Tabak (2004) and Cajueiro, Tabak and Andrade (2009)[2].

2 Available in: http://prorum.com/?qa=2173/calcular-expoente-exponent-dependencia-dependence-temporais

4 ANALYSIS AND DISCUSSION OF THE RESULTS

4.1 Market Overview

As previously stated, the time series were subdivided into five periods, thus aiming to identify moments of significant changes in the market over the years to assess how market efficiency behaves. Table 2 brings the descriptive statistics of the cryptocurrency market index to illustrate the characteristics of each period.

Table 2 - Descriptive Statistics by Period of the CRIX's Return

Period	CRIX					
	Annualized Average Retrun	Annualized Standard Deviation	Minimum	Annualized Median	Maximum	Total Return
Period 1	0.42	0.59	-0.2	0.48	0.21	0.51
Period 2	14.77	0.82	-0.21	13.79	0.18	9.51
Period 3	0.93	0.94	-0.22	1.63	0.22	0.23
Period 4	0.48	0.72	-0.18	0.42	0.17	0.32
Period 5	4.37	0.72	-0.36	3.34	0.15	2.92
Total Period	1.63	0.73	-0.36	1.46	0.22	100.16

Source: Elaborated by the authors.

According to the historical series of the analyzed cryptocurrencies and CRIX, it can be seen that during Period 2 there was a considerable increase in the cryptocurrency quotations, as well as the risk embedded in the operations. In the two following periods, there is a downward movement in prices together with a reduction in risk, as well as a reduction in the price fluctuation window, thus characterizing a more stable environment.

During Period 4, there was a reduction in average returns and volatility, however, the drop in return was greater than that in risk, so that the risk/return analysis indicates this period as the riskiest. Finally, during Period 5, risk was maintained with an increase of more than eight times the value of the last period, so that its coefficient of variation is 0.16, a value which historically is lower only than Period 2. Moreover, it is possible to identify a bear subperiod due to the pandemic. However, the downward scenario soon had a reversal and started a new scenario of euphoria and speculation.

To better understand the market movements, it is important to analyze the profile of cryptocurrency investors. So far, there is no database on the profile of crypto-assets users, nor many academic papers on the topic. A Coindesk article[3] indicated that in 2015 the population of cryptocurrency investors was approximately 20% up to 24 years old, 39% from 25 to 34 years old, 22% from 34 to 44 years old, and 18% over 45 years old. By 2020, according to a study by Cointelegraph[4], these percentages have risen to approximately 10%, 21%, 27%, and 42%, respectively.

Thus, it is possible to verify the lower concentration of novice investors in this market over the years. In addition, to complement the analysis of the investors' profile, Gurdgiev and O'Loughlin (2020) and Hasso, Pelster, and Breitmayer (2019) identified that crypto-assets investors have a high degree of risk aversion and at times of rising prices, there is an increase in the volume of users trading these assets.

These insights corroborate with results from the technical analyses of cryptocurrencies and CRIX performed in this study. At times of market upswings, increases in speculation are noticeable. However, at down times, there is a drop in the trading volume of these assets. Thus, one can assume the suitability of the AMH for the cryptocurrency market since there are evidences of modulations in the EI over the periods, demonstrating that in moments of euphoria, there is a greater presence

3 Available in: https://www.coindesk.com/new-coindesk-report-reveals-who-really-uses-bitcoin

4 Available in: https://cointelegraph.com/news/data-suggests-bitcoin-price-will-rise-as-investor-demographics-shift

of behavioral biases, which ends up generating speculative bubbles, this effect being corrected in bearish moments of the market. However, to confirm this hypothesis, the following analyses of the EI's and the elements that compose it were performed.

4.2 Fractal Analysis

This subsection discusses the results of the Hurst exponents and fractal dimensions of the selected cryptocurrencies. Starting with the Hurst exponents, which measure the long-term correlation of cryptocurrencies, Table 3 shows the average results of the indicators calculated via DMA, DFA and HHCA.

