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Modeling the effect of memory on the propagation of fake news between two populations that share information

Modelagem do efeito da memória na propagação de fake news entre duas populações que compartilham informações

Adriano De Cezaro[I], **Fabiana Travessini De Cezaro**[I], **Luverci do Nascimento Ferreira**[I]

[I]Institute of Mathematics, Statistics and Physics, Federal University of Rio Grande, RS, Brazil

ABSTRACT

The dissemination of information can have a huge impact on public opinion, especially if it is false (fake news). In this work, we propose and analyze a reinterpretation of the SIR model that takes into account the effect of memory on the dynamics of fake news diffusion, resulting from use of fractional derivatives, between two distinct population groups. From a theoretical point of view, we will show that the proposed model is well-posed. Furthermore, we will present simulated scenarios showing that the presence of memory reduces the proportion and propagation time of fake news in both populations that interact. Furthermore, we show numerically that the inverse of the rigidity radius cannot be used as a measure of the rate at which fake news disappears.

Keywords: Fake news; Mathematical modeling; Fractional derivatives; Memory; Interacting populations

RESUMO

A divulgação de informações pode ter um enorme impacto na opinião pública, especialmente no caso em que estas são falsa (fake news). Neste trabalho propomos e analisamos uma reinterpretação do modelo SIR que leva em conta o efeito da memória na dinâmica de difusão de notícias falsas, devido ao uso de derivadas fracionárias, entre dois grupos populacionais distintos. Do ponto de vista teórico, mostraremos que o modelo proposto é bem posto. Além disso, apresentaremos cenários simulados mostrando que a presença de memória reduz a proporção e o tempo de propagação de notícias falsas em ambas populações que interagem. Ainda, mostramos numericamente que o inverso do raio de rigidez não pode ser usado como uma medida de valocidade de desaparecimento das noticias falsas.

Palavras-chave: Notícas falsas; Modelagem matemática; Derivadas fracionárias; Memória; Populações que interagem

1 INTRODUCTION

Dissimulating real facts to obtain some type of advantage has always been an object of human behavior; see Vosoughi et al. (2018) and references. However, the advent of social networks has brought another dimension to the volume and ease of dissemination of such information, either of a political nature or even to profit from the number of online visitors to the published material. Those who adopt this tactic to gain an advantage have in their favor the fact that misinformation (Fake News) normally circulates more widely. The chance of Fake News being shared is estimated to be 70% higher than true news Vosoughi et al. (2018).

In light of the prevalence and impact of misinformation on shaping societal behavior, it is evident that a significant contemporary challenge lies in understanding the mechanisms of Fake News propagation and curbing its dissemination to defend the truth. Mathematical modeling emerges as a potential approach to address this issue. This area has garnered recent interest in D'Ambrosio et al. (2022); D'Ambrosio et al. (2021,2); Mahmoud (2020); Travessini De Cezaro et al. (2023) and related works.

Assuming that the dissemination of Fake News operates similarly to an epidemic, the authors in D'Ambrosio et al. (2022); D'Ambrosio et al. (2021) introduced a reinterpretation of the SIR-type compartmental models used in the spread of diseases Allen et al. (2007) to simulate the propagation of Fake News. By employing a linear approximation based on initial conditions and parameters reflecting social progress, D'Ambrosio et al. (2022); D'Ambrosio et al. (2021) linked the rigidity index of the dynamics of the SIR model to the velocity of the spread of fake news within a population. Their numerical analysis demonstrated that a higher rigidity index corresponds to a quicker restoration of truth. In (Travessini De Cezaro et al., 2023), the authors extended the modeling approach proposed in (D'Ambrosio et al., 2022; D'Ambrosio et al., 2021) to encompass two populations that exchange information. The computational findings in (Travessini De Cezaro et al., 2023) indicated that variations between populations can significantly influence the dynamics of fake news dissemination, affecting both the intensity and speed of its spread.

This paper aims to examine how memory influences the dissemination of Fake News among two interconnected populations. The spread of Fake News is likened to an epidemic, with memory effects incorporated through the use of a Caputo-type fractional-order derivative in the dynamics. Specifically, we introduce and investigate a fractional SIR model, extending the modeling frameworks introduced in previous work such as (D'Ambrosio et al., 2022; D'Ambrosio et al., 2021; Travessini De Cezaro et al., 2023). Our finds indicate that the existence of memory in a populations (given by the fractional-order derivative in the dynamics) reduces the proportion, the duration, and speed of propagation of Fake News compared to the dynamics with integer-order derivatives. As a consequence, memory acts as a control strategy in the Fake News propagation.

