The GILTT analytical solution applied in a puff model

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Abstract

A new puff model is presented where the horizontal diffusion is described by two Gaussian functions, but the vertical one it is expressed by an analytical solution.

Introduction

Puff models were introduced to simulate the behaviour of pollutants in non homogeneous and non-stationary meteorological and emission conditions [1]. The emission is discretised in a temporal succession of puffs, each of which shifts into the area of calculus thanks to a three-dimensional wind field that is time variable (see Figure 1). Puff models assume that each emission of pollutants in a time interval Δt releases into the atmosphere a mass of pollutants $\Delta M = Q\Delta t$, where Q is the emission rate, which is variable in time. A puff release scenario assumes that the release time and sampling times are very short compared to the travel time from the source to the receptor.

Each puff contains the mass ΔM and its baricenter is transported by the wind, which may vary in space and time. In a puff model the wind velocity influences the calculation of the concentration only in the density of puffs in the region of diffusion (the lower the wind velocity, the closer the puffs emitted by a source). For this reason puff models can be used to simulate diffusion in calm wind conditions. Puff models are also suitable for the simulation of diffusion over complex orographies.

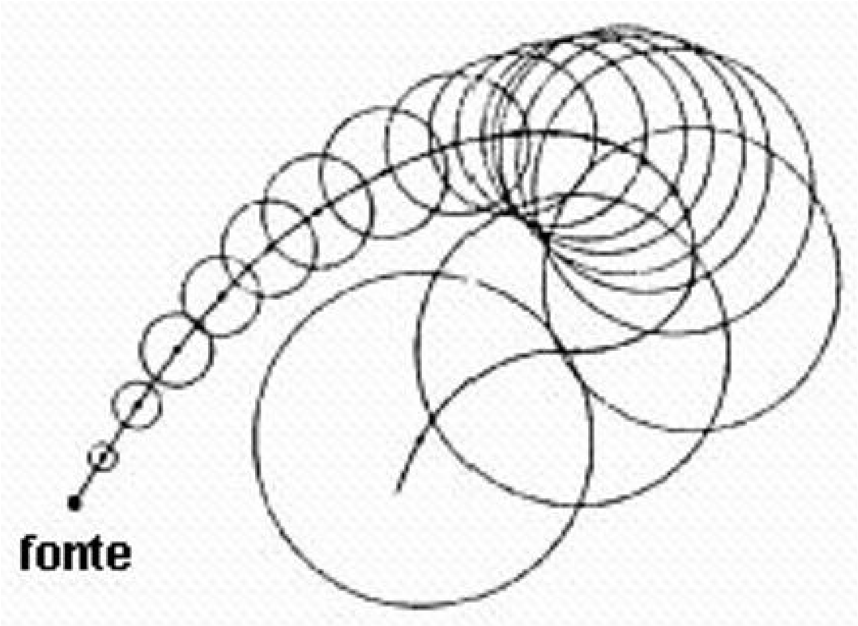

Figura 1. Simulation of a plume through a series of puffs released at a certain time interval from the source.

The puff model

The puffs considered here are emitted in time intervals Δt_1 and the calculation of the concentration of pollutants of each one is made in a time interval Δt_2. Each puff is carried in accordance with the trajectory from its baricenter, which is determined by the velocity vector of the local wind, while it is enlarged in the time by means of the dispersion coefficients. In particular, in our model, the vertical diffusion of the material carried in accordance with the trajectory of puff is non-Gaussian and it is described by general solutions of the equation:

$$\frac{\partial c(z,t)}{\partial t} = \frac{\partial}{\partial z}\left(K(z) \frac{\partial c(z,t)}{\partial z} \right) + S \qquad (1)$$

where, c(z,t) represents the average concentration,
z is the vertical coordinate,
K(z) is the vertical eddy diffusion,

S is the source term ($S = Q\delta(t-t_0)\delta(z-H_S)$),
H_S is the height of the source (located in x=0);
δ means Delta Dirac function.

We assume that, at the beginning of pollutants release the dispersion region is not polluted. Moreover, we introduce the boundary conditions of zero flux at the ground and at the boundary layer top zi (0<z<zi), t>0.

To solve equation (1) the GILTT method (Generalized Integral Laplace Transform Technique) is used [2]. Once the solution for single puff is know, we claim that the puff solution reads like the summation of the contribution of all puffs emitted by the source. This procedure leads to the solution:

$$C_{n_{puff}}(z,t) = \sum_{p=1}^{P} \Delta M_p \left\{ \int_{t=0}^{\infty} C_n(z,t)\, H(t - t_{0_p})\, dt \right\}$$

for p=1:P, where P is the total number of puffs emitted,

ΔM_p is the mass carried out for the p^th puff,

H is the Heaviside function

and $C_{n_{puff}}$ is the one-dimensional puff solution.

The horizontal dispersion is shaped by a Gaussian (Ψ_{puff}) characterizing the diffusion in this direction defined as:

$$\psi_{puff}(x,y,t) = \frac{1}{2\pi\,\sigma_y\,\sigma_x} \exp\left[-\frac{1}{2}\left(\frac{x-x_0}{\sigma_x}\right)^2\right] \exp\left[-\frac{1}{2}\left(\frac{y-y_0}{\sigma_y}\right)^2\right] \qquad (2)$$

where $x_0 = u\Delta t$, $y_0 = v\Delta t$ *and* (x_0, y_0) is the puff centre (u and v are average wind velocity) and σ_x , σ_y the lateral dispersion parameters. Henceforth, the three-dimensional puff solution, assuming point source at co-ordinate origin, has the form:

$$C_{puff}(x,y,z,t) = C_{n_{puff}}(z,t)\,\psi_{puff}(x,y,t) \qquad (3)$$

Preliminary results against experimental data confirm the applicability of the approach proposed and are promising for future work.

References

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