According to the results for the Hurst exponents, it can be seen that the total sample period, cryptocurrencies showed values close to 0.44, thus indicating weak positive dependence for long-term prices. There are three exceptions to these analyses. The first two, Stellar (XLM) and Ripple (XRP) showed a higher level of long-term dependence, with an exponent for the total period around 0.3971 and 0.3972, respectively. Stellar was developed by the same creator as Ripple, and in both cases, the mining of the cryptocurrency does not exist, as the coins were distributed for free during their creation. In addition, studies have found similarities about the security of their protocols and the centralization of their nodes, as well as having a history of being negatively correlated with the other altcoins, which may justify results that are more negatively correlated with past values than the others (Bracciali, Grossi & Hann, 2021; Cagli, 2019; Hsieh, Vergne & Wang, 2017).

Table 3 - Average Hurst Exponents of Selected Cryptocurrencies

Cryptocurrency	Period					Total Period	Average
	1	2	3	4	5		
ADA	NA	0.572	0.5383	0.5262	0.5098	0.461	0.5366
BCH	NA	0.4694	0.5064	0.5197	0.2842	0.4832	0.445
BNB	NA	0.4479	0.3993	0.5823	0.375	0.4573	0.4511
BTC	0.4948	0.4147	0.4465	0.4796	0.4838	0.4803	0.4639
BTS	0.5294	0.5276	0.5125	0.5753	0.4242	0.4703	0.5138
DASH	0.2741	0.535	0.5406	0.5585	0.3421	0.4931	0.4501
EOS	NA	0.6225	0.5382	0.5166	0.2644	0.4541	0.4854
ETH	0.5048	0.5821	0.5047	0.4826	0.4792	0.5029	0.5107
LINK	NA	0.533	0.4311	0.4033	0.3629	0.5037	0.4326
LTC	0.5809	0.5783	0.5283	0.5343	0.343	0.4662	0.513
NEO	0.3929	0.5612	0.5366	0.4858	0.3802	0.5022	0.4713
TRX	NA	0.5926	0.4608	0.4488	0.3269	0.4077	0.4573
USDT	0.07	0.0606	0.0686	0.2188	0.036	0.1928	0.0908
XLM	NA	0.4165	0.3873	0.467	0.461	0.3971	0.433
XRP	0.634	0.5998	0.3864	0.3634	0.3633	0.3972	0.4694
Average	0.4351	0.5009	0.4524	0.4775	0.3624	0.4446	0.4457
CRIX	0.5807	0.5542	0.4942	0.4782	0.5141	0.5035	0.5243

Note: NA indicates that the cryptocurrency did not exist in the analyzed period.
Source: Elaborated by the authors.

As for Tether (USDT), its value for the total period was the lowest compared to the others, at 0.1928. This is because, unlike traditional cryptocurrencies, Tether aims to match the US dollar exchange rate. In this sense, it is unfeasible to expect that its Hurst exponent would approach its theoretical value in an efficient market, since its quotation is not random, but deterministic and linked to the dollar. Studies such as Silva, Klotzle, Pinto & Gomes (2019) highlight some particularities of tether several effects, such as behavioral biases and level of contagion from Bitcoin's effects.

Performing an analysis of the evolution of the exponents over the periods of analysis, it can be seen that during the period of market consolidation, cryptocurrencies that have existed for longer, such as Bitcoin, Bitshares, and Litecoin, had already presented values consistent with market efficiency. During Period 2, with the creation of several altcoins and the great euphoria of investors in this market, an increase in inefficiency can be observed for Bitcoin and Etheriun, two of the largest altcoins, due to the increase in long-term memory, which corroborates the analyses of Nguyen et al. (2019). However, for the other cryptocurrencies, an approximation of the values of the exponents with what is expected in an efficient market is noticeable. In the following period, characterized by the fall of crypto-assets values, can be noticed the inversion of the positive correlation with long-term prices, thus reflecting the end of investors' euphoria and the beginning of a scenario with greater pessimism and lower efficiency.