In Section 2, we present the suggested model and demonstrate its well-posedness. Furthermore, we motivate the memory effect. In Section 3, simulations are presented to demonstrate the impact of model-imposed memory on the spread of Fake News in both population groups. Section 4 provides a summary of the findings and discusses potential future directions.

2 A MODELING FOR THE PROPAGATION OF FAKE NEWS BETWEEN TWO POPULATIONS THAT SHARE INFORMATION

In this work, we propose an alternative reinterpretation and a generalization that includes memory to the dynamics of the SIR-type compartmental model Allen et al. (2007), to describe the spread of Fake News in a scenario composed of two distinct populations of individuals (say P_i, for $i = 1, 2$) that share information. The proposed reinterpretation consists of assuming that each population P_i is proportionally subdivided into compartments of individuals $S_i(t)$ susceptible to believing in fake news shared by individuals in the compartment $C_i(t)$ who are already convinced that the fake news is true and want to take advantage and decide to share this information with other individuals in their own population or with individuals from another population, whose share rate is given by β_{ij}, for $i, j = 1, 2$ and $i \neq j$. Finally, there are individuals in the $R_i(t)$ compartment who managed to restore the veracity of information after believing in Fake News, whose rate is given by γ_i, after having been in the $C_i(t)$ compartment for $i = 1.2$ at some time $t \geq 0$. Using the laws of mass action Allen et al. (2007), we see that the dynamics must follow the coupled system of fractional differential equations

$$
\begin{aligned}
&D^{\alpha_j}S_j(t) = S_j(t)(-\beta_{jj}^{\alpha_j}C_j(t) - \beta_{ij}^{\alpha_j}C_i(t))\\
&D^{\alpha_j}C_j(t) = S_j(t)(\beta_{jj}^{\alpha_j}C_j(t) + \beta_{ij}^{\alpha_j}C_i(t)) - \gamma_j^{\alpha_j}C_j(t) \qquad (1)\\
&D^{\alpha_j}R_j(t) = \gamma_j^{\alpha_j}C_j(t)\,,
\end{aligned}
$$

for $i, j = 1, 2$ and $j \neq i$. In the model (1), we assume that the parameters β_{ij} and γ_i are associated with the economic development indices of the population $j = 1, 2$, namely, the *internet penetration index* IPI_j and *human development index* IDH_j. In particular, $\beta_{ij} = \sigma_{ij}IPI_j$ and $\gamma_j = \theta_j HDI_j$, for $i, j = 1, 2$, where $\sigma_{ij},\ theta_j \in]0, 1]$ (proportions to be specified). In general, $\sigma_{ij} > \alpha_i$ since Fake News spreads faster than attempts to restore truth Vosoughi et al. (2018). This interpretation of model parameters was previously used by (D'Ambrosio et al., 2021,2; Travessini De Cezaro et al., 2023). The operator $D^{\alpha_j}(X)(t)$ is the Caputo fractional derivative operator[1] (see (Diethelm, 2010)) of order $\alpha_j \in]0, 1]$, given by

$$
D^{\alpha_j}X(t) := \frac{1}{\Gamma(1-\alpha_j)}\int_0^t \frac{X'(s)}{(t-s)^{\alpha_j}}ds\,. \qquad (2)
$$

The parameters of the model (1) are raised to the respective orders of the derivatives α_j, to keep the time in unit t^{-1} Diethelm (2013). Furthermore, the model (1) will be considered with the following initial conditions

$$
S_j(0) = P_j - C_j(0) \geq 0, C_j(0) \geq 0, R_j(0) \geq 0\,. \qquad (3)
$$

2.1 Well-posedness and memory effect

In this subsection, we prove the well-posedness (existence and uniqueness of a solution $X(t) = (S_1(t), C_1(t), R_1(t), S_2(t), C_2(t), R_2(t))^T$) for the model (1) with initial conditions (3). Furthermore, we will show the memory effects in the model arising from fractional order derivatives.

Lemma 2.1. *Let $P(t) = P_1(t) + P_2(t)$, where $P_j(t)$ is the total population of individuals in the population $j = 1, 2$. Then $P(t)$ is constant for any $t \geq 0$.*

[1]Alternative definitions for fractional derivatives, such as the Riemann-Liouville derivative, also exist (refer to (Diethelm, 2010)). However, ensuring the well-posedness of the model (1) with these definitions necessitates initial conditions that are complex to interpret (see Diethelm (2010)). This is the main reason for chosen the Caputo definition see (Diethelm, 2010).

