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Chemistry

Analysis of the influence of the soil retardation coefficient on the dispersion of pollutants in landfills

Análise da influência do coeficiente de retardamento do solo na dispersão de poluentes em aterros sanitários

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ABSTRACT

The increase in the consumption of disposable products, the excessive use of packaging and the presence of goods scheduled for rapid obsolescence are factors that have directly driven the growth in the production of municipal solid waste (MSW). When these wastes are not treated properly, their negative impacts on the environment become evident, including contamination of soil and water resources. Thus, the present work presents a study of a two-dimensional model of the dispersion of pollutants in landfills, considering the governing equation in dimensionless form. The problem under study is hypothesized to be the continuous and uniform leakage of an MSW storage cell. The adopted model is solved by the Generalized Integral Laplace Transform Technique (GILTT) method, whose solution is analytical, except for the truncation error in the infinite series. The results obtained show that the proposed method is effective in reproducing the physical characteristics of the problem and when compared with other results available in the literature, highlighting the influence of the delay factor parameter in the study.

Keywords: Sanitary landfills; Dispersion of pollutants; Retardation factor; GILTT; Mathematical modeling

RESUMO

O aumento no consumo de produtos descartáveis, a utilização excessiva de embalagens e a presença de bens programados para rápida obsolescência são fatores que têm diretamente impulsionado o crescimento da produção de resíduos sólidos urbanos (RSU). Quando esses resíduos não são tratados de forma adequada, seus impactos negativos no meio ambiente se tornam evidentes, incluindo a contaminação do solo e dos recursos hídricos. Desta forma, o presente trabalho apresenta um estudo de um modelo bidimensional da dispersão de poluentes em aterros sanitários, considerando a equação governante na forma adimensional. O problema em estudo tem como hipótese o vazamento contínuo e uniforme de uma célula de armazenamento de RSU. O modelo adotado é resolvido pelo método da *Generalized Integral Laplace Transform Technique* (GILTT), cuja solução obtida é analítica, exceto pelo erro de truncamento na série infinita. Os resultados obtidos mostram que o método proposto é eficaz para reproduzir as características físicas do problema e quando comparado com outros resultados disponíveis na literatura, destacando a influência do parâmetro fator de retardamento no estudo.

Palavras-chave: Aterros sanitários; Dispersão de poluentes; Fator de retardamento; GILTT; Modelagem matemática

1 INTRODUCTION

The significant generation of urban solid waste (MSW) is the result of several factors intrinsically linked to population growth, contemporary lifestyle and consumption patterns. These phenomena reflect the growing demand for materials and packaging, as well as the challenge of dealing with products that end up being designed to become obsolete quickly, encouraging constant replacement and generating more waste.

According to the report published by ABRELPE (2022), progress was observed in the adequate disposal of MSW, and in 2022, the final disposal in landfills reached a rate of 61%, highlighting that the remaining 39% are still destined to controlled dumps and landfills, which remain active throughout the country, as in these locations there is no layer of impermeable material to protect the soil, in addition to adequate treatment for the released gases and leachate, as in a sanitary landfill. Therefore, when MSW is sent to these inappropriate means, it constitutes a permanent source of pollution and environmental degradation, with considerable impacts on the health of the population.