With the non-explosive bull movements of Period 4, there was a new approximation of the movements expected in efficient markets. Finally, in the last period, which is composed of both the fall in prices due to the pandemic and the subsequent explosive resumption of the same, it is possible to characterize the scenario as possessing negative long-term memory. These effects were also found in the study by Mnif, Jarboui and Mouakhar (2020).

In this work, the authors characterize this phenomenon as being caused due to the herd effect, in which investors in this market follow patterns of other investors' movements, ignoring their own information and expectations on CRIX, it can be seen that its Hurst exponents tend to indicate smaller long-term memory biases than individual cryptocurrencies. Furthermore, it is verified that moments of explosive price drops or increases are compatible with lower degrees of efficiency as a result of long-term memory, corroborating the analyses of studies such as those of Patil and Rastogi (2020) for instance.

Other variables used in the analysis were the fractal dimensions, measured by the Periodogram, Wavelet, Genton, and Hall-Wood methods. Table 4 shows the results of the fractal dimension averages. During the entire period, cryptocurrencies had an average value of 1.0263, which allows us to verify local persistence movements. Once again, it can be seen that Tether, while having the objective of pairing with the dollar, presented the highest degree of persistence than the others.

Regarding the CRIX, it can be seen that it presents values close to the average during each period and during the total period. Besides this, one can notice a peak in its efficiency during Period 4, characterized by the retraction of prices in a first moment and then a slowly price recovery, returning to a less efficient level in the following period. This trend can also be observed in the specific case of Bitcoin.

Table 4 - Average Fractal Dimensions of Selected Cryptocurrencies

Cryptocurrency	Period					Total Period	Average
	1	2	3	4	5		
ADA	NA	1.0169	1.1139	1.2487	1.0716	1.0886	1.1128
BCH	NA	1.024	1.0222	1.0116	1.0507	1.0373	1.0271
BNB	NA	1.0452	1.0255	1.0058	1.0108	1.0213	1.0218
BTC	1.0081	1.0095	1.0075	1.0186	1.0131	1.0101	1.0114
BTS	1.0067	1.0076	1.0023	1.1114	0.8655	0.967	0.9987
DASH	1.0031	1.0027	1.0064	1.0083	1.0095	1.0069	1.006
EOS	NA	1.0127	1.0049	1.0057	1.0411	1.0278	1.0161
ETH	1.0075	1.0077	1.0034	1.0022	1.0097	1.0073	1.0061
LINK	NA	1.0075	1.0468	1.0468	1.0042	1.02	1.0263
LTC	1.0066	1.0057	1.0042	1.003	1.0452	1.0239	1.013
NEO	1.0204	1.0212	1.0027	1.0031	1.0179	1.0145	1.0131
TRX	NA	1.249	1.2031	1.2029	1.154	1.1929	1.2022
USDT	0.8594	0.7508	0.8582	0.9381	0.8658	0.8773	0.8545
XLM	NA	1.0391	1.0762	1.0498	1.1519	1.1117	1.0793
XRP	1.0097	1.0194	1.0098	1.0336	1.0953	1.0504	1.0336
Average	0.9902	1.0146	1.0258	1.046	1.0271	1.0263	1.0207
CRIX	1.0057	1.0065	1.005	1.0051	1.0136	1.0089	1.0072

Note: NA indicates that the cryptocurrency did not exist in the analyzed period.
Source: Elaborated by the authors.

Analyzing the other cryptocurrencies, one can see that on average, during Periods 4, characterized by and non-explosive price resumes, there was a peak in efficiency as a result of short-term memory, with a drop at the beginning of the pandemic and a resumption of efficiency gains in the following period. Moreover, one notices a movement of efficiency loss at times of explosive gains, which corroborates the analyses done in the study by Selmi, Tiwari, and Hammoudeh (2018) as well the analyses of the Hurst exponent.

4.3 Efficiency Indicator Analysis (EI)

Based on Hurst exponents, fractal dimensions and first order correlation, the EI's proposed by Kristoufek and Vosvrda (2013) were constructed. According to the methodology, the maximum value of the EI constructed with eight elements would be close to 2, indicating a completely inefficient market, and with a minimum value of 0, indicating a market in full efficiency. Table 5 shows the results of the EI's for the selected cryptocurrencies.

