CIÊNCIA e NATURA

ISSN 2179-460X

OPEN ACCESS

Ci. e Nat., Santa Maria, v. 46, n. spe. 1, e87212, 2024 • https://doi.org/10.5902/2179460X87212

Submitted: 03/22/2024 • Approved: 06/25/2024 • Published: 11/07/24

ERMAC e ENMC

Structural Damage Identification via Bayesian Inference with a New Hierarchical Modeling and Spike-and-Slab Prior

Identificação de Danos Estruturais via Inferência Bayesiana com uma Nova Modelagem Hierárquica e Priori Spike-and-Slab

Josiele da Silva Teixeira[I], **Helio dos Santos Migon**[II], **Leonardo Tavares Stutz**[I], **Diego Campos Knupp**[I], **Antônio José da Silva Neto**[I],

[I]Department of Mechanical Engineering and Energy, Polytechnic Institute, Rio de Janeiro State University, IPRJ/UERJ, Brazil

[II]Institute of Mathematics, Federal University of Rio de Janeiro, Rio de Janeiro, Brazil

ABSTRACT

The present work aims to formulate and solve the inverse problem of structural damage identification using Bayesian Inference. In the solution of the direct problem, the Finite Element Method (FEM) is considered. The modeling of the damage field is performed through the cohesion parameter, which continuously describes the integrity of the structure. The damage identification problem is formulated as an inverse parameter estimation problem, where the posterior probability distribution of the cohesion parameters is sampled using the Adaptive Markov Chain Monte Carlo method and a Spike-Slab prior, adopting a novel hierarchical modeling approach for the inverse problem and an appropriate prior distribution that naturally models the available information about the parameters of interest.

Keywords: Bayesian Inference, Adaptive Markov Chain Monte Carlo Method, Spike-and-Slab Prior

RESUMO

O presente trabalho tem como propósito formular e resolver o problema inverso de identificação de danos estruturais empregando Inferência Bayesiana. Na solução do problema direto é considerado o Método de Elementos Finitos (MEF). A modelagem do campo de dano é realizada por meio do parâmetro de coesão, o qual descreve continuamente a integridade da estrutura. O problema de identificação de danos é formulado como problema inverso de estimação de parâmetros, onde a distribuição de probabilidade *a posteriori* dos parâmetros de coesão é amostrada empregando o Método de Monte Carlo com Cadeias de Markov Adaptativo com priori do tipo Spike-Slab, onde foi adotada uma nova modelagem hierárquica do problema inverso e uma distribuição a priori apropriada que modela naturalmente as informações disponíveis sobre os parâmetros de interesse.

Palavras-chave: Inferência Bayesiana, Método de Monte Carlo com Cadeias de Markov Adaptativo, Priori Spike-and-Slab

1 INTRODUCTION

The interest of the scientific community in developing advanced methodologies for damage assessment and identification in structures of great importance to society is both growing and crucial, as it aims to ensure the safety, durability, and efficiency of a wide range of civil infrastructures, including buildings, bridges, dams, airplanes, and others. These structures are daily subjected to a variety of factors that can impair their performance, such as excessive loads, fatigue, corrosion, and impacts. These factors can lead to progressive damage that is not visible to the naked eye and can even evolve into tragic scenarios with loss of life and significant material losses. In this context, the development and refinement of precise and non-invasive techniques for early-stage structural damage assessment are of paramount importance (Link and Weiland, 2009; Pandey and Biswas, 1994; Teixeira et al., 2020). They not only contribute to proactive maintenance and extension of the service life of structures through the preventive implementation of efficient maintenance and repair actions, but also play a crucial role in preventing catastrophic failures, saving financial resources, and promoting resilient and sustainable infrastructure for future generations.

Given the relevance of continuous structural monitoring, this work proposes the use of Bayesian Inference as a technique for solving the inverse problem of structural damage identification (Migon et al., 2014; Tanner, 1993). This approach has been widely applied to solve problems in various research areas, including engineering, medicine, biology, astrophysics, meteorology, and others (Albani et al., 2023; Taghizadeh et al., 2020; Völkel et al., 2021). Its main advantage lies in its ability to treat with statistical rigor the uncertainties inherent in problem modeling and the experimentation process, as well as the ease of incorporating all prior information about the parameters to be estimated into the damage identification process.