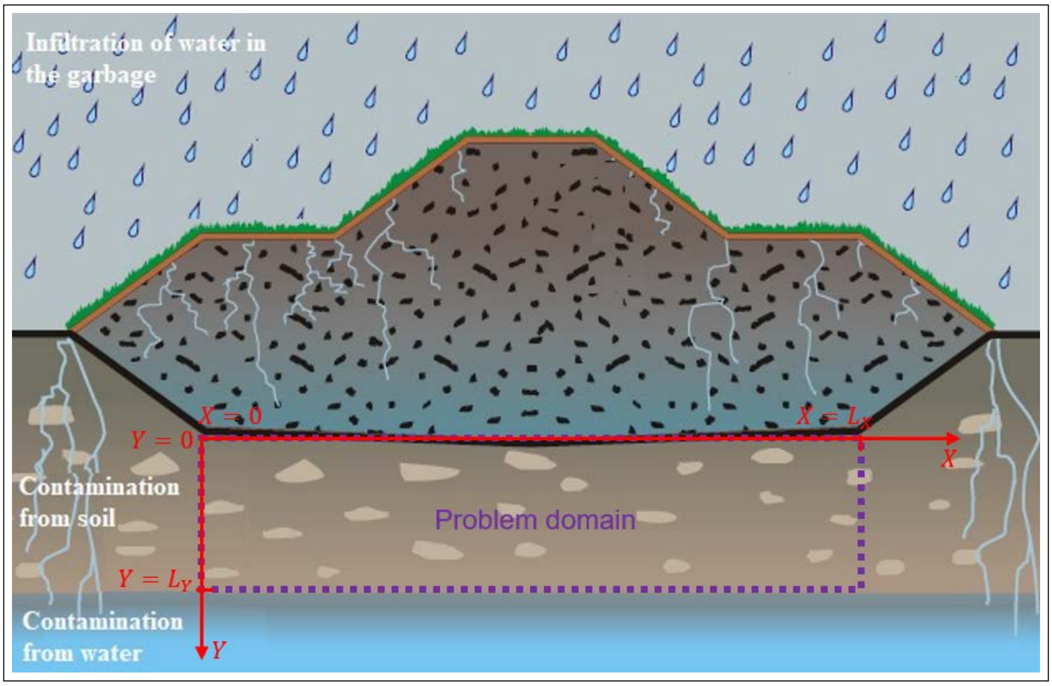
The purpose of this work is to present the solution of a two-dimensional model of contaminant transport in a landfill and, through simulations, observe the importance and influence of the soil retardation factor. For this purpose, the dimensionless form of the effluent dispersion model in the porous medium will be used, considering the continuous and uniform leakage of pollutants from an MSW storage cell.

To solve the model, the Generalized Integral Laplace Transform Technique (GILTT) will be used, which combines the expansion of the contaminant in a series in terms of eigenfunctions determined with the support of an auxiliary Sturm-Liouville problem and obtaining an ordinary differential equation (ODE) in matrix form, which is solved by the Laplace transform (Buske, 2008), thus obtaining the solution of the analytical form of the transient model of mass transport in a saturated porous medium.

2 METHODOLOGY

The figure 1 represents the two-dimensional model used in this work, depicting the simplified scheme of a solid waste storage cell. This is where the transport of contaminant concentration occurs through the porous medium until it reaches the groundwater table, where $Y = 0$ represents the boundary between the landfill and the soil, and $Y = L_Y$ represents the limit between the soil and the groundwater table.

Figure 1 – Cross-section of a landfill



Source: Adapted from CONDER (2017)

The equation (1), written in dimensionless form, describes the transport of pollutants in the saturated porous medium and can be found in the work of Albuquerque (2018):

$$R\frac{\partial C}{\partial \tau} = L^*\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} - Pe\frac{\partial C}{\partial Y}, \tag{1}$$

where R represents the soil retardation factor, C is the concentration of the contaminant in the liquid phase, τ represents time, L^* is the ratio of dimensions in X and Y of the problem $\left(L^* = (\frac{L_Y}{L_X})^2\right)$, and Pe is the Peclet number.

The initial condition of the problem is given by:

$$C(X,Y,0) = C_0, \tag{2}$$

where C_0 is the initial concentration of the contaminant in the solid waste storage cell.

The boundary conditions in dimensionless form and in the X direction are given by:

$$\frac{\partial C}{\partial X}(0,Y,\tau) = 0, \quad \frac{\partial C}{\partial X}(1,Y,\tau) = 0, \tag{3}$$

where zero flux conditions are used at the boundaries of the domain in X.