Starting the analyses by the total period, one can see that all cryptocurrencies presented on average around 55% inefficiency. By comparing the results of the CRIX index with those of individual assets, one can observe an approximation between the observed inefficiency levels. Based on the analyses of the components used in the construction of the index, individual made earlier, one can infer that the effects of long-term memory accounts for about 22.03% of the inefficiency, while short-term memory accounts for about 41.98% and the first-order autocorrelation coefficient 36.08%.

Table 5- Efficiency Ratios of Selected Cryptocurrencies

Cryptocurrency	Period					Total Period	Average
	1	2	3	4	5		
ADA	NA	1.1026	1.1151	1.1131	1.1281	1.1148	1.1147
BCH	NA	1.0729	1.0973	1.1044	1.0776	1.115	1.088
BNB	NA	1.0979	1.1382	1.1297	1.126	1.118	1.1229
BTC	1.127	1.1525	1.1383	1.1188	1.1232	1.1124	1.1319
BTS	1.128	1.1378	1.1342	1.1247	1.0272	1.1143	1.1104
DASH	1.2106	1.1273	1.1111	1.1089	1.1096	1.1171	1.1335
EOS	NA	1.2151	1.1163	1.1134	1.0797	1.1168	1.1311
ETH	1.1398	1.1328	1.121	1.1184	1.109	1.1182	1.1242
LINK	NA	1.134	1.0673	1.0764	1.1623	1.111	1.11
LTC	1.1383	1.1403	1.1118	1.1145	1.0644	1.118	1.1139
NEO	1.0645	1.095	1.1139	1.1186	1.0691	1.116	1.0922
TRX	NA	1.1214	1.0327	1.0277	1.1049	1.1419	1.0717
USDT	NA	1.0443	1.1079	0.9312	0.9732	1.1113	1.0141
XLM	NA	1.1112	1.0564	1.065	1.1107	1.1284	1.0858
XRP	1.1219	1.1173	1.1373	1.0999	1.1092	1.1401	1.1171
Average	1.1329	1.1202	1.1066	1.091	1.0916	1.1196	1.1084
CRIX	1.1529	1.1615	1.1219	1.1169	1.1176	1.113	1.1342

Note: NA indicates that the cryptocurrency did not exist in the analyzed period.
Source: Elaborated by the authors.

Performing the analysis per period, it can be seen that in Period 1 the EI's values were around 1.1290, while the market index presented a value close to 1.1170. During the euphoria of Period 2, there was an average increase of 0.31% in the index values, thus reflecting the increase in behavioral effects. In the following period, there was a correction of inefficiency, which can be justified by the reduced presence of the herd effect in the short and long term as a result of the maturation of the market and its investors. This conclusion is consistent with the expectations of Tran and Leirvik (2019) and with the results of the profile of crypto-assets investors, which indicates that as the market matures, less inexperienced investors are active.

During Period 4, one can verify the maintenance of efficiency gains, thus signaling the increase in decisions free of behavioral biases and dependence on past prices in the long run. Finally, during the period of the resumption of the expressive increase in cryptocurrency prices (Period 5), some cryptocurrencies showed expressive positive variations in EI, such as Chainlink, Tronix and Theterr, signaling efficiency losses, while Litecoin, Neo, EOS, Bitcoin cash and Bitcoin showed efficiency gains. For other cryptocurrencies, variations were limited to below 2%.

Evaluating the total EI variations, Eos, Bitcoin and Dash presented the greatest reduction in inefficiencies, while Cardano, Chainlink and Binance coins presented the greatest efficiency loss. Going further, it is interesting to evaluate which cryptocurrencies presented the greatest efficiency oscillation. In this regard, it can be highlighted that, except for the specific case of Tether, Eos, Tronix and Bitshares presented greater volatility when it comes to efficiency level, while Cardano, Ethereum and Bitcoins, presented greater constancy in this regard. Tether, as explained earlier, despite being a cryptocurrency does not have random price variations, as its aim is to pair up with the dollar. In this sense, its degree of efficiency from a fractal point of view is null, since its quotes have an almost perfect correlation with the North American currency. In this sense, the effects captured by the calculated EI are not related to the currency itself, but to the dollar variation.

