# Lorenz maps of a simple intermittent model – part 2

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**Resumo**

Neste trabalho são apresentadas as análises dos mapas de Lorenz dos casos discutidos na primeira parte deste estudo.

## 1. Introduction

The Lorenz maps were introduced by Lorenz (1963), when he intends to predict the next local maxima of the solutions, of his model, ($x_{t_0+1}$) for the previous value ($x_{t_0}$) at a time $t_0$, and it has proved to be a very useful tool to analyze chaotic systems. In this work we show the Lorenz maps for the cases presented in the part 1.

## 2. Results and discussion

Figure 1, shows the Lorenz maps for different number of levels and conditions indicated in each panel, for the cases where the system has periodic solutions with period 3. In spite of the different numbers of levels, the structure shown in the cases of period 3 is similar, resembling a triangle. It shows that when there is a small turbulence peak, the next one will be a bit more intense, and the next will be the most intense. Figure 1 also shows that for $n=3$ the period 3 pattern is nearly perfect, however for $n=2$, $n=4$ and $n=5$, there are some small variations around the largest peak, that, in the last two cases, are mainly caused by inertial oscillations due to the long time of the simulations (700 hours).

Figure 2 shows the Lorenz maps for different wind intensities. For these parameters we observe structures in the return map, which surround the region delimited by the period 3 cases. Figure 2 indicates that there are parameter values leading to low-dimensional chaotic attractors, in agreement with Li and Yorke (1975).

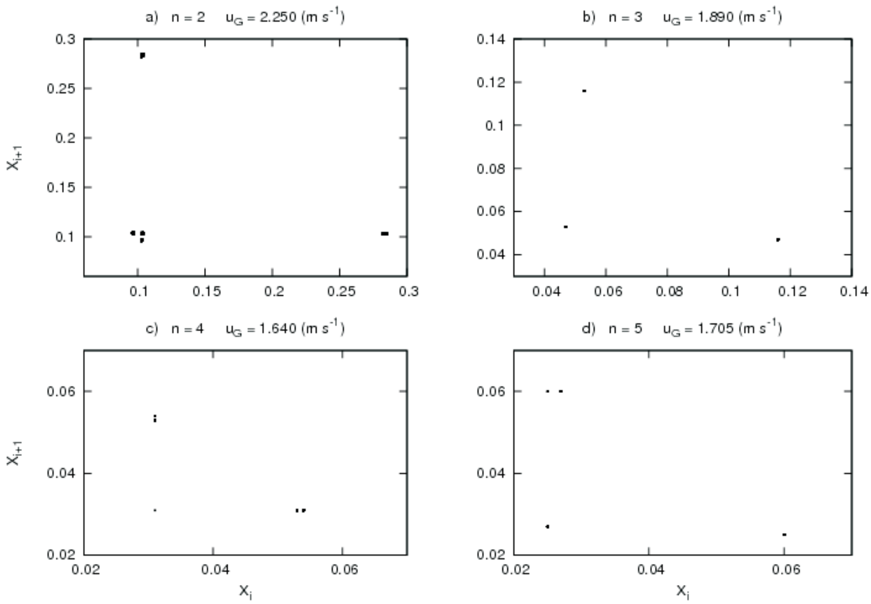


**Figure 1.** Lorenz maps for different conditions and number of levels that as indicated at the top of the panels, for cases where the model has periodic solutions with period 3.

Figure 3, shows the Lorenz maps for large geostrophical wind velocities ($u_G$). In this case, one can see that more complicated structure arise. The cases shown in Figure 3 are those where the bifurcation diagrams are mostly cloudy, with relatively stronger geostrophical winds, that are equivalent to the cloudy region in the bifurcation diagram (Figure 1 – part 1), before the system reaches the connected state. The absence of structures may be caused by the fact that for those parameters the attractor is high dimensional. However, such affirmation depends on a more detailed analysis.

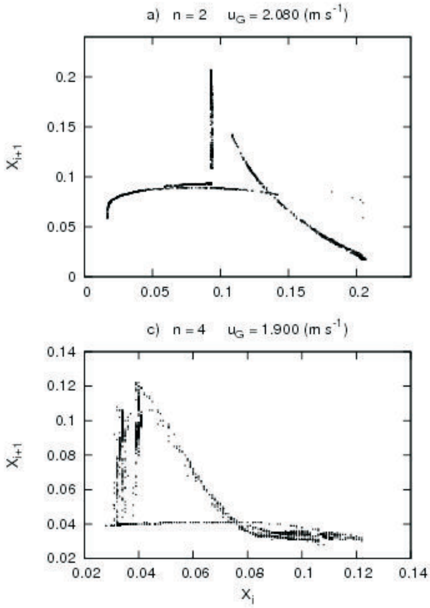


**Figure 2.** Same as Figure 1 for different values of $u_G$.

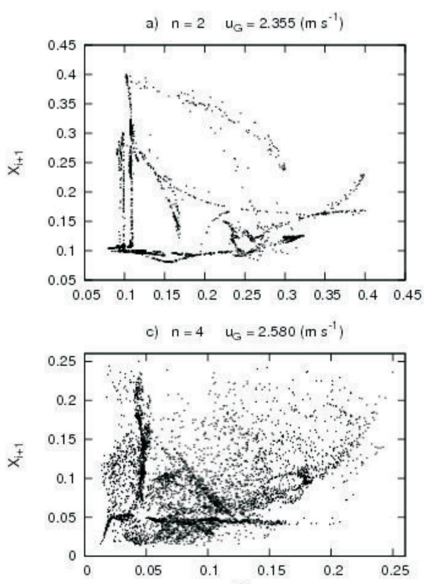


**Figure 3.** Same as Figure 1, for large geostrophical wind velocities ($u_G$).

## 3. Conclusions

The Lorenz maps show that for some conditions the solutions present well defined structures, suggesting that the solutions are chaotic with a low dimensional attractor. However, for stronger geostrophical winds the Lorenz maps do not show any defined structure, indicating that the attractor may be high dimensional in those cases. In a future work, the solutions will be analyzed more deeply focusing on the dimensionality of the attractor as well as on the positive Lyapunov exponents.

## 4. References


Li, T. Y., Yorke, J. A.. Period Three Implies Chaos**. The American Mathematical Monthly**, 1975.

Lorenz, E.. Deterministic nonperiodic flow. **Journal of the Atmospheric Sciences**, 1963.