The boundary conditions in the Y direction are given by:

$$C(X,0,\tau) = 1, \tag{4}$$

equation (1) is subject to the interface condition (4), which corresponds to continuous and uniform leachate leakage in a solid waste storage cell, in addition to:

$$\frac{\partial C}{\partial Y}(X,1,\tau) + BiC(X,1,\tau) = 0, \tag{5}$$

where Bi is the Biot number, and this condition represents the convective flow at the contact point between the soil and the groundwater table.

With the dimensionless equations represented by equations (1) - (5), the resolution of the problem begins by applying the Superposition Method (Hahn and Özisik, 1993):

$$C(X,Y,\tau) = C^*(X,Y,\tau) + C_E(Y), \tag{6}$$

where C^* is an auxiliary function that carries the homogeneous boundary condition, and C_E is the solution for the steady-state problem. Thus, substituting equation (6) into the expression governing the transport of pollutants in the porous medium equation (1):

$$R\frac{\partial C^*}{\partial \tau} = L^*\frac{\partial^2 C^*}{\partial X^2} + \frac{\partial^2 C^*}{\partial Y^2} + \frac{d^2 C_E}{dY^2} - Pe\left(\frac{\partial C^*}{\partial Y} + \frac{dC_E}{dY}\right). \tag{7}$$

From equation (7), two differential equations are obtained, one ordinary differential equation (ODE) and one partial differential equation (PDE).

The ODE extracted from equation (7) is represented by the following expression:

$$\frac{d^2C_E}{dY^2} - Pe\frac{dC_E}{dY} = 0, \quad (8)$$

and the boundary conditions are given by the following equations:

$$C_E(0) = 1, \qquad \frac{dC_E(1)}{dY} + BiC_E(1) = 0. \quad (9)$$

Using the boundary conditions given by (9), the analytical solution of equation (8) is obtained:

$$C_E(Y) = \frac{e^{Pe}(Pe + Bi) - Bie^{PeY}}{e^{Pe}(Pe + Bi) - Bi}. \quad (10)$$

The PDE obtained from equation (7) is given by:

$$R\frac{\partial C^*}{\partial \tau} = L^*\frac{\partial^2 C^*}{\partial X^2} + \frac{\partial^2 C^*}{\partial Y^2} - Pe\frac{\partial C^*}{\partial Y}, \quad (11)$$

and the boundary conditions in the X direction are provided by the following equations:

$$\frac{\partial C^*}{\partial X}(0, Y, \tau) = 0, \qquad \frac{\partial C^*}{\partial X}(1, Y, \tau) = 0, \quad (12)$$

and the boundary conditions in the Y direction are given by:

$$C^*(X, 0, \tau) = 0, \qquad \frac{\partial C^*}{\partial Y}(X, 1, \tau) + BiC^*(X, 1, \tau) = 0, \quad (13)$$

and the initial condition is given by:

$$C^*(X, Y, 0) = C_0 - C_E(Y). \quad (14)$$

To obtain the solution $C^*(X, Y, \tau)$ of equation (11), the GILTT method is used.

Firstly, the auxiliary Sturm-Liouville problem is considered in the X direction:

$$\frac{d^2\varphi}{dX^2} + \frac{\lambda^2}{L^*}\,\varphi = 0, \qquad \frac{d\varphi(0)}{dX} = 0, \qquad \frac{d\varphi(1)}{dX} = 0, \tag{15}$$

where the differential equation has as a solution the eigenfunctions (Hahn and Özisik, 1993):

$$\varphi_n(X) = \cos\left(\frac{\lambda_n}{\sqrt{L^*}}X\right), \tag{16}$$

applying the boundary conditions leads to the eigenvalues associated with each eigenfunction:

