A derivation of a variable vertical mesh spacing for LES models: application to a CBL

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1. Introduction

In Large Eddy Simulation (LES), the energy-containing eddies of the turbulent motion are explicitly resolved and the effect of the smaller, more isotropic eddies (typical eddies of the inertial subrange), needs to be parameterized. Modeling these residual turbulent motions, which are also called as subfilter scale (SFS) motions is in large part a phenomenological procedure based on heuristic arguments. The resolved turbulent flow time-dependent random variables are obtained by the application of a low-pass spatial filter of characteristic width Δ, the turbulent resolution length scale smaller than the scales of the resolved turbulent motions. Thusly, there is a clear expectation that an adequate physical derivation, of the parameter Δ provides robust fluxes of heat and momentum from resolved-scale motions and that the contributions of the unresolved subfilter motions are responsible by the dissipation of the resolved turbulence. However, most geophysical flows present boundary regions where the turbulence-scale reduces and becomes smaller than the spatial filter scale width. Indeed, in the convective boundary layer (CBL), the wavelength of the peak in the vertical velocity spectrum $(\lambda_m)_w$ decreases with the proximity of the ground (Kaimal et al., 1976) and this surface presence constrains our capacity to accomplish high-Reynolds-number LES. Near to the ground, the filter cutoff wavelength λ_f employed in LES modeling is of the same order than and consequently the SFS fluxes in LES become important and their participation to the total flux (resolved and filtered eddies) grows with the proximity to the surface. Recent testing of SFS models using observational data shows that the contribution of the SFS turbulent energy modes to the total turbulence is dominant when $(\lambda_m)_w/\lambda_f \leq 1$ while the resolved motions are the increasing important for $(\lambda_m)_w / \lambda_f >> 1$. A commonly used filter in LES is the sharp Fourier cutoff. The sharp spectral filter annihilates all Fourier modes of wave number $|k|$ greater than the cutoff wave number $k_f \equiv \pi / \Delta$, whereas it has no effect on lower wave number modes. It is important to note that in practical applications of LES the parameter Δ is expressed in terms of Δx and normally it is substituted by this horizontal mesh spacing. Observations show that a CBL presents a homogeneous character in the horizontal directions. Therefore, throughout the CBL, both *u* (longitudinal velocity) and *v* (lateral velocity) spectra display a well-established filter z-less limiting wavelength λ_f for inertial subrange. This horizontal homogeneity imposes large values of $\Delta x = \Delta y \approx 0.05 z_i$ in the surface regions of a CBL and as a consequence decreases the magnitude of k_f and favours the condition $k_f \leq k_e$. This experimental evidence points out that a filter wave number, such as $k_f = \pi / \Delta x$, is physically not adequate to be employed in LES applications to reproduce turbulent fluxes in the proximity to the surface. Thusly, to obtain a possible solution that avoids this condition and assures that $k_f >> k_e$ will be necessary to define the sharp spectral filter as $k_f = \pi / \Delta z$. However, differently of *u* and *v*, the vertical velocity component *w* exhibits the largest height dependence among the three velocity components. This vertical inhomogeneity makes the inertial subrange limiting wavelength vary strongly with height in the lower layers of the CBL. Therefore, for the turbulent vertical velocity, observational values of λ_f or k_f valid in the surface CBL cannot be clearly determined. This lack of definition of λ_f for the vertical turbulent velocity does not allows that the magnitudes of Δz and consequently of Δ can be established for the vertical region near to the ground in a CBL. Therefore, motivated by the selection of a filter cutoff wave number provided by $k_f = \pi / \Delta z$ the purpose of the present investigation is to develop a methodology that provides values for Δz in the lower layers of the CBL.

2. Methodology to obtain a vertical mesh spacing near to the ground in a CBL

The equation for Eulerian velocity spectra under unstable conditions can be expressed as a function of convective scales as follows

$$\frac{nS_i}{w_*^2} = \frac{1.06 c_i f \psi^{2/3} (z/z_i)^{2/3}}{(f_m)_i^{5/3} \left(1+1.5\left(f/(f_m)_i\right)\right)^{5/3}} \tag{1}$$

From the Eq. (1), the vertical turbulent velocity spectrum can be written as

$$\frac{nS_w}{w_*^2 \psi^{2/3}} = \frac{1.06 c_w (nz/U) \psi^{2/3} (z/z_i)^{2/3}}{(f_m)_w^{5/3} \left(1+1.5(nz/U)/(f_m)_w\right)^{5/3}} \tag{2}$$

where $c_w = 0.36$. From the Ashchurch and Minnesota experiments, for $z < 0.1 z_i$, the following relation can be used $(f_m)_w = z/(\lambda_m)_w = z/5.9z = 0.17$, which is precisely the free convection limit , observed in the Kansas data (Caughey, 1982). Substituting this relation into equation (2) yields

$$\frac{nS_w}{w_*^2 \psi^{2/3}} = \frac{7.32 f_i z (z/z_i)^{5/3}}{\left(1+8.85 f_i (z/z_i)\right)^{5/3}} \tag{3}$$

Furthermore, the asymptotic behavior of Eq. (3) for large values of f_i, in the inertial subrange of the spectrum, can be described as $nS_w / w_*^2 \psi^{2/3} = 0.19 f_i^{-2/3}$.Comparing equation (3) with their asymptotic behavior, results $f_f = 3.66 (z/z_i)^{-1}$ and $\lambda_f = 0.27 (z/z_i) z_i$. Furthermore, k_f can be written as $k_f = 2\pi / \lambda_f$ and therefore a comparison of this ratio with $k_f = \pi / \Delta z$ leads to $\Delta z = \lambda_f / 2 \cong 0.14 (z/z_i) z_i$. Thusly, $\lambda_f = 0.27 (z/z_i) z_i$ imparts a physical spatial constraint (obtained from observed data), which helps the correct choice of the dimension of the vertical mesh spacing in LES models to lower layers of the CBL.

3. Acknowledgement

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4. References

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