In this approach, the variables of interest of the damage identification problem and all available prior information are modeled as random variables through probability density functions (PDFs) (Malakoff, 1999; Teixeira et al., 2016). The appropriate choice of the prior plays a fundamental role in solving the inverse problem, allowing for more accurate and reliable estimates. This prior information, often derived from previous knowledge, experience, or well-founded theories, allows for the incorporation of a solid probabilistic structure into the statistical model, resulting in a more accurate posterior distribution that reflects uncertainty more appropriately, making the estimate more robust and reliable. Additionally, incorporating prior information can help reduce the dimension of the search space, increasing computational efficiency and avoiding improper estimates. In the context of damage identification, Spike-and-Slab priors are particularly useful, as they are able to model prior knowledge about the cohesion parameters in a simple and natural way (Andersen et al., 2017; Hernández-Lobato et al., 2013; Mitchell and Beauchamp, 1988). At the end of the damage identification process, the estimates are obtained via sampling of the posterior PDF of the parameters of interest. In this work, the Finite Element Method (FEM) is used to solve the direct problem, which is in turn parameterized by the cohesion parameter of the structure.

Bayesian Inference will be employed to solve the inverse problem of damage identification, using both conventional Markov Chain Monte Carlo method (MCMC) and adaptive MCMC method with Spike-and-Slab prior (adpMCMC-SSP). Although conventional MCMC methods are very efficient in solving inverse problems, they may experience some difficulties in adequately exploring the parameter search space. To overcome such problems, the literature shows the use of adaptive techniques that significantly improve the efficiency of MCMC methods. These adaptations are performed via careful adjustment of the auxiliary probability distribution used in the Metropolis-Hastings algorithm. Therefore, the adaptive MCMC method with Spike-and-Slab prior (adpMCMC-SSP) will also be employed. The results obtained, as well as the computational cost of the employed methodologies, will be compared to each other to verify their effectiveness in solving the structural damage identification problem. A set of numerical results is presented, considering two damage scenarios and one noise level for the adopted solution methodologies.

2 DIRECT PROBLEM MODELING

In modeling the direct problem, the structural integrity is considered to be continuously described, in the body domain, by a structural parameter called the cohesion parameter $\boldsymbol{\beta}(x)$ (Stutz et al., 2005,1; Teixeira et al., 2020). This parameter describes the bonding between material points of the structure and can be interpreted as a measure of the local cohesion state of the material, where $\beta \in [0, 1]$. When all bonds between material points are preserved, $\beta = 1$ is obtained. When a local rupture occurs, $\beta = 0$ is obtained, and when there is damage in the structure, $0 < \beta < 1$ is obtained. It is widely accepted in the literature that structural damage significantly alters only the stiffness properties of the structure. Therefore, the bending stiffness of a beam is defined as

$$E(x)I(x) = \beta(x)E_0I_0 \tag{1}$$

where E_0 and I_0 are, respectively, the nominal Young modulus and the nominal area moment of inertia of the cross section. Therefore, the stiffness matrix of the structure may be written as

$$\mathbf{K}(\beta) = \int_\Omega \beta(x)E_0I_0\mathbf{H}^T(x)\mathbf{H}(x)d\Omega \tag{2}$$

where $\mathbf{H}$ is the discretized differential operator and β represents the cohesion field in the elastic domain Ω of the structure. Therefore, the cohesion parameter represents any change, caused by the presence of structural damage, in the bending stiffness of the structure. The nodal cohesion parameter vector is defined as $\boldsymbol{\beta} = [\beta_1, \beta_2, \ldots, \beta_{n_p}]^T$, where n_p is the total number of cohesion parameters in the model.

Considering a system with n_p degrees of freedom, the equation of motion obtained by the Finite Element Method (Reddy, 1984) is given by

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{D}\dot{\mathbf{u}}(t) + \mathbf{K}(\boldsymbol{\beta})\mathbf{u}(t) = \mathbf{f}(t) \tag{3}$$

where $\mathbf{u}$ is the vector of generalized coordinates, $\mathbf{M}$ is the mass matrix, $\mathbf{D}$ is the damping matrix, $\mathbf{K}(\boldsymbol{\beta})$ is the stiffness matrix, and $\mathbf{f}$ is the loading vector.