$$\lambda_n = n\pi\sqrt{L^*} \qquad\qquad n = 0, 1, 2, ... \tag{17}$$

In addition, the concentration is expanded as a series in terms of the eigenfunctions and then substituted into equation (11). By applying the integral operator $\int_0^1 (.)\varphi_m(X)dX$ on both sides of the equation, noting that, by equation (15), $\varphi_n''(X) = -\lambda_n^2\varphi_n(X)$, an equation is obtained where all summations have zero terms, except when $m = n$. Therefore, the following expression is obtained:

$$R\frac{\partial\overline{C}_n(Y,\tau)}{\partial\tau} = -\lambda_n^2\overline{C}_n(Y,\tau) + \frac{\partial^2\overline{C}_n(Y,\tau)}{\partial Y^2} - Pe\frac{\partial\overline{C}_n(Y,\tau)}{\partial Y}. \tag{18}$$

For the resolution of the PDE (18), the auxiliary Sturm-Liouville problem is first solved in Y:

$$\frac{d^2\psi}{dY^2} + \beta^2\psi = 0, \qquad \psi(0) = 0, \qquad \frac{d\psi(1)}{dY} + Bi\psi(1) = 0, \tag{19}$$

where the differential equation has as a solution the eigenfunctions (Hahn and Özisik, 1993):

$$\psi_k(Y) = \mathrm{sen}(\beta_k Y), \tag{20}$$

applying the boundary conditions leads to the eigenvalues associated with each eigenfunction, which must satisfy a transcendental equation, where the roots are calculated by the Newton-Raphson method.

Next, the solution of the PDE (18) is expanded as a series in terms of the eigenfunctions and then the substitution in the expansion in equation (18) is performed. Also, applying the integral operator $\int_0^1 (.)\psi_l(Y)dY$ on both sides of the equation, noting that, by equation (19), $\psi_k''(Y) = -\beta_k^2\psi_k(Y)$ and regrouping the terms, the obtained equation can be rewritten in matrix form:

$$A \cdot Z'(\tau) + B \cdot Z(\tau) = 0, \tag{21}$$

where $Z(\tau) = \widetilde{C}k$, with $k = 0, 1, 2, ...$; $A = ak, l$, where $a_{k,l} = R\int_0^1 \psi_k(Y)\psi_l(Y)dY$; and $B = b_{k,l}$, where $b_{k,l} = (\lambda_n^2 + \beta_k^2)\int_0^1 \psi_k(Y)\psi_l(Y)dY + Pe\int_0^1 \psi_k'(Y)\psi_l(Y)dY$. Also, considering $F = A^{-1} \cdot B$, equation (21) is rewritten as:

$$Z'(\tau) + F \cdot Z(\tau) = 0. \tag{22}$$

The initial condition of the matrix differential equation, equation (22), is obtained by applying the same procedures performed in the PDE on equation (14), and thus, the initial condition is well defined.

To solve the matrix ODE (22), Laplace transform is applied on both sides. In this problem, it is assumed that the matrix F is diagonalizable, and $F = X \cdot D \cdot X^{-1}$ is written, where D is the diagonal matrix whose elements are the eigenvalues of F, X is the matrix whose columns constitute the linearly independent eigenvectors of F, and X^{-1} is its inverse. Therefore, it is concluded that the solution of the matrix ODE (22) is:

$$Z(\tau) = X \cdot G(\tau) \cdot X^{-1} \cdot Z(0). \tag{23}$$

Thus, the solution of the two-dimensional model of pollutant dispersion in porous media, represented by equations (1) - (5), is given by:

$$C(X, Y, \tau) = \sum_{n=0}^{N} \varphi_n(X)\left[\sum_{k=0}^{K} \psi_k(Y)\widetilde{C}_k(\tau)\right] + C_E(Y), \tag{24}$$

where $\varphi_n(X)$ is defined by equation (16), $\psi_k(Y)$ is defined in equation (20), and $\widetilde{C}_k(\tau)$ is defined by (23).