Furthermore, it can be seen that this market, based on EI averages or by CRIX, is moving in a way to reduce its degree of inefficiency, which corroborates the principles of AMH. It can also be seen that this pattern is maintained both at times of significant falls in the market and at times of non-explosive price movements. However, at times of extreme growth, as seen in the last period, there is a small loss in the degree of efficiency. However, it is important to point out that this increase in inefficiencies did not cause the EI to oscillate to values far from the value obtained in Period 4.

Reading the evolution of the cryptocurrency market from Bitcoin's point of view, it can be assumed that in times of price corrections, such as Periods 2, the exit of investors from this market ends up correcting behavioral effects and, consequently, increasing efficiency. Analyzing CRIX and the average of the indicators, it can be seen that this trend is followed by the cryptocurrency market, although it is noteworthy that the corrections of altcoins is smoother than that of Bitcoin. Considering the trading volume and market share of this crypto-assets, it is coherent to verify that the level of their corrections will be more expressive, since they have higher volumes of investors who end up leaving the market at times of falling prices. These oscillatory movements of market inefficiency as a result of investor entry and exit as well as strategy adaptation and reduction of behavioral effects is consistent with Lo's (2004) AMH. From an empirical point of view, this conclusion about the inefficiency of the crypto-assets market as well as the temporal oscillation of the efficiency of this market is consistent with the findings made by works, such as those by Mnif, Jarboui and Mouakhar (2020), Kristoufek and Vosvrda (2019), Tran and Leirvik (2019) and Selmi, Tiwari and Hammoudeh (2018). Moreover, considering the performance of cryptocurrencies in times of euphoria, can create a new speculative bubble, given the increased volatility of asset returns as well as the growth in trading volume, and this conclusion is corroborated with studies, such as by Cagli (2019) and Kristoufek (2013) on the behavior of crypto assets.

5 FINAL REMARKS

In this study we try to identify the efficiency of the cryptocurrency market, based on fractal element analysis. In methodological terms we based on the study by Kristoufek and Vosvrda (2013) and analyze a sample of the 15 most traded cryptocurrencies in this market, besides the benchmark developed by Trimborn and Härdle (2018), the CRIX.

The analysis of the Hurst exponents allowed us to verify the existence of negative long-term memory on average. The analysis of the fractal dimensions, on the other hand, allowed us to verify the occurrence of a positive short-term dependence on average, which together with the first-order autocorrelation coefficients, allow us to verify the existence of a strong short-term dependence. From these metrics, the EI's were calculated, based on the difference between the value obtained from each metric with its theoretical value in an efficient market.

Based on the calculated EI's, it can be verified market inefficiency, which corroborates previous studies, such as those of Kristoufek and Vosvrda (2019). However, based on the analyses of the evolution of the indices over time, it can be verified the oscillation of efficiency according to market moments: in moments of upswings, efficiency presented a drop; and in moments of downswings or stability, it can be verified efficiency gains. Such findings corroborate studies, such as those of Mnif, Jarboui, and Mouakhar (2020) and Tran and Leirvik (2019) and Selmi, Tiwari, and Hammoudeh (2018), as well as can be explained by Lo's (2004) AMH, which preaches a process of evolution of investors regarding their rationality and, consequently, the evolution of market efficiency.

In this sense, it is possible to verify that at times of market downturns, there is a greater approximation to the EMH. Thus, in bullish moments there are more chances for arbitrage aiming at abnormal profits. The study also found that in bullish moments it is possible to identify patterns of speculative bubbles, so as to confirm the high degree of risk of these investments. Thus, individual and institutional investors should be aware of this risk in order to correctly calibrate their portfolios given the potential for losses.