3 INVERSE PROBLEM FORMULATION

Inverse problem modeling starts from experimental observation and aims to estimate unknown quantities by adjusting computational models. This approach is a very effective tool and has been increasingly used to solve inverse problems in the most diverse areas (Ozisik and Orlande, 2021). From the Bayesian perspective, the solution of an inverse problem, given the posterior experimental observations $\boldsymbol{Z}_E$, is a probability density function of $\boldsymbol{\beta}$, which may be written, according to Bayes' theorem, as (Tanner, 1993)

$$p(\boldsymbol{\beta}|\mathbf{Z}_E) = \frac{p(\mathbf{Z}_E|\boldsymbol{\beta})p(\boldsymbol{\beta})}{p(\mathbf{Z}_E)} \tag{4}$$

where $p(\boldsymbol{\beta})$ is the prior probability density function of β, $p(\mathbf{Z}_E)$ is the marginal density of the experimental data and $p(\mathbf{Z}_E|\boldsymbol{\beta})$ is the likelihood function. Samples from the posterior distribution of interest, which is intractable to simulate directly, may be obtained via Markov Chain Monte Carlo (MCMC) methods. The general idea of MCMC methods is to simulate random samples in the parameter domain $\boldsymbol{\beta}$, such that the stationary distribution of the samples converges to the posterior distribution $p(\boldsymbol{\beta}|\mathbf{Z}_E)$. Specific algorithms are used for this purpose. In this work, the Metropolis-Hastings algorithm was used, which makes use of an auxiliary probability density function q, from which it is easy to obtain sample values. Assuming that the chain of a given cohesion parameter is in a state $\boldsymbol{\beta}^{t-1}$, a new candidate value $\boldsymbol{\beta}^*$ will be generated from the auxiliary distribution $q(\boldsymbol{\beta}^*|\boldsymbol{\beta}^{t-1})$, given the current state of the chain $\boldsymbol{\beta}^{t-1}$. The new value $\boldsymbol{\beta}^*$ may be accepted with probability given by the Hastings Ratio (Hastings, 1970).

$$\gamma = \min\left[1, \frac{p(\boldsymbol{\beta}^*|\mathbf{Z}_E)q(\boldsymbol{\beta}^{t-1}|\boldsymbol{\beta}^*)}{p(\boldsymbol{\beta}^{t-1}|\mathbf{Z}_E)q(\boldsymbol{\beta}^*|\boldsymbol{\beta}^{t-1})}\right] \tag{5}$$

3.1 Hierarchical Modeling with Spike-and-Slab Priors

Spike-and-Slab priors were initially proposed by Mitchell and Beauchamp (1988) and are generally defined as mixtures of probability distributions. They consist of a mixture of a point mass at x_p called the ”Spike” and another distribution, known as the ”Slab.” In the inverse problem modeling adopted in this work, the cohesion parameters $\boldsymbol{\beta}$ are considered as unobservable quantities in the interval $[0, 1]$, but with a mass point at $\beta = 1$, characterizing the regions of the structure that are intact. In this sense, a spike-and-slab prior is very appropriate and can be naturally used to identify which cohesion parameters are associated with intact or damaged regions. That is, if $z_j = 1$ then node j has no structural damage, so $p(\beta_j = 1|z_j = 1) = 1$. On the other hand, if $z_j = 0$ then there is a probability of the parameter being associated with a damaged region given by $p(0 < \beta_j < 1|z_j = 0) = Be(\beta_j; a, b)$. Therefore, the hierarchical model for the damage identification problem considering a Spike-and-Slab prior is given by

$$\begin{aligned} \mathbf{Z}_E &\sim N[\mathbf{F}(\boldsymbol{\beta}), \phi^{-1}\mathbf{I}] \\ \beta_j|z_j &\sim (1 - z_j)\, \mathit{Be}(\beta_j; a, b) \,+\, z_j\, \delta_j \\ \boldsymbol{z}|\pi &\sim \mathit{Ber}(\pi) \\ \pi &\sim \mathit{Be}(e, d) \end{aligned}$$