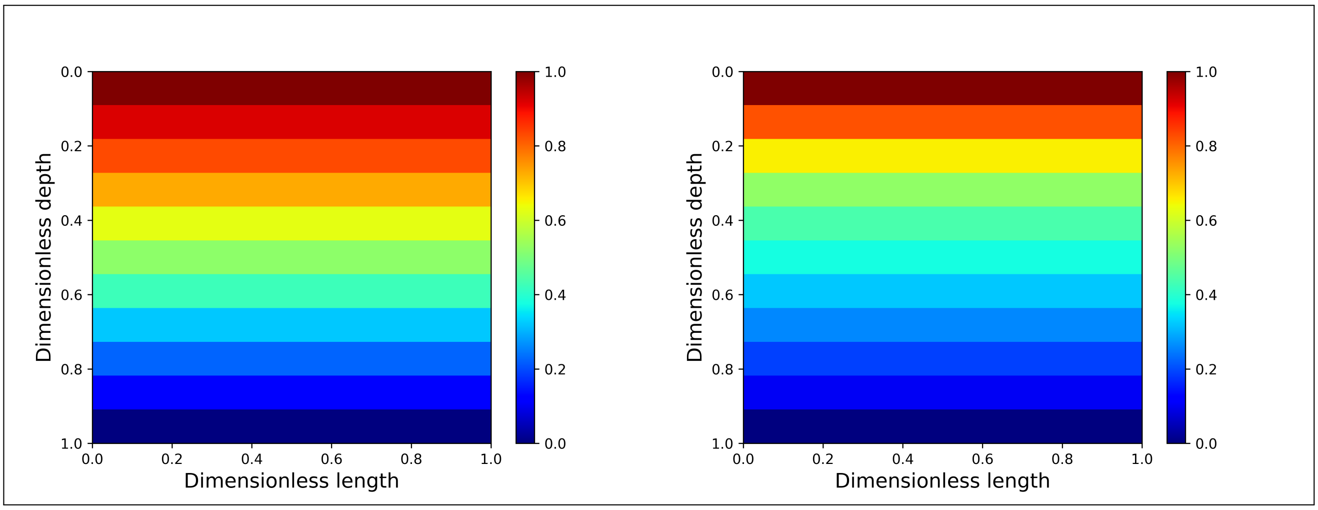
3 RESULTS AND DISCUSSION

Based on the solution of the pollutant transport model in a two-dimensional porous medium, the results obtained will be presented and analyzed using the online environment Google Colaboratory, in Python language. For the analysis, the influence of the R parameter on the concentration field stands out. Furthermore, it should be noted that in the simulations, the value $Bi = 2000$ was used.

The parameter R is used to evaluate the retention capacity of the soil, being a characteristic of the soil, which depends on the substance considered, the activity of the porous medium, the initial concentration of the substance in the contaminated solution and among other factors (Lange et al., 2002).

Figures 2, 2, 3 and 3 present the distribution of the dimensionless concentration of the pollutant considering fixed $Pe = 2$, for two different values of the retardation factor, $R = 1$ and $R = 5$, in addition to $\tau = 0.10$ and $\tau = 0.50$.

