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Two-wheeled self-balancing car, modeling, nonlinear system simulation and linearization

Carro com auto-equilíbrio em duas rodas, modelagem, simulação do sistema não linear e linearização

Pedro Henrique Rodrigues da Rosa [I], **Vandrei Rubin Steffanello** [I], **André Francisco Caldeira** [I], **Charles Rech** [I], **Cristiano Frandalozo Maidana** [I], **Simone Ferigolo Venturini** [I]

[I] Universidade Federal de Santa Maria, Campus Cachoeira do Sul, RS, Brazil

ABSTRACT

Applying the signals and systems theory and control theory studied in electrical engineering and mechanical engineering courses requires sophisticated, expensive and delicate handling devices. The mathematical tools that are taught in the disciplines of signals and systems and control theory in the undergraduate course are often abstract, especially in terms of practical application in an industrial setting, which makes it necessary to use didactic control plants to complement teaching and hands-on experimentation. This paper presents a low-cost two-wheeled self-balancing car system as a teaching tool for engineering. The two-wheeled self-balancing car is a dynamic benchmark system designed to control the car in a vertical position. The car is able to remain balanced through calculations that analyze the angle of the gyroscope and the position of the wheels. DC motor drivers are used to compensate for tilt and balance the car. As the purpose of the article is to serve as support material for undergraduate students who are taking their first steps in the study of signals and systems and control theory, detailed modeling by the laws of physics are presented, together with the simulation of the nonlinear model of the system and a complete linearization of the model.

Keywords: Signals and systems, Control theory, Nonlinear system, Linearization, Simulation

RESUMO

A aplicação da teoria de sinais e sistemas e da teoria de controle estudadas nos cursos de engenharia elétrica e engenharia mecânica requer dispositivos de manuseio sofisticados, caros e delicados. As ferramentas matemáticas que são ensinadas nas disciplinas de sinais e sistemas e teoria de controle no curso de graduação são muitas vezes abstratas, principalmente em termos de aplicação prática em ambiente industrial, o que torna necessária a utilização de plantas de controle didáticas para complementar o ensino e a experimentação prática. Este artigo apresenta um sistema de carro com autoequilíbrio de duas rodas de baixo custo como ferramenta de ensino para engenharia. O carro com equilíbrio automático de duas rodas é um sistema de referência dinâmico projetado para controlar o carro na posição vertical. O carro consegue se manter equilibrado por meio de cálculos que analisam o ângulo do giroscópio e a posição das rodas. Drivers de motor CC são usados para compensar a inclinação e equilibrar o carro. Como o objetivo do artigo é servir de material de apoio para alunos de graduação que estão dando os primeiros passos no estudo de sinais e sistemas e teoria de controle, são apresentadas modelagens detalhadas pelas leis da física, juntamente com a simulação do modelo não linear do sistema e uma linearização completa do modelo.

Palavras-chave: Sinais e sistemas, Teoria de controle, Sistemas não lineares, Linearização, Simulação