Proof. Given $j = 1, 2$, it can be deduced from the linearity property of Caputo's fractional order derivatives (refer to Diethelm (2010)) and the summation of the rows in system (1), that

$$D^{\alpha_j} P_j(t) = D^{\alpha_j} S_j(t) + D^{\alpha_j} C_j(t) + D^{\alpha_j} R_j(t) = 0\,.$$

Therefore, $P_j(t)$, is constant, see Diethelm (2010).

Lemma 2.2. *Let $X(t)$ denote a possible solution of the model* (1) *with initial conditions* (3)*. Then $X(t)$ is uniformly bounded by $P(0) := P_1(0) + P_2(0)$. In particular, each coordinate of $X(t)$ is uniformly bounded.*

Proof. Let $\|\cdot\|_1$ be $1 - norm$ in $\mathbb{R}^6$. It follows that $\|X(t)\|_1 \leq \|P(t)\|_1$, where $P(t) = P_1(t) + P_2(t)$, for any $t \geq 0$. As Lemma 2.1 implies that $P(t)$ is constant, the lemma statement is true.

Proposition 2.1. *Let $F(t, X(t))$ be the vector function that represents the right-hand side of the model* (1)*. Then:*

i) $F(t, X(t))$ *is continuous with* $t \geq 0$*.*

ii) There are constants w_1 and w_2 such that $\|F(t, X(t))\| \leq w_1 + w_2\|X(t)\|\,.$

iii) $F(t, \cdot)$ *is Lipschitz continuous with respect to the second coordinate.*

Proof. Item i) follows directly from the fact that each coordinate of $F(t, X(t))$ is given by the sum and products of continuous functions.

As each coordinate of $X(t)$ is uniformly bounded (see Lemma 2.2), the assertion of item ii) follows directly from the above fact and the definition of $F(t, X(t))$.

Since the Jacobian matrix of $F(t, X(t))$ is given by

$$JF(t, X(t)) = \begin{bmatrix} a_{11} & -\beta_{11}^{\alpha_1} S_1(t) & 0 & 0 & -\beta_{21}^{\alpha 1} S_2(t) & 0 \\ -a_{11} & \beta_{11}^{\alpha_1} S_1(t) - \gamma_1^{\alpha_1} & 0 & 0 & \beta_{21}^{\alpha_1} S_2(t) & 0 \\ 0 & \gamma_1^{\alpha_1} & 0 & 0 & 0 & 0 \\ 0 & -\beta_{12}^{\alpha_2} S_1(t) & 0 & a_{44} & -\beta_{22}^{\alpha_2} S_2(t) & 0 \\ 0 & \beta_{12}^{\alpha_2} S_2(t) & 0 & -a_{44} & \beta_{22}^{\alpha_2} S_2(t) - \gamma_2 & 0 \\ 0 & 0 & 0 & 0 & \gamma_2^{\alpha_2} & 0 \end{bmatrix}$$

where $a_{11} = (-\beta_{11}^{\alpha_1} C_1(t) - \beta_{21}^{\alpha_1} C_2(t))$ and $a_{44} = (-\beta_{22}^{\alpha_2} C_2(t) - \beta_{12}^{\alpha_2} C_1(t))$.

Thus, it follows from Lemma 2.1 that there exists a constant $L > 0$ such that $\||JF(t, U(t))|\| < L$. Therefore, the Mean Value Theorem implies that item iii) is true. □

The next result shows the existence, uniqueness, and continuous dependence of a solution $X(t)$ for the model (1)-(3). Moreover, we prove the consistency of the multi-fractional model (1), in the sense that all components of the solution $X(t)$ remain positive for all $t \geq 0$, according to what the application suggests, given that the components of $X(t)$ are proportions of the population. In order to accomplish this, define $\Omega_+ := \{U(t) : \text{ all components of } U(t) \text{ are non-negative}\}$.

Theorem 2.1. *Consider the model* (1) *with initial conditions* (3)*. Then:*

i) [Existence and Uniqueness] There is a unique solution $X \in C([0, h(T)], \mathbb{R}^6)$ for the model (1)-(**??**)*, for some $h(T) > 0$.*

ii) [Extension] Such a solution can be continuously extended for the entire positive real line.

iii) [Continuous dependence] $X(t)$ depends continuously on the initial data (3)*, the model parameters and the order of the derivatives $\alpha_j \in]0, 1]$, for $j = 1, 2$.*

iv) [Consistency] $X(t) \in \Omega_+$, for all $t \geq 0$.