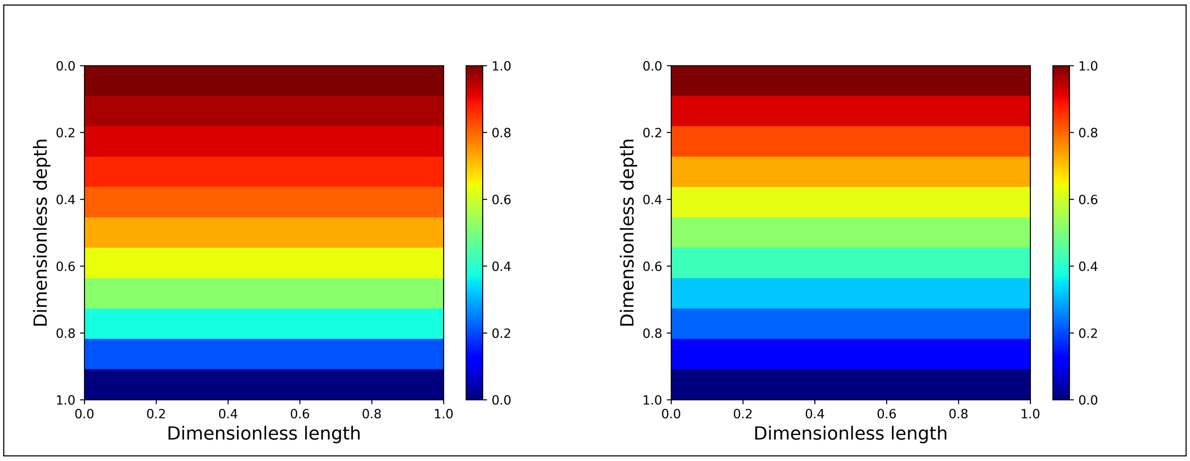
Figure 2 – Dimensionless concentration distribution of the pollutant



Source: Authors

It can be seen that for the time of $\tau = 0.10$ or $\tau = 0.50$, in both cases considered, $R = 1$ and $R = 5$, it occurs that the delayed from the soil reduces the concentration of the contaminant by 20%. In other words, in the case of figure 2, it can be observed that the pollutant reaches around 40% of the soil with contaminant levels considered high, around 0.7. When raising the parameter to $R = 5$, it is clear that the concentration levels are different, in this case the pollutant reaches around 20% of the soil with the same level of contaminant concentration.

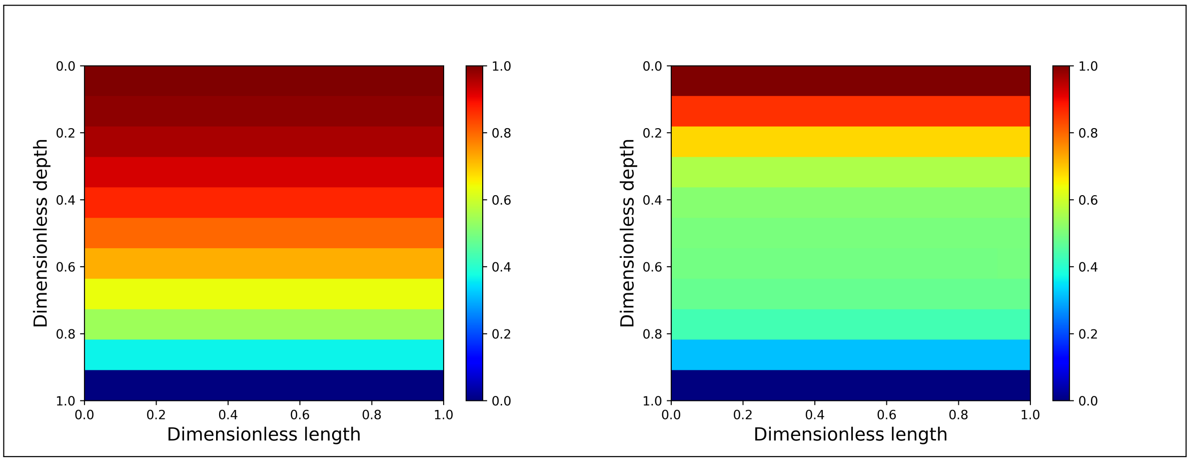
Figure 3 – Dimensionless concentration distribution of the pollutant



Source: Authors

Furthermore, the cases of concentration distribution were analyzed, with $Pe = 10$ being fixed, for two different values of the delay factor, $R = 1$ and $R = 5$, in addition to $\tau = 0.05$ and $\tau = 0.20$.