1 INTRODUCTION

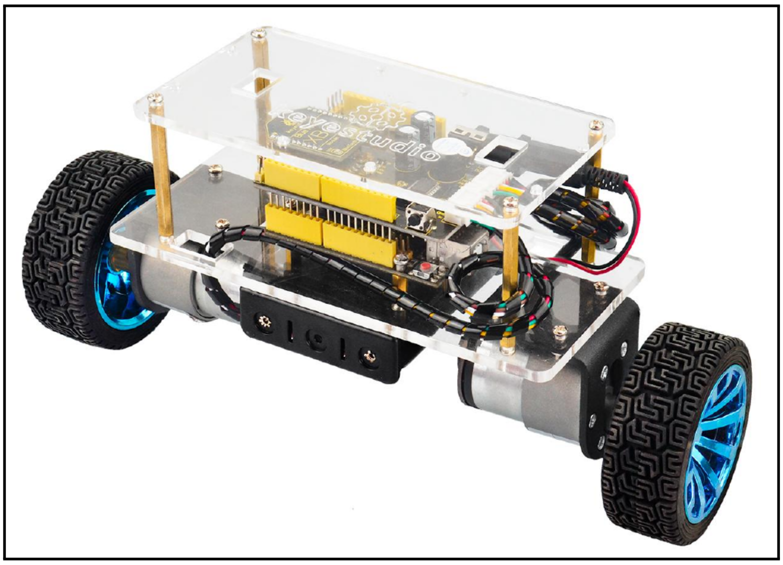
The mathematical modeling and control of the two-wheeled self-balancing car (TWSBC) has received great attention from researchers in recent decades, due to the fact that it is a non-linear electromechanical system, composed of two side wheels that come into contact with the floor surface, and by be an unstable system by nature. The wheels are independently driven to balance at the center of gravity above the axis of rotation of the wheels Grasser et al. (2002); Kocaturk (2015).

The wheels are driven by two motors coupled to each of the they. The motors can be of DC nature and are controlled by an H-bridge and PWM (Pulse-Width Modulation) electrical signals through a control system based on reading the voltage and speed of its center of gravity. Its operation is similar to the classic inverted pendulum system.

The movement of the TWSBC is governed by an under-actuated configuration, i.e., the number of control inputs is less than the number of degrees of freedom to be stabilized, which makes it difficult to control the system. Therefore, the TWSBC is an excellent benchmark for study and investigating of various control techniques and controller efficiency for application in the design and development of control systems for cars, spacecraft, domestic transportation, military transport, among others.Jeong and Takahashi (2008); Jiménez et al. (2020); Johnson and Nasar (2017); Shino et al. (2015)

The open source community is full of low-cost prototypes, instructions and code snippets for studying the modeling and control of the TWSBC, the vast majority use some open source micro controller from the Arduino ecosystem. An example is the open source Kit Keyestudio Self-balancing Car as seen in Figure 1.

Figure 1 – The open Source self-balancing Car Kit supplie by Keyestudio



Source: Keyestudio

The motivation for working with the TWSBC are: small size, simple and compact structure, action flexibility, good maneuverability and low cost Galicki (2016); Takei et al. (2009). The fact that the prototype presents unstable and non-linear dynamics makes the two-wheeled balanced self-balancing car an excellent challenge for engineering students to revisit the literature related to concepts of modeling systems with nonlinear dynamics, stability analysis, linearization, feedback control systems, robotics application and design of control systems. Jiménez et al. (2020)

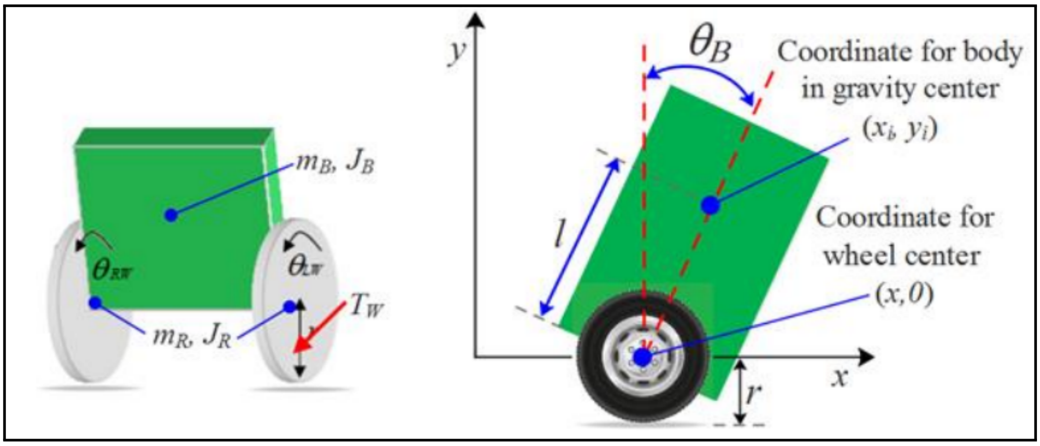
In recent years, various design of controllers and analysis technique had been proposed by numerous researchers to control the TWSBC such that the car able to balance itself. In Arvidsson and Karlsson (2012), a comparison between PID and LQR has been presented while the heading angle of the robot was also studied in the dynamic equation that was derived using Lagrangian method. In Majczak and Wawrzyński (2015) a linear stabilizing Proportional Integral Derivative (PID) and Linear Quadratic Regulator (LQR) controller was derived by a planar model without considering robot's heading angle. In Fang (2014) dynamics was derived using a Newtonian approach and the control was design by the equations linearized around an operating point.