where $\mathbf{F}(\boldsymbol{\beta})$ is the solution of Eq. 3 obtained via Finite Element Method (FEM); δ_j is a Dirac delta function, which characterizes a point mass at $\beta_j = 1$. In other words, $\delta_j = 1$ if $\beta_j = 1$ and $\delta_j = 0$ if $0 < \beta_j < 1$; *Be*$(\beta_j; a, b)$ represents a Beta distribution in the interval $(0, 1)$ with parameters a and b, with mean $\mu_\beta = \frac{a}{a+b}$ and variance $\sigma^2_\beta = \mu_\beta(1-\mu_\beta)/(a+b+1)$; *Ber*$(\pi)$ is a Bernoulli distribution with success probability π. In this work, success means the nodal point j of the structure is intact. Therefore, π represents the proportion of undamaged parameters ($\beta_k = 1$, $k = 1, \cdots, n_{un}$, where n_{un} is the number of undamaged parameters) relative to the total number of unknown parameters. Finally, $Be(e, d)$ is another Beta distribution in the interval $(0, 1)$ with parameters e and d with mean $\mu_\pi = \frac{e}{e+d}$ and variance $\sigma^2_\pi = \mu_\pi(1 - \mu_\pi)/(e + d + 1)$.

It is important to highlight that the inverse problem modeling approach for structural damage identification used in this work is recent. Furthermore, a hierarchical statistical model that considers the impulse response of the structure, in the time domain, and a Spike-and-Slab distribution mixture to naturally characterize the available prior information on the cohesion parameters has not yet been found in the specialized literature.

3.2 Adaptive Markov Chain Monte Carlo Method

Recent literature has shown that the effectiveness of the Markov Chain Monte Carlo method can be significantly improved through careful adjustment of the support of the auxiliary distribution q used in the Metropolis-Hastings algorithm (Teixeira et al., 2020).

In this work, for each cohesion parameter, a normally distributed random walk proposal was considered. To implement this, the cohesion parameter values in the interval $[0,1]$ were transformed to the real line, that is, $\xi_j = \ln\left(\frac{\beta_j}{1-\beta_j}\right)$ with $j = 1, ..., n_p$. Therefore, $\boldsymbol{\xi}^k = \boldsymbol{\xi}^{k-1} + \boldsymbol{\epsilon}$, where $\boldsymbol{\xi}$ is an n_p-dimensional vector, $\boldsymbol{\epsilon} \sim N[\mathbf{0}, \mathbf{V}]$, k is the iteration index, and **V** is the variance-covariance matrix. Furthermore, **V** is assumed to be diagonal, with each diagonal element ν_j, with $j = 1, ..., n_p$, representing the variance of the j-th auxiliary distribution used to sample the respective cohesion parameter. In this work, an adaptation of the **V** matrix was used, which starts from t_0 states and is given as follows

$$\nu_j^t = \begin{cases} \nu_j^0 & \text{se} \quad t < t_0 \\ \gamma_{sd}^t ({\xi_j}^{t-1} - M_j)^2 & \text{se} \quad t_0 \leq t \leq N_{burn-in} \end{cases} \tag{6}$$

for $j = 1, ..., n_p$, where $t-1$ is the current state of the chain, γ_{sd}^t is a real constant, M_j is the average value of the j-th cohesion parameter, calculated considering the last m iterations of Markov chain. In order to keep the acceptance rate (τ) within the interval $\bar{\tau} \in [10\%, 50\%]$, an adaptation was incorporated into the constant γ_{sd}, which starts at $\gamma_{sd}^0 = 1$ and is adapted after t_0 states as follows

$$\gamma_{sd}^t = \begin{cases} 1 & \text{se} \quad t < t_0 \\ \gamma_{sd}^{t-1}(1-\lambda^t) & \text{se} \quad \bar{\tau} < 0,1 \quad \text{e} \quad t_0 \leq t \leq N_{burn-in} \\ \gamma_{sd}^{t-1}(1+\lambda^t) & \text{se} \quad \bar{\tau} > 0,5 \quad \text{e} \quad t_0 \leq t \leq N_{burn-in} \end{cases} \tag{7}$$

Additionally, it was considered $\lambda^t = L_{low} + (L_{up} - L_{low})r^j$, where $L_{low} = 0.05$, $L_{up} = 0.15$, and r^t is a random number between 0 and 1. After the burn-in period, both adaptations are no longer performed, and the standard deviation remains constant and equal to the deviation obtained up to the burn-in.

