An analytical formulation to simulate contaminant dispersion in the atmosphere

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Resumo

Neste trabalho, mostramos a existência e unicidade de uma analítica solução que permite a simulação de dispersão de poluentes na atmosfera, resolvendo a equação tridimensional de advecção-difusão dependente de tempo, através da combinação GITT e Transformada de Laplace.

Summary

In this work, we show the existence and uniqueness of an analytical solution which allows the simulation of pollutant dispersion in atmosphere by solving the three-dimensional, time-dependent, advection-diffusion equation by combined GITT and Laplace Transform techniques.

1. Introduction

The advection-diffusion equation has been widely applied in operational atmospheric dispersion models to predict the mean concentration of pollutant in the Planetary Boundary Layer (PBL). Its solution is quite general, i.e. is valid for a wide class of problems dealing with transfer processes, besides air pollution problems. To the best of our knowledge analytical solutions are available only for specialized problems, where strong assumptions are made on the eddy diffusivity coefficient and wind profiles except for some stationary problems. Despite the fact that the profiles assumed are not physically realistic, these solutions are used for operative applications and environmental management, because they can give a description of diffusion processes within the PBL.

In this work we report on an analytical solution for the time-dependent, three-dimensional diffusion equation using the GITT and the Laplace Transform techniques. We mention also that errors inherent to mathematical models are due to idealizations of the physical phenomenon and numerical errors. The latter ones are minimized, if an analytical solution is at hand. In contrast to the existing analytical solutions, the solution presented here is not limited to the shape of vertical profiles, so that it is possible to simulate more realistic scenarios. The novelty of this work relies on the existence and uniqueness of an analytical solution for the time-dependent, three-dimensional advection-diffusion equation which allows to simulate the pollutant concentration dispersion in the atmosphere.

2. The analytical solution

To construct the analytical solution of the three-dimensional, time-dependent equation we start from the conventional advection-diffusion model with arbitrary source term defined in a cartesian domain. Taking the symmetry of the solution in a 2-D subdomain we assume zero fluxes at the boundaries and the initial condition of null concentration. Here, we also consider that the eddy diffusivity coefficients have a continuous dependence on the vertical variable.

The GITT approach, by the use of orthogonalization relations permits to cast the original problem into a set of independent partial differential equations. We solve the second order partial differential matrix equation applying the Laplace transform technique in the time variable. The resulting ordinary second order differential equation may be cast into two first order differential equations which may be solved using an orthogonalization procedure and yields the coefficient of the GITT procedure. Now Laplace inversion is applied with the inversion integral calculated using the Gaussian quadrature approach.

3. Conclusion

In deriving the closed form solution for the 3-D time dependent advection-diffusion equation we showed the existence and uniqueness of an analytical solution by virtue of the Cauchy-Kowaleski theorem, which as a result allows the simulation of pollutant dispersion in atmosphere. In addition we emphasize underline that this solution due its analytical character is appropriated for symbolic computation.

Thus the found solution is valid for any diffusion coefficients and wind profiles depending on the vertical variable and, moreover, for any arbitrary source. In a future work we will complete the mathematical analysis of the reported solution proving the convergence of the truncated series to the exact solution as well extending this solution to problems requiring time and spatial variation of the parameters in a mathematically more rigorous way. We focus our future attention in this direction as well as to the task of applying this methodology to the simulation of pollutant dispersion in atmosphere for actual physical scenarios.