Figure 4 – Dimensionless concentration distribution of the pollutant

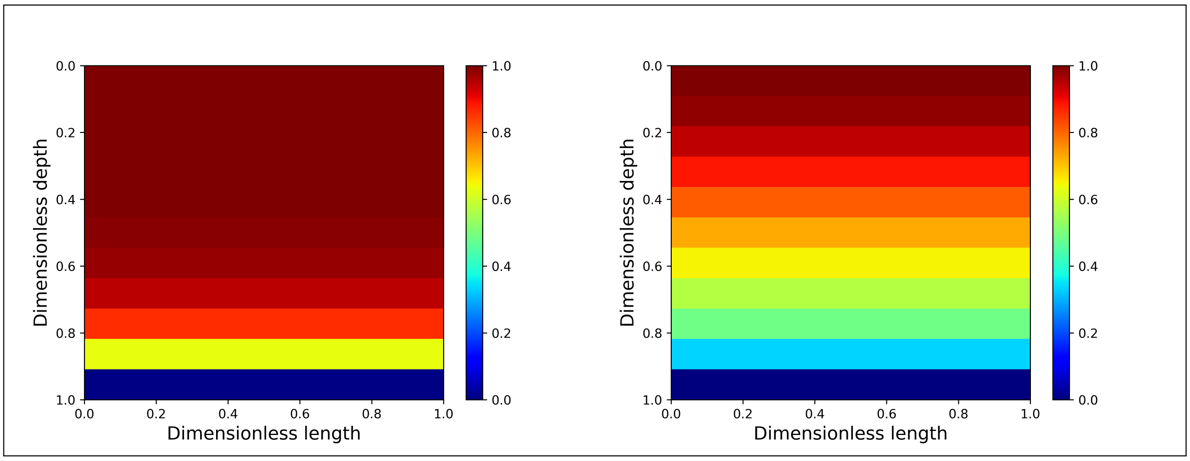


Source: Authors

When evaluating figure 4 and 4 the significant difference in the distribution of contaminant concentration is evident. When considering the parameter $R = 5$, high levels of contamination remain at, at most, 20% of the soil. While, for $R = 1$, the pollutant reaches around 60% of the porous medium.

It is noted that in figure 5, the contaminant reached 90% of the soil, at a level of 0.7 on the contaminant concentration scale. When the same case is evaluated, but considering $R = 5$, figure 5, it is observed that the pollutant reaches 60% of the soil. Demonstrating the influence of the R parameter on the distribution of pollutant concentrations.

Figure 5 – Dimensionless concentration distribution of the pollutant



Source: Authors

Furthermore, it can be highlighted that evaluating $Pe = 10$, that is, a significant difference to $Pe = 2$, it is clear that the pollutant originating from the MSW cell contaminates the soil more significantly in a shorter time, due to the fact that the contaminant flows at a greater speed and contaminates the soil less, that is, the flow is predominantly advective. Therefore, the possible causes of the groundwater can be significant, due to the concentration of pollutants that can reach it, and the soil retardation factor plays an important role due to the retention capacity of this contaminant.

4 CONCLUSIONS

The study of groundwater contamination is essential to understand how polluting agents, in this case, the leachate generated by a landfill cell, can infiltrate the soil and reach groundwater, compromising its quality and availability.

With the simulations carried out, it can be concluded that as the value of R increases, the dimensionless concentration will be lower, that is, the contaminant concentration is retained by the soil porosity. Therefore, the soil retardation factor parameter is essential to understand and manage soil and groundwater contamination, aiming to protect human health and the environment. Furthermore, in both cases, it is observed, as expected, that considering a value of Y closer to the surface and consequently to the landfill, it is clear that the dimensionless concentration is a value close to 1, maximum concentration. However, when considering a value of Y close to the water table, it is clear that the concentration of the contaminant is practically zero. In this way, it can be concluded that the results obtained are consistent with the dynamics of the problem.

As the next steps in the research, we intend to study the influence of other parameters on the spread of contaminants and whether they reach the groundwater table in a more significant way.

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