In this paper the information was organized in the following way: In section II, a complete modeling of the TWSBC system is described using the Lagrange mechanical dynamics equations using the simile with the dynamic behavior of the inverted pendulum. Section III describes the nonlinear simulation of the open-loop TWSBC system based on the analysis of Nonlinear Ordinary Differential Equations using Matlab®/Simulink™ software . Section IV presents the digital PID controller that will be used later in future work. And to conclude, in section V the system linearization around the equilibrium point is presented, and a transfer function is obtained.

2 MATHEMATIC MODEL OF THE TWSBC SYSTEM

The TWSBC system is dynamically unstable and has a behavior equivalent to an inverted pendulum on car with wheels, therefore, its modeled using as a reference to the inverted pendulum system. Kung (2019); Mateşică et al. (2016). The TWSBC system can be considered as a mechanical platform composed of two coupling subsystems: the main body (pendulum) and the assembled rotation system (pendulum cart). The main body resembles the inverted pendulum and integrates the chassis of the TWSBC along with the control cards, drivers, battery and DC motors. The main body is coupled to the rotation system and its mass m_2 is concentrated in its center of gravity. The rotation system is equivalent to the inverted pendulum cart The rotation system is equivalent to the inverted pendulum cart and is made up of the wheels and rotors of the DC motors, as shown in the diagram in in Figure 2(a).

Figure 2 – Free-body diagram of the TWSBC system



a) Balanced b) Inclined
Source: Keyestudio

The coordinate system and the notation of the physical properties of the TWSBC system are illustrated in the free-body diagram in Figure 2(b) where the platform moves along a horizontal axis x. The tilt angle of the TWSBC is θ_B is the angle formed by the line that intersects the mass center of the main body and the axis of the wheel, with the y axis. The masses are measured using an electronic bascule and the individual and total moments of inertia are found using the Steiner theorem or parallels axis's theorem Jiménez et al. (2020).

In the free-body diagram in Figure 2(a) it is observed that a torque is applied to each T_W wheel due to the movement of the DC motors, making that the main body mass m_B of the TWSBC to shift, presenting the tilt angle θ_B.

2.1 Dynamic model of the TWSBC system

The motion equations that define the behavior of the TWSBC system can be obtained using the Lagrangian dynamics.

The kinetic energy of the rotation system due to angular displacement can be represented as:

$$K \cdot E_r = \frac{1}{2} m_r \cdot \dot{x}^2 \tag{1}$$

Where x_i and y_i coordinates indicate the position of the center of gravity of gravity of the main body:

$$\begin{aligned} x_i &= x + l \cdot \sin(\theta_B) \\ y_i &= l \cdot \cos(\theta_B). \end{aligned} \tag{2}$$

Calculating the derivatives of the previous position (x_i, y_i) with respect to time, the respective velocities were found as:

$$\begin{aligned} v_{x_i} &= \dot{x} + l \cdot \cos(\theta_B) \\ v_{y_i} &= -l \cdot \sin(\theta_B). \end{aligned} \tag{3}$$