Furthermore, given the possibility of arbitrage and market inefficiencies, models that consider market efficiency assumptions should be avoided. Thus, empirical models, such as those used by econophysicists and time series, which may have greater flexibility to deal with this breakdown of assumptions and absorb the effects of inefficiency more sparingly in their modeling, would be preferable.

As limitations of the present work, we highlight: 1. the use of daily data, which limits the sample for each period; 2. the metrics used consider mainly short and long-term memory effects, disregarding a wide range of other effects studied in the theory of behavioral finance; 3. the use of the cryptocurrency quotation only in dollars, since the work of Kristoufek and Vosvrda (2019) shows evidence that EI's suffer changes when considering the quotation based on Bitcoin.

Finally, as a suggestion for future work, it could expand the database with the use of intraday effects, thus allowing both increasing the sample analyzed, and also capturing behavioral effects that disappear when considering daily data. Another suggestion would be the incorporation of other behavioral metrics, such as loss aversion and weekday effects, or even indicators to capture delay effects of information absorption. It is also suggested to perform analyses considering the Bitcoin quotation of cryptocurrencies to verify possible changes in changes when disregarding the link to the dollar. However, it is emphasized that by using this approach, information about Bitcoin's behavior is lost. Finally, another possibility of research on the subject would be to analyze the changes in the profile of crypto-assets investors, given the absence of studies with this purpose.

REFERENCES

Barabási, A. L., Szépfalusy, P., & Vicsek, T. (1991). Multifractal spectra of multi-affine functions. *Physica A: Statistical Mechanics and its Applications*, *178*(1), 17-28.

Białkowski, J. (2020). Cryptocurrencies in institutional investors' portfolios: Evidence from industry stop-loss rules. *Economics Letters*, 191, 108834.

Bracciali, A., Grossi, D., & de Haan, R. (2021). Decentralization in open quorum systems: limitative results for Ripple and Stellar. In *2nd International Conference on Blockchain Economics, Security and Protocols (Tokenomics 2020*). Schloss Dagstuhl-Leibniz-Zentrum für Informatik.

Briére, M., Oosterlinck, K., & Szafarz, A. (2015). Virtual currency, tangible return: portfolio diversification with bitcoin. *Journal of Asset Management*, *16*(6), 365-373.

Brito, J., Shadab, H. B., & Castillo O'Sullivan, A. (2014). Bitcoin financial regulation: securities, derivatives, prediction markets, and gambling. *Columbia Science and Technology Law Review,*16,144-221.

Cagli, E. C. (2019). Explosive behavior in the prices of Bitcoin and altcoins. *Finance Research Letters*, *29*, 398-403.

Cajueiro, D. O. & Tabak, B. M. (2004). The Hurst exponent over time: testing the assertion that emerging markets are becoming more efficient. *Physica A: Statistical Mechanics and its Applications*, 336(3-4), 521-537.

Cajueiro, D. O., Tabak, B. M., & Andrade, R. F. (2009). Fluctuations in interbank network dynamics. *Physical Review E,* 79(3), 037101.

Calvet, L. E., & Fisher, A. J. (2013). Extreme risk and fractal regularity in finance. *Contemporary Mathematics*, 601, 65-94.

Caporale, G. M., Gil-Alana, L., Plastun, A., & Makarenko, I. (2016). Long memory in the Ukrainian stock market and financial crises. *Journal of Economics and Finance*, 40(2), 235-257.

Chan, G., Hall, P., & Poskitt, D. S. (1995). Periodogram-based estimators of fractal properties. *The Annals of Statistics*, 1684-1711.

Charfeddine, L., Benlagha, N., & Maouchi, Y. (2020). Investigating the dynamic relationship between cryptocurrencies and conventional assets: implications for financial investors. *Economic Modelling*, *85*, 198-217.

Davies, S., & Hall, P. (1999). Fractal analysis of surface roughness by using spatial data. *Journal of the Royal Statistical Society: Series B (Statistical Methodology),* 61(1), 3-37.

Di Matteo, T., Aste, T., & Dacorogna, M. M. (2003). Scaling behaviors in differently developed markets. *Physica A: Statistical Mechanics and its Applications*, *324*(1-2), 183-188.