4 NUMERICAL RESULTS

To assess the effectiveness of the proposed approach, a simply supported Euler-Bernoulli beam was considered. The both conventional Markov Chain Monte Carlo (MCMC) method and adaptive MCMC with Spike-and-Slab prior (adpMCMC-SSP) method were employed to solve the inverse problem of damage identification. The beam was spatially discretized into 24 finite elements. The physical and geometrical properties of the beam are given in the table 1. The experimental data was simulated

Table 1 – Physical and geometrical properties of the beam

Length (L)	Height (h_0)	Width (w)	Mass density (ρ)	Young Modulus (E)
1.46 m	0.008 m	0.0762 m	7.85×10^3 kg/m^3	207 GPa

Source: the authors (2024)

from the system impulse response, given by the FEM in terms of acceleration, for a prescribed value of $\boldsymbol{\beta}$. The added noise was calculated considering that the sensor precision is given by

$$\phi^{-1} = \max\left(s \cdot \left|\mathbf{F}(\boldsymbol{\beta}_p)\right|\right) \tag{8}$$

where s is a percentage that varies in the interval $[0.005, 0.02]$, $\mathbf{F}(\boldsymbol{\beta}_p)$ is the structural response, Eq. 3, considering a prescribed value for the cohesion vector $\boldsymbol{\beta}$.

In this work, only one noise level with $s = 0.01$ was used for the simulations. To obtain the time-domain data used in the damage identification process, a Dirac impulse was considered at $x = 0.2433$ m and the impulse response, given in terms of acceleration, was measured at the same position. In the present work, the damage was modeled by means of a V-shaped notch in the beam, where the damage position corresponds to the notch tip position (Teixeira et al., 2020). Therefore, considering Eq. (1), and assuming that structural damage affects only the geometrical properties of the beam, i.e., $E(x) = E_0$, the cohesion parameter at a point x is given by

$$\beta(x) = \left(\frac{h(x)}{h_0}\right)^3 \tag{9}$$

where $h(x)$ is the thickness of the beam at point x and h_0 is the corresponding nominal thickness. Two damage scenarios were considered. The first consists of the presence of two damage at the positions $x = 0.547$ m and $x = 1.277$ m, which correspond to the nodes 10 and 22 of the FE mesh, with relative height $h(x)/h_0 = 0.9$. The second scenario consists of the presence of two damages at positions $x = 1.156$ m and 1.277 m, which correspond to nodes 20 and 22 of the FE mesh, respectively, and both with a relative height $h(x)/h_0 = 0.9$. Thus, 3 case studies were defined, as shown in the Table 2.

Table 2 – Analyzed cases

Case	Method	Scenario
1	MCMC	1
2	adpMCMC-SSP	1
3	adpMCMC-SSP	2

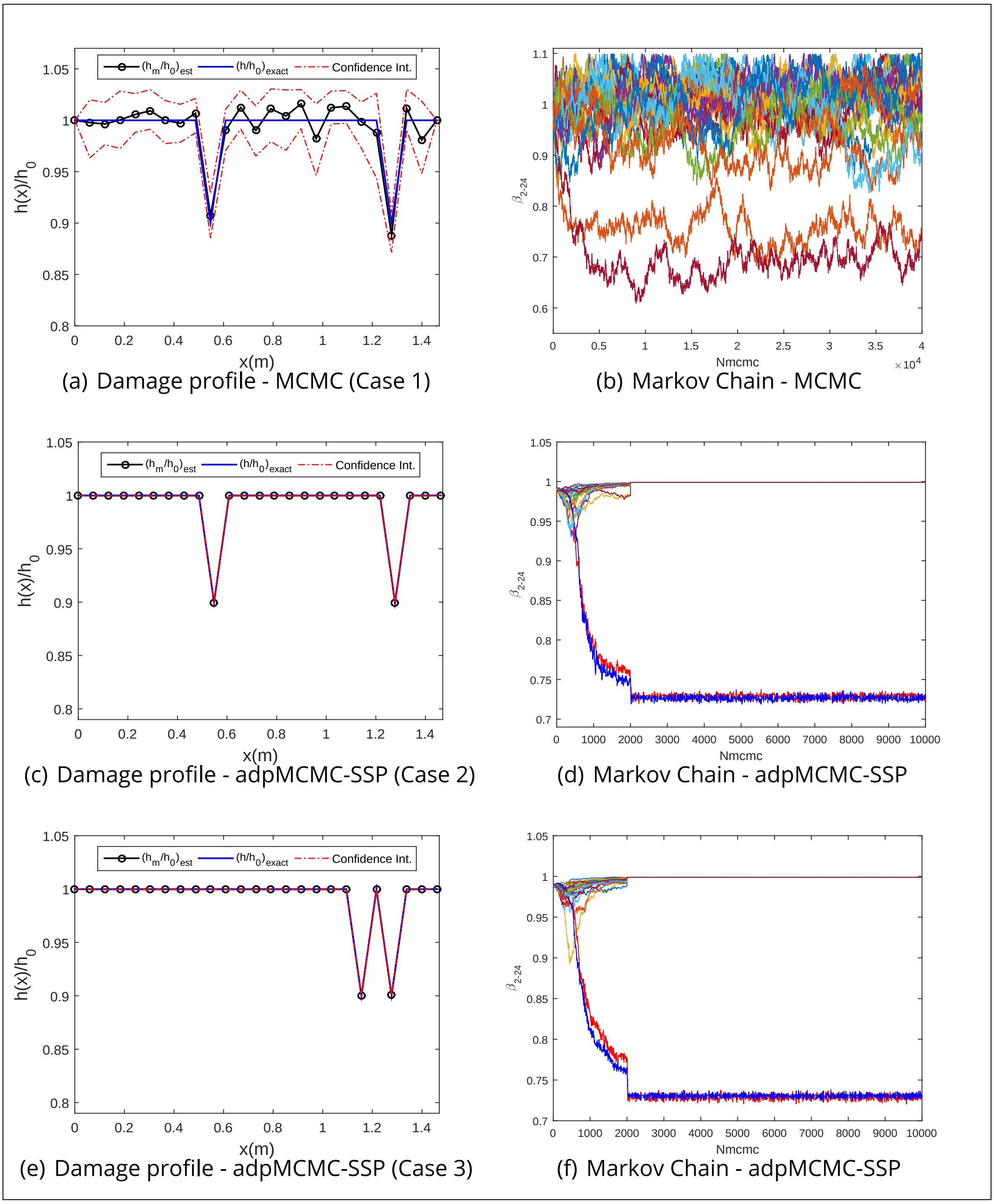
Source: the authors (2024)