Proof. It follows from Proposition 2.1 that $F(t, X(t))$ is continuous with respect to t and Lipschitz continuous with respect to $X(t)$. Then, the results in (Diethelm, 2010, Theorems 8.7 - 8.11) are true, which guarantees item i). Furthermore, from Proposition 2.1 item ii), the hypotheses of Theorem 3.1 in (Lin, 2007) are satisfied. This implies ii). Item iii) follows from Proposition 2.1 and the results in (Diethelm, 2010, Theorems 8.7 - 8.11). As $X(0) \in \Omega_+$, what remains to be shown in item iv) is that Ω_+ is a dynamically invariant set of the system (1). This fact follows from the same arguments presented in (Almeida, 2018, Proposition 2.1).

In the following, we aim to justify the presence of memory in the fractional dynamics model (1). In addition, we will take advantage of this opportunity to present the equations that will be used for numerical simulations to estimate the solution in Section 3.

By integrating every line of the model (1) with respect to the respective order α_j for $j = 1, 2$, we derive the Volterra equation system

$$x_i(t) = \sum_{l=1}^{6} x_i(0)\frac{t^l}{l!} + \frac{1}{\Gamma(\alpha_j)}\int_0^t (t-s)^{\alpha_j - 1} f_i(s, x_1(s), ..., x_6(s))ds. \tag{4}$$

where f_i and $x_i(t)$ represent the i-th coordinate of $F(t, X(t))$ and $X(t)$, for $i = 1, \cdots, 6$, corresponding to the right side of the model (1) and respective initial conditions (3). Thus, for $\alpha = min\{\alpha_1, \alpha_2\}$ the equation (4) can be rewritten as

$$x_i(t) = \sum_{l=1}^{6} x_j(0)\frac{t^l}{l!} + \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha - 1} \widehat{f}_i(s, x_1(s), ..., x_6(s))ds. \tag{5}$$

for $i = 1, \cdots, 6$, where, $\widehat{f}_i(s, x_1, ..., x_6) := \frac{\Gamma(\alpha)}{\Gamma(\alpha_j)}(t-s)^{\alpha_j - \alpha} f_i(s, x_1, \cdots, x_6)$.

It follows from (5), that for any coordinate $x_i(t)$ of the solution $X(t)$ for the model (1)-(3) is such that, for any $t_1 \leq t_2$,

$$\begin{aligned} x_i(t_2) - x_i(t_2) = &\frac{1}{\Gamma(\alpha_j)}\int_0^{t_1} [(t_2 - s)^{\alpha-1} - (t_1 - s)^{\alpha - 1}]\widehat{f}_i(s, x_1(s), ..., x_6(s))ds \\ &+ \frac{1}{\Gamma(\alpha_j)}\int_{t_1}^{t_2} (t_2 - s)^{\alpha-1}\widehat{f}_i(s, x_1(s), ..., x_6(s))ds\,. \end{aligned} \tag{6}$$

Note that if $\alpha = \alpha_1 = \alpha_2 = 1$, (the model has integer-order derivatives only), then the term in brackets in (6) is zero and the first of the integrals in (6) is null. Therefore, the value of the solution $X(t)$ in $t = t_2$ depends only on the values of $X(t_1)$ and the functions $f_i(X(t))$. On the other hand, if any of the $\alpha_j \neq 1$, then the first of the integrals is not identically zero, in general. Therefore, the history of the dynamics from $t = 0$ to $t = t_2$ is being evaluated when we calculate $X(t_2)$. We call this phenomenon memory.

3 SIMULATED SCENARIOS

In this section, we will show the results of several numerical simulations designed to illustrate the memory effects induced in model (1) by Caputo's fractional order derivatives. In all subsequent scenarios, the solution $X(t)$ provided by (4) of model (1) with initial conditions (3) is numerically approximated using a trapezoidal method suggested in (Garrappa, 2015), with a constant step size of $h = 10^{-1}$. This method represents a numerical integration approach for the equation (4), implemented in the Python 3.9 programming language.

In what follows, we will only present the results for the case in which the populations are symmetric $P_1 = P_2 = 1$. The parameters[2] are given by $\beta_{jj} = 0.072$ and $\gamma_j = 0.008$, for $j = 1, 2$. Furthermore, $\beta_{12} = \beta_{21} = \beta_{jj}/10$.