Dubovikov, M. M., Starchenko, N. V., & Dubovikov, M. S. (2004). Dimension of the minimal cover and fractal analysis of time series. *Physica A: Statistical Mechanics and its Applications*, 339(3-4), 591-608.

Elliott, R. N. (1994). *R. N. Elliott's Masterworks*. Prechter, Robert R., Jr. (ed.) Gainesville, GA: New Classics Library. pp. 70, 217, 194, 196.

Fama, E. F. (1970). Efficient capital markets: a review of theory and empirical work. *The Journal of Finance*, *25*(2), 383-417.

Fama, E. F. (1991). Efficient Capital Markets: II. *The Journal of Finance*,45 (5), 1575-1617.

Galbraith, J. K. (1994). *A short history of financial euphoria*. 1 ed. London: Penguin Books.

Genton, M. G. (1998). Highly robust variogram estimation. *Mathematical Geology*, *30*(2), 213-221.

Gurdgiev, C., & O'Loughlin, D. (2020). Herding and anchoring in cryptocurrency markets: Investor reaction to fear and uncertainty. *Journal of Behavioral and Experimental Finance*, 25, 100271.

Hall, P., & Wood, A. (1993). On the performance of box-counting estimators of fractal dimension. *Biometrika*, *80*(1), 246-251.

Hasso, T., Pelster, M., & Breitmayer, B. (2019). Who trades cryptocurrencies, how do they trade it, and how do they perform? Evidence from brokerage accounts. *Journal of Behavioral and Experimental Finance*, 23, 64-74.

Hsieh, Y. Y., Vergne, J. P., & Wang, S. (2017). *The internal and external governance of blockchain-based* *organizations: Evidence from cryptocurrencies*. In: Campbell-Verduyn, M. (2017). Bitcoin and Beyond: Cryptocurrencies, Blockchains and Global Governance. New York, NY, pp. 48-68.

Jovanovic, F., & Schinckus, C. (2013). The emergence of econophysics: a new approach in modern financial theory. *History of Political Economy*, 45(3), 443-474.

Kahneman, D., & Tversky, A. (2013). *Prospect theory: an analysis of decision under risk*. In: MacLean, L. C., & Ziemba, W. T. (2013). Handbook of the fundamentals of financial decision making: Part I. World Scientific, 99-127.

Kimura, H. (2005). The financial market from the fractal optics perpsective. *Revista de Administração de Empresas*, 45(4), 124-125.

Kristoufek, L. (2013). BitCoin meets Google Trends and Wikipedia: quantifying the relationship between phenomena of the Internet era. *Scientific Reports*, *3*, 3415.

Kristoufek, L., & Vosvrda, M. (2013). Measuring capital market efficiency: global and local correlations structure. *Physica A: Statistical Mechanics and its Applications*, *392*(1), 184-193.

Kristoufek, L., & Vosvrda, M. (2019). Cryptocurrencies market efficiency ranking: not so straightforward. *Physica A: Statistical Mechanics and its Applications*, *531*, 120853.

Lánský, J. (2017). Bitcoin system. *Acta Informatica Pragensia*, *6*(1), 20-31.

Lo, A. W. (2004). The adaptive markets hypothesis. *The Journal of Portfolio Management*, 30(5), 15-29.

Lo, A. W. (2005). Reconciling efficient markets with behavioral finance: the adaptive markets hypothesis. *Journal of Investment Consulting*, 7(2), 21-44.

Mandelbrot, B. (1963). New methods in statistical economics. *Journal of Political Economy,* 71(5), 421-440.

Mandelbrot, B. B. (2005). The inescapable need for fractal tools in finance. *Annals of Finance*, 1(2), 193-195.

Mantegna, R. N. (1991). Lévy walks and enhanced diffusion in Milan stock exchange. *Physica A: Statistical Mechanics and its Applications*, 179(2), 232-242.

Mantegna, R. N., & Kertész, J. (2011). Focus on statistical physics modeling in economics and finance. *New Journal of Physics*, 13(2), 025011.