For the conventional MCMC method, constant variances $\nu_j = 0.005, j = 2, ..., 23$, and for the AdpMCMC-SSP method, the adaptation process starts after $t_0 = 5,000$ states using Eq. (6)

In the composition of the Spike-and-Slab mixture distribution, defined as the prior distribution of the cohesion parameters, a Beta distribution with parameters $a = 3,\ b = 1.61538$ was considered and consequently mean $\mu_\beta = 0.65$. For the prior of π, another Beta distribution with parameters $e = 4.5,\ d = 0.5$ was considered, where $\mu_\pi = 0.91304$ represents the proportion of intact cohesion parameters.

Figure 1 presents the exact and estimated damage fields, together with the corresponding 95% credible intervals and Markov chains for the estimated parameters. Table 3 presents the statistical properties of the posterior distribution of the estimated cohesion parameters. The estimated mean (μ_{est}), standard deviation (σ_{est}), relative error between the estimated mean value and the exact value, acceptance rate (τ) and also the Root Mean Squared Error (E_{RMS}) are also presented. Table 4 presents the estimated 95% credibility interval (CI). Markov chains with $40,000$ states were considered for the MCMC and burn-in of $20,000$ states. For the adpMCMC-SSP method, chains with $10,000$ states, a burn-in of $5,000$ states, and an adaptation starting at $t_0 = 2,000$ states were considered. This significant difference in the chain lengths is associated with the convergence speed obtained by each methodology adopted.

Figure 1 – Damage Identification results and Markov Chains for Cases 1 and 2



Source: the authors (2024)

For Case 1, it may be observed from Figure 1 and Tables 3 and 4 that although the conventional MCMC method successfully located the actual damage in the structure, it also identified false damage in the structure. Additionally, it may be observed that even with a Markov chain four times larger than those used by the

Table 3 – Estimated statistical properties ($\beta_{exact} = 0.729$)

Method	Case	β	μ_{est}	σ_{est}	E_r	τ	E_{RMS}
MCMC	1	β_{10}	0.7466	0.0286	0.0241	31%	0.0305
		β_{22}	0.7002	0.0351	0.0395		
adpMCMC-SSP	2	β_{10}	0.7283	0.0027	0.0009	20%	0.0009
		β_{22}	0.7268	0.0026	0.0030		
adpMCMC-SSP	3	β_{20}	0.7289	0.0026	0.0001	21%	0.0009
		β_{22}	0.7305	0.0021	0.0021		

Source: the author (2024)