To examine the impact of memory on the dynamics of fake news spread, the parameters outlined earlier remain constant in all simulated scenarios, with the only variation being the order of the derivatives $\alpha_j \in]0, 1]$.

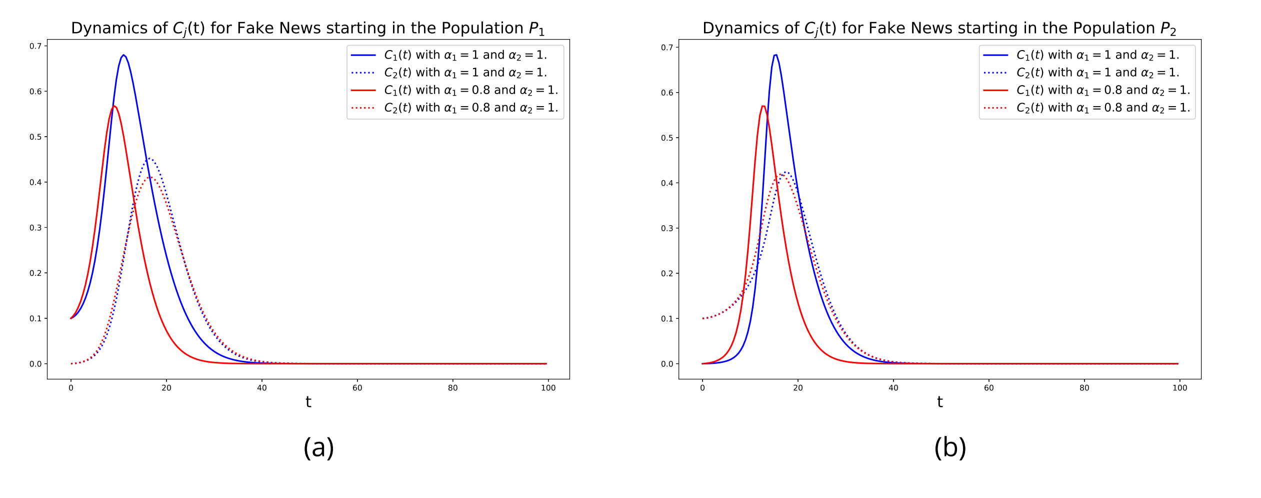
Simulated Scenario 1: This situation replicates the dynamics of false information spreading, where Population P_1 retains memory, while Population P_2 does not. For this purpose, we set $\alpha_1 = 0.8$ and $\alpha_2 = 1$. This setup is contrasted with the scenario where there is no memory, represented by $\alpha_1 = \alpha_2 = 1$.

On the left side of Figure 1, we can see the scenario in which false news originates in Population P_1 while Population P_2 is not affected at the initial time. It corresponds to the initial conditions (3) with $C_1(0) = 0.1$ and $C_2(0) = 0$ as presented in Figure 2. On the right side of Figure 1, we can observe how Fake News spreads when it originates in Population P_2, with Population P_1 not affected at the initial time. It corresponds to the initial conditions (3) with $C_1(0) = 0$ and $C_2(0) = 0.1$ as presented in Figure 1.

The results presented in Figure 1 show that the existence of memory in a population reduces the total number of people who believe in fake news, in a population that has memory (population P_1), while having little impact in a population that does not have, in this case Population P_2. Furthermore, in the case where the fake news starts in P_2 the dynamics suffers a shift to the right and a small decrease in the number of people in Population 1 who believe in the fake news (see Figure 1), compared to the case where the fake news starts in population P_1 (see Figure 2(a)). This least result is consistent with Theorem 2.1.

[2]The parameters IPI_j and IDH_j were obtained from (D'Ambrosio et al., 2021, Table 1) and represent information sourced from the United Nations Development Program in 2019 for Brazil.

Figure 1 – Simulations of model (1) in Simulated Scenario 1



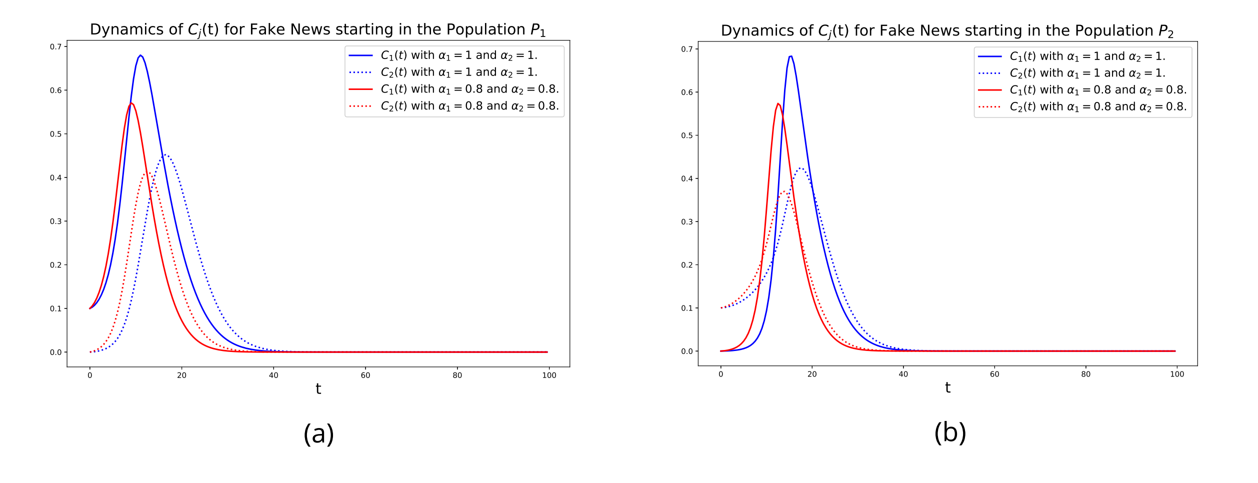
Source: The authors (2024)
Caption: The (a) figure displays the simulations of model (1) in Simulated Scenario 1 with the initial conditions (3) with $C_1(0) = 0.1$ and $C_2(0) = 0$ and (b) shows the simulations of model (1) in Simulated Scenario 1 with the initial conditions (3) with $C_1(0) = 0$ and $C_2(0) = 0.1$

Simulated Scenario 2: In this case, we investigate the dynamics concerning the comparison of scenarios where both populations lack memory ($\alpha_1 = \alpha_2 = 1$) and where both populations possess identical memory levels ($\alpha_1 = \alpha_2 = 0.8$). The simulations are presented in Figure 2.

In the scenario depicted in Figure 2(a), fake news appears in Population P_1, while Population P_2 is unaffected in the initial time (this corresponds to the initial condition 3 with $C_1(0) = 0.1$ and $C_2(0) = 0$). In Figure 2(b), we can see how Fake News spreads when it originates in Population P_2, with Population P_1 is unaffected in the initial time (this corresponds to the initial condition 3 with $C_1(0) = 0$ and $C_2(0) = 0.1$).

Since we assume that the populations are symmetric, the results presented in Figure 2 show that there is only a shift to the left or to the right in the dynamics, depending on where the fake news starts. The results in Figure 2 also show that the dynamics of the spreading of fake news in P_2 is less impacted by memory than in P_1 (compare the results in Figure 1 with Figure 2.)

Figure 2 – Simulations of model (1) in Simulated Scenario 2



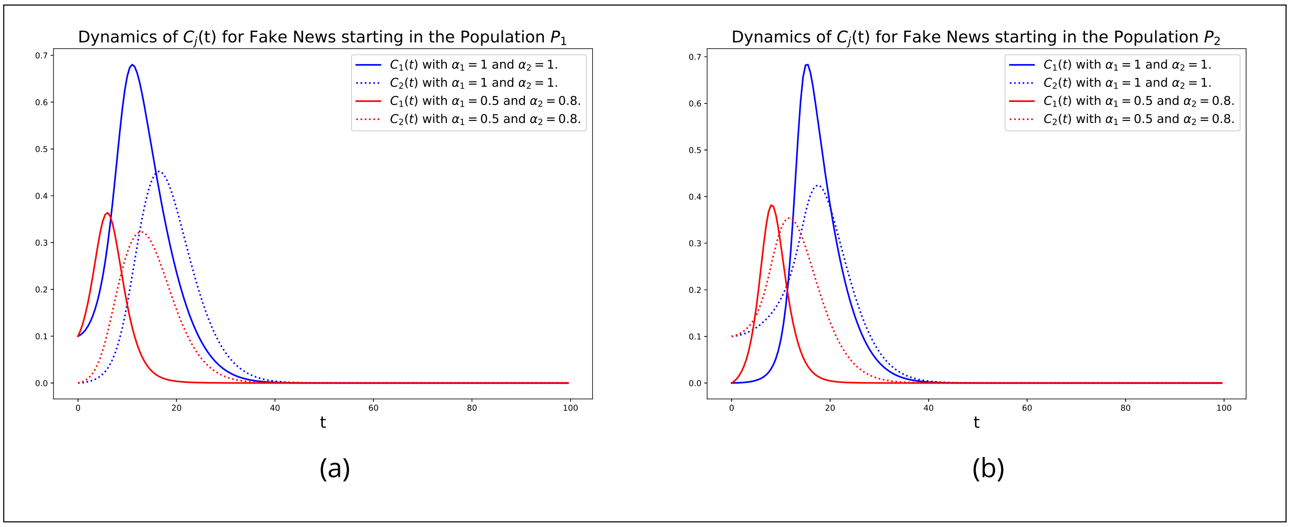
Source: The authors (2024)
Caption: The (a) figure displays the simulations of model (1) in Simulated Scenario 2 with the initial conditions (3) with $C_1(0) = 0.1$ and $C_2(0) = 0$ and (b) shows the simulations of model (1) in Simulated Scenario 2 with the initial conditions (3) with $C_1(0) = 0$ and $C_2(0) = 0.1$