Mnif, E., Jarboui, A., & Mouakhar, K. (2020). How the cryptocurrency market has performed during COVID 19? A multifractal analysis. *Finance Research Letters*, 36, 101647.

Mosteanu, N. R., & Faccia, A. (2021). Fintech Frontiers in Quantum Computing, Fractals, and Blockchain Distributed Ledger: Paradigm Shifts and Open Innovation. *Journal of Open Innovation: Technology, Market, and Complexity*, 7(1), 19.

Nakamoto, S. (2008). *Bitcoin: a peer-to-peer electronixic cash system*. Disponível em: https://bitcoin.org/bitcoin.pdf.

Nguyen, T. V. H., Nguyen, B. T., Nguyen, T. C., & Nguyen, Q. Q. (2019). Bitcoin return: impacts from the introduction of new altcoins. *Research in International Business and Finance*, *48*(C), 420-425.

Patil, A. C., & Rastogi, S. (2020). Multifractal analysis of market efficiency across structural breaks: Implications for the adaptive market hypothesis. *Journal of Risk and Financial Management*, 13(10), 248.

Peng, C. K., Buldyrev, S. V., Goldberger, A. L., Havlin, S., Simons, M., & Stanley, H. E. (1993). Finite-size effects on long-range correlations: Implications for analyzing DNA sequences. *Physical Review E*, *47*(5), 3730.

Peters, E. E. (1994) *Fractal Market analysis: applying caos theory to investment and economics***.** New York: Willey.

Peters, E. E. (1996). *Chaos and order in the capital markets: A new view of cycles, prices, and market volatility.* New York, NY: John Wiley and Sons, Inc.

Reed, J. (2017). *Litecoin: an Introduction to Litecoin cryptocurrency and Litecoin mining.* North Charleston, United States.

Schinckus, C. (2011). What can econophysics contribute to financial economics? *International Review of Economics*, 58(2), 147-163.

Selmi, R., Tiwari, A., & Hammoudeh, S. (2018). Efficiency or speculation? A dynamic analysis of the Bitcoin market. *Economics bulletin*, 38(4), 2037-2046.

Serroukh, A., Walden, A. T., & Percival, D. B. (2000). Statistical properties and uses of the wavelet variance estimator for the scale analysis of time series. *Journal of the American Statistical Association*, 95(449), 184-196.

Silva, P. V. J. G., Klotzle, M. C., Pinto, A. C. F., & Gomes, L. L. (2019). Herding behavior and contagion in the cryptocurrency market. *Journal of Behavioral and Experimental Finance*, 22, 41-50.

Tran, V., & Leirvik, T. (2019). Efficiency in the markets of crypto-currencies. *Finance Research Letters*, 35, 101382.

Trimborn, S., & Härdle, W. K. (2018). CRIX an index for cryptocurrencies. *Journal of Empirical Finance*, *49*, 107-122.

Urquhart, A. (2016). The inefficiency of Bitcoin. *Economics Letters*, *148*(C), 80-82.

Urquhart, A., & McGroarty, F. (2014). Calendar effects, market conditions and the Adaptive Market Hypothesis: Evidence from long-run US data. *International Review of Financial Analysis*, 35, 154-166.

Wei, W. C. (2018). Liquidity and market efficiency in cryptocurrencies. *Economics Letters*, 168, 21-24.

Yermack, D. (2013). Is Bitcoin a real currency? An economic appraisal (No. w19747). *National Bureau of Economic Research*, 36(2), 843-850.

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1. Definition of research problem	√		
2. Development of hypotheses or research questions (empirical studies)	√	√	√
3. Development of theoretical propositions (theoretical work)	√		
4. Theoretical foundation / Literature review	√	√	
5. Definition of methodological procedures	√	√	
6. Data collection	√		
7. Statistical analysis	√	√	
8. Analysis and interpretation of data	√		√
9. Critical revision of the manuscript	√	√	√
10. Manuscript writing	√		√
11. Other (please specify)			

Conflict of Interest
The authors have stated that there is no conflict of interest.

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