Taking the squares of the main body velocities (pendulum):

$$|{v_i}^2| = {v_{x_i}}^2 + {v_{y_i}}^2 = \dot{x}^2 + 2l\dot{x}\dot{\theta}_B\cos(\theta_B) + l^2\left(\dot{\theta}_B\right)^2\cos^2(\theta_B) + l^2\left(\dot{\theta}_B\right)^2\sin^2(\theta_B). \tag{4}$$

Simplifying (4) using trigonometric identity:

$$|{v_i}^2| \;=\; \dot{x}^2 + 2l\dot{x}\dot{\theta}_B\cos(\theta_B) \;+\; l^2\big(\dot{\theta}_B\big)^2. \tag{5}$$

Then the kinetic energy of the main body due to the linear displacement of the TWSBC is obtained as:

$$K\cdot E_B \;=\; \frac{1}{2}m_B\cdot\dot{x}^2 \;+\; m_B l\dot{x}\dot{\theta}_B\cos(\theta_B) \;+\; \frac{1}{2}m_B l^2\big(\dot{\theta}_B\big)^2. \tag{6}$$

Adding (1) and (6) the total kinetic energy of TWSBC is obtained in (7).

$$E_T \;=\; \frac{1}{2}(m_B+m_r)\cdot\dot{x}^2 \;+\; m_B l\dot{x}\dot{\theta}_B\cos(\theta_B) \;+\; \frac{1}{2}(J_B+m_B l^2)\big(\dot{\theta}_B\big)^2. \tag{7}$$

The general Lagrange equations for the velocity of the rotation system and the main body of the TWSBC are calculated as follows:

$$\frac{d}{dt}\left(\frac{\partial E_T}{\partial\dot{x}}\right)-\frac{\partial E_T}{\partial x} \;=\; Q_R. \tag{8}$$

$$\frac{d}{dt}\left(\frac{\partial E_T}{\partial\dot{\theta}}\right)-\frac{\partial E_T}{\partial\theta} \;=\; Q_B. \tag{9}$$

Deriving (7) to find Q_R

$$\frac{\partial E_T}{\partial\dot{x}} \;=\; (m_B+m_r)\cdot\dot{x} \;+\; m_B l\dot{\theta}_B\cos(\theta_B) \tag{10}$$

$$\frac{d}{dt}\left(\frac{\partial E_T}{\partial\dot{x}}\right) \;=\; (m_B+m_r)\cdot\ddot{x} \;+\; m_B l\ddot{\theta}_B\cos(\theta_B) \tag{11}$$

$$\frac{\partial E_T}{\partial x} \;=\; 0 \Longrightarrow Q_R \;=\; T_W - f_R\cdot\dot{x}. \tag{12}$$

Similarly, deriving (7) to find Q_B:

$$\frac{\partial E_T}{\partial \dot{\theta}} = m_B l \dot{x} \cos(\theta_B) + (J_B + m_B l^2)\dot{\theta}_B. \tag{13}$$

$$\frac{d}{dt}\left(\frac{\partial E_T}{\partial \dot{\theta}}\right) = m_B l \ddot{x} \cos(\theta_B) + (J_B + m_B l^2)\ddot{\theta}_B - m_B l \dot{x} \left(\dot{\theta}_B\right)^2 \sin(\theta_B). \tag{14}$$

$$\frac{\partial E_T}{\partial \theta} = -m_B l \dot{x} \dot{\theta}_B \sin(\theta_B). \tag{15}$$

$$Q_B = -m_B g l \sin(\theta_B) - f_B \cdot \dot{\theta}_B. \tag{16}$$

Substituting the previous values in the Lagrange equations (8) and (9) is obtained the motion equations:

$$\ddot{x} = -\frac{m_B l}{(m_B + m_r)}\left(\ddot{\theta}_B \cos(\theta_B) + \left(\dot{\theta}_B\right)^2 \sin(\theta_B)\right) + \frac{1}{(m_B + m_r)}\left(T_W - f_R \cdot \dot{x}\right) \tag{17}$$