Simulated Scenario 3: In the situation presented in Figure 3, we examine the impact on the dynamics of the spread of fake news when Population P_1 has a memory parameter of $\alpha_1 = 0.5$, which is significantly higher than the memory of Population P_2, which is characterized by $\alpha_2 = 0.8$.

In the simulations presented in Figure 3(a), the Fake News originates in Population P_1 while Population P_2 is unaffected in the initial time (this corresponds to the initial condition 3 with $C_1(0) = 0.1$ and $C_2(0) = 0$). In Figure 3(b), we can see how false news spreads when it starts in population P_2, with population P_1 unaffected at the initial times (this corresponds to the initial condition 3 with $C_1(0) = 0$ and $C_2(0) = 0.1$).

The findings illustrated in Figure 3 indicate that having high levels of memory in communities leads to a notable decrease in the total number of individuals who endorse fake news. Furthermore, when the misinformation originates in P_2, there is a downward shift in the dynamics and a slight increase in the proportion of individuals in Population 2 who fall for the false news, as shown in Figure 3(b), in contrast to when the false news originates in Population P_1, as shown in Figure 3(a).

Figure 3 – Simulations of model (1) in Simulated Scenario 3



Source: The authors (2024)
Caption: The (a) figure displays the simulations of model (1) in Simulated Scenario 3 with the initial conditions (3) with $C_1(0) = 0.1$ and $C_2(0) = 0$ and (b) shows the simulations of model (1) in Simulated Scenario 3 with the initial conditions (3) with $C_1(0) = 0$ and $C_2(0) = 0.1$

3.1 The inverse of the stiffness ratio and the speed of the spread of fake news

In D'Ambrosio et al. (2022); D'Ambrosio et al. (2021), the authors link the rate at which fake news spreads and recovers to the reciprocal of the stiffness ratio of the Jacobian matrix of their model, which varies over time. Here, we demonstrate through numerical analysis in this section that the assertions made by the authors in D'Ambrosio et al. (2022); D'Ambrosio et al. (2021) hold valid solely when dealing with a single homogeneous population.

Let $JF(t, X(t))$ denote the Jacobian matrix of model (1), as specified in Proposition 2.1. The inverse of the stiffness ratio that varies with time is expressed as

$$\tau(t, X(t)) := \frac{|\lambda_{min}(t, X(t))|}{|\lambda_{max}(t, X(t))|}. \tag{7}$$

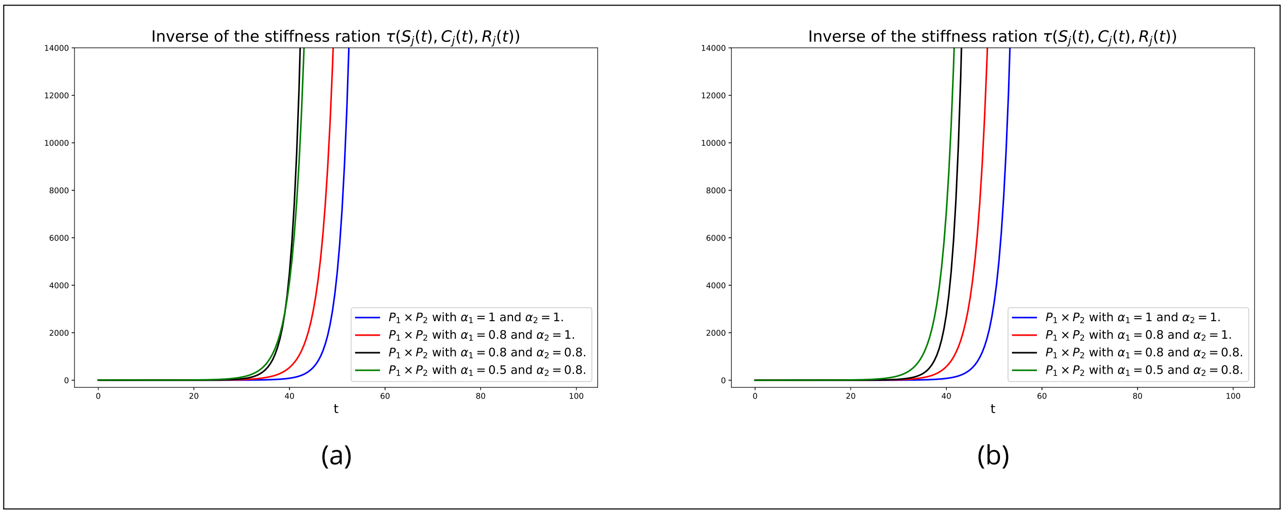
The quantities $\lambda_{max}(t, X(t))$ and $\lambda_{min}(t, X(t))$ represent the highest and lowest non-zero eigenvalues of the Jacobian matrix $JF(t, X(t))$ at any time $t \geq 0$.

In Figure 4, we show the corresponding evolution of $\tau(t, X(t))$ for the interacting populations and parameters as in **Simulated Scenarios 1 to 3**.

The findings illustrated in Figure 4 indicate that, in contrast to the scenario examined in D'Ambrosio et al. (2022); D'Ambrosio et al. (2021), when dealing with populations that interact and possess memory, higher values of $\tau(t, X(t))$ are initially observed, depending on the initial location of the fake news. In fact, the values of $\tau(t, X(t))$ increase monotonically with the level of memory in the case where the fake news starts in the population P_2, as depicted in Figure 4(b), but are not valid for the scenarios where the fake news starts in Figure 4.

Consequently, the utility of $\tau(t, X(t))$ as an indicator of the speed of fake news diffusion is not applicable to interacting populations (also refer to Travessini De Cezaro et al. (2023)).

Figure 4 – The inverse of the stifness ration $\tau(t, X(t))$, with interacting populations for the **Simulated Scenarios 1 to 3**



Source: The authors (2024)
Caption: The (a) figure displays the $\tau(t, X(t))$ for the **Simulated Scenario 1 to 3** with the fake news starting in population P_1 and (b) shows the $\tau(t, X(t))$ for the **Simulated Scenario 1 to 3** with the fake news starting in population P_2

4 CONCLUSIONS AND FUTURE PERSPECTIVES

In this contribution, we propose a reinterpretation of a multi-fractional SIR-type compartmental model to describe the dynamics of Fake News dissemination behavior between two distinct populations that share information. The results obtained in the simulations presented indicate that the memory effect, when compared to the entire case (without memory), is preponderant in reducing the proportion and speed of propagation of Fake News. In particular, Scenario 1 suggests that the memory factor impacts the relationship between the Fake News dynamics between the Contaminated

$C_j(t)$ between these two populations. The simulations presented in Scenario 2 indicate that the memory effect does not depend on the initial conditions. Finally, in Scenario 3 we observe that the greater the memory of a population, the propagation interval, and the proportion of fake news are smaller than in populations that have a great memory. Moreover, we have shown that the inverse of the stifness ratio cannot be used as a measure for the velocity in with the fake news that vanishes in model with interacting populations with memory.

The study with non-symmetric populations and multi-population models will be the result of future investigations.

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Author contributions

1 – Adriano De Cezaro (Corresponding Author)
Mathematician
https://orcid.org/0000-0001-8431-9120 • decezaromtm@gmail.com
Contribution: Conceptualization; Methodology; Writing – Original Draft Preparation

2 – Fabiana Travessini De Cezaro
Mathematician
https://orcid.org/0000-0001-9401-5315 • fabi.travessini@gmail.com
Contribution: Conceptualization; Methodology; Writing – Original Draft Preparation

3 – Luverci do Nascimento Ferreira
Mathematician
https://orcid.org/0000-0002-0662-9112 • luverci@gmail.com
Contribution: Conceptualization; Methodology; Writing – Original Draft Preparation

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