Table 4 – Credibility intervals

Method	Case	β	CI 95%
MCMC	1	β_{10}	[0.6945; 0.8015]
		β_{22}	[0.6621; 0.7536]
adpMCMC-SSP	2	β_{10}	[0.7235; 0.7325]
		β_{22}	[0.7226; 0.7313]
adpMCMC-SSP	3	β_{20}	[0.7247; 0.7332]
		β_{22}	[0.7272; 0.7341]

Source: the authors (2024

adpMCMC-SSP, the conventional method did not present chain convergence, this demonstrates the difficulty that the conventional method has in adequately and efficiently traversing the parameter search space. On the other hand, for Case 2, it is evident that the adpMCMC-SSP method obtained significantly more accurate results, where a much faster chain convergence may be observed, around $2,000$ states, demonstrating the need for a smaller chain and consequently a much lower computational cost, approximately 3 minutes and 21 seconds, four times less than the adpMCMC-SSP time, which was around 12 minutes and 45 seconds. Furthermore, it presented the lowest values of E_{RMS} and relative errors E_r, indicating that the estimated values are closer to the exact values. Good acceptance rates are also observed for both methodologies. From Table 4, it can be verified that, for all cases, the exact values are contained within the intervals obtained with 95% credibility. However, the intervals obtained by adpMCMC-SSP are smaller, indicating with a high degree of credibility that the probability of the exact values belonging to the support of the estimated PDFs is higher.

For Case 3, where two damages located close to each other were considered, only the adpMCMC-SSP method was employed, as the conventional MCMC method faced difficulties in adequately estimating the damage field, even considering chains with more states. Thus, it is noticeable that even for this more challenging estimation case, adpMCMC-SSP continued to present excellent results, with rapid chain convergence, approximately 2,000 states, accurate identification of both the location and intensity of the damages, and low values of E_{RMS} and relative errors E_r, in addition to the exact values contained within the credibility intervals, where their lengths are of the order of 10^{-3} for parameters associated with damaged regions.

It is important to emphasize that, based on the results presented in this work, it may be concluded that the proposed adaptive technique with Spike-and-Slab prior was highly successful in overcoming the difficulties presented by the conventional MCMC method. This demonstrates its effectiveness in identifying structural damage and reducing computational cost.

5 CONCLUSIONS

In this study, the structural damage identification problem was solved using a Bayesian approach. The formulation of the direct problem was presented, and its solution was obtained through the Finite Element Method (FEM). The structural damage field was modeled using the structural cohesion parameter $\boldsymbol{\beta}$. The inverse problem solution was obtained by the conventional Markov Chain Monte Carlo method and adaptive Markov Chain Monte Carlo method with Spike-and-Slab prior. The method proposed here combined hierarchical modeling, an adaptive technique, and a Spike-and-Slab mixture distribution as prior information for the parameters of interest. The results obtained by this method were more accurate compared to the conventional MCMC method, successfully identifying the existing damage in the structure. A high convergence speed of the Markov chains was also observed, implying the need for fewer states, which reduced its execution time by approximately 75%

compared to the conventional MCMC method. This demonstrates its attractiveness for applications in problems with high associated costs.

ACKNOWLEDGEMENTS

The authors acknowledge the financial support provided by the Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ), the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001.

References

Albani, R. A., Albani, V. L., Gomes, L. E. S., Migon, H. S., and Silva Neto, A. J. (2023). Bayesian inference and wind field statistical modeling applied to multiple source estimation. *Environmental Pollution*, 321:121061.1–11.

Andersen, M. R., Vehtari, A., Winther, O., and Hansen, L. K. (2017). Bayesian inference for spatio-temporal spike-and-slab priors. *Journal of Machine Learning Research*, 18:1–58.

Hastings, W. K. (1970). Monte Carlo sampling methods using Markov Chain and their applications. *Biometrika*, 57:97–109.

Hernández-Lobato, D., Hernández-Lobato, J. M., and Dupont, P. (2013). Generalized spike-and-slab priors for bayesian group feature selection using expectation propagation. *Journal of Machine Learning Research*, 14:1891–1945.

Link, M. and Weiland, M. (2009). Damage identification by multi-model updating in the modal and in the time domain. *Mech. Syst. Signal Process*, 23:1734–1746.

Malakoff, D. M. (1999). Bayes offers “new” way to make sense of numbers. *Science*, 268:1460–1464.

Migon, H. S., Gamerman, D., and Louzada, F. (2014). *Statistical Inference - An Integrated Approach*, volume 1. FL: Chapman Hall, Boca Raton, 2 edition.