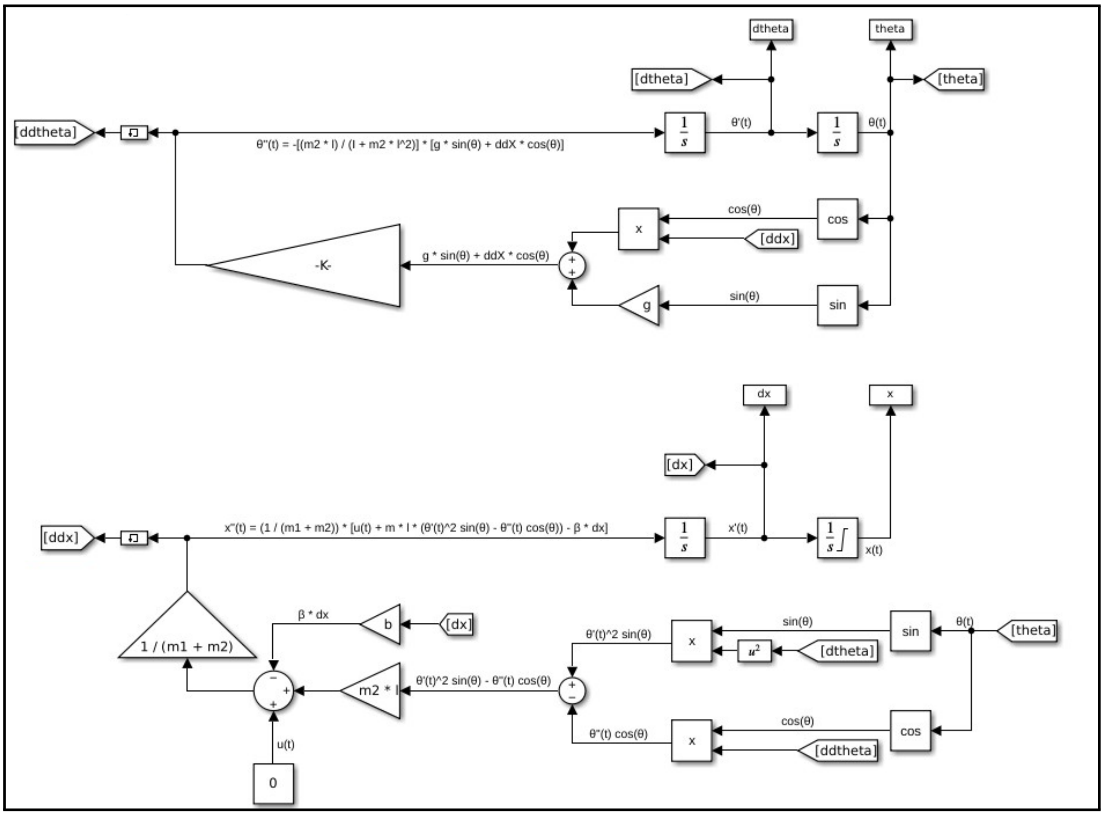
$$\ddot{\theta}_B = -\frac{m_B l}{(J_B + m_B l^2)}\left(\ddot{x} \cos(\theta_B) + g \sin(\theta_B)\right) - \frac{f_B}{(J_B + m_B l^2)} \tag{18}$$

Where (17) and (18) defines the non-linear dynamic behavior of the TWSBC system. Equation (17) was obtained from the assembled rotation system, while (18) was obtained from the main body. The inputs of the TWSBC system are the torque T_{LW} and T_{RW} applied to the left and right wheel assemblies by the DC motors, which are assumed to be similar, that is: $T_W = T_{LW} = T_{RW}$ Jiménez et al. (2020).

3 SIMULATION OF THE NONLINEAR MODEL OF THE TWSBC SYSTEM