Mitchell, T. J. and Beauchamp, J. J. (1988). Bayesian variable selection in linear regression. *Journal of the American Statistical Association*, 83:1023–1032.

Ozisik, M. N. and Orlande, H. R. B. (2021). *Inverse Heat Transfer*. CRC Press, Boca Raton, 2 edition.

Pandey, A. K. and Biswas, M. (1994). Damage detection in structures using changes in flexibility. *Journal of Sound and Vibration*, 169(1):3–17.

Reddy, J. N. (1984). *An Introduction to the Finite Element Method*. McGraw-Hill.

Stutz, L. T., Castello, D. A., and Rochinha, F. A. (2005). A flexibility-based continuum damage identification approach. *Journal of Sound and Vibration*, 279:641–667.

Stutz, L. T., Rangel, I. C. S. S., Rangel, L. S., Corrêa, R. A. P., and Knupp, D. C. (2018).

Structural damage identification built on a response surface model and the flexibility matrix. *Journal of Sound and Vibration*, 434:284–297.

Taghizadeh, L., Karimi, A., Stadlbauer, B., Weninger, W. J., Kaniusas, E., and Heitzinger, C. (2020). Bayesian inversion for electrical-impedance tomography in medical imaging using the nonlinear poisson–boltzmann equation. *Computer Methods in Applied Mechanics and Engineering*, 365:112959.

Tanner, M. A. (1993). *Tools for Statistical Inference*. Springer Verlag, New York.

Teixeira, J. S., Stutz, L., Knupp, D. C., and Silva Neto, A. J. (2016). Structural damage identification via time domain response and Markov Chain Monte Carlo method. *Inverse Problems in Science and Engineering*, 25(6):909–935.

Teixeira, J. S., Stutz, L. T., Knupp, D. C., and Neto, A. J. S. (2020). A new adaptive approach of the metropolis-hastings algorithm applied to structural damage identification using time domain data. *Applied Mathematical Modelling*, 82:587–606.

Völkel, S. H., Krüger, C. J., and Kokkotas, K. D. (2021). Bayesian inverse problem of rotating neutron stars. *Physical Review D*, 103:083008.

Author contributions

1 – Josiele da Silva Teixeira (Corresponding Author)
Postdoctoral Researcher at Polytechnic Institute of Rio de Janeiro State University, D.Sc. in Computational Modeling
https://orcid.org/0000-0002-1400-7938 • jteixeira@iprj.uerj.br
Contribution: Conceptualization, Investigation, Methodology, Software, Validation, Visualization, Writing – Original Draft, Writing – Review & Editing

2 – Helio dos Santos Migon
Professor Emeritus at Federal University of Rio de Janeiro, Ph. D. in Statistics
https://orcid.org/0000-0003-4913-3210 • migon@im.ufrj.br
Contribution: Conceptualization, Investigation, Methodology, Visualization, Writing – Review & Editing

3 – Leonardo Tavares Stutz
Associate Professor at Polytechnic Institute of Rio de Janeiro State University, Ph.D. in Mechanical Engineering
https://orcid.org/0000-0003-3005-765X • ltstutz@iprj.uerj.br
Contribution: Conceptualization, Investigation, Methodology, Visualization, Writing – Review & Editing

4 – Diego Campos Knupp
Assistant Professor at Polytechnic Institute of Rio de Janeiro State University, Ph.D. in Mechanical Engineering
https://orcid.org/0000-0001-9534-5623 • diegoknupp@iprj.uerj.br
Contribution: Conceptualization, Investigation, Methodology, Visualization, Writing – Review & Editing

5 – Antônio José da Silva Neto
Full Professor at Polytechnic Institute of Rio de Janeiro State University, Ph.D. in Mechanical Engineering
https://orcid.org/0000-0002-9616-6093 • ajsneto@iprj.uerj.br
Contribution: Conceptualization, Investigation, Methodology, Visualization, Writing – Review & Editing

How to cite this article

Teixeira, J. S., Migon, H. S., Stutz, L. T., Knupp, D. C., & Neto, A. J. S. (2024). Structural Damage Identification via Bayesian Inference with a New Hierarchical Modeling and Spike-and-Slab Prior. *Ciência e Natura*, Santa Maria, v. 46, spe. 1, e87212. https://doi.org/10.5902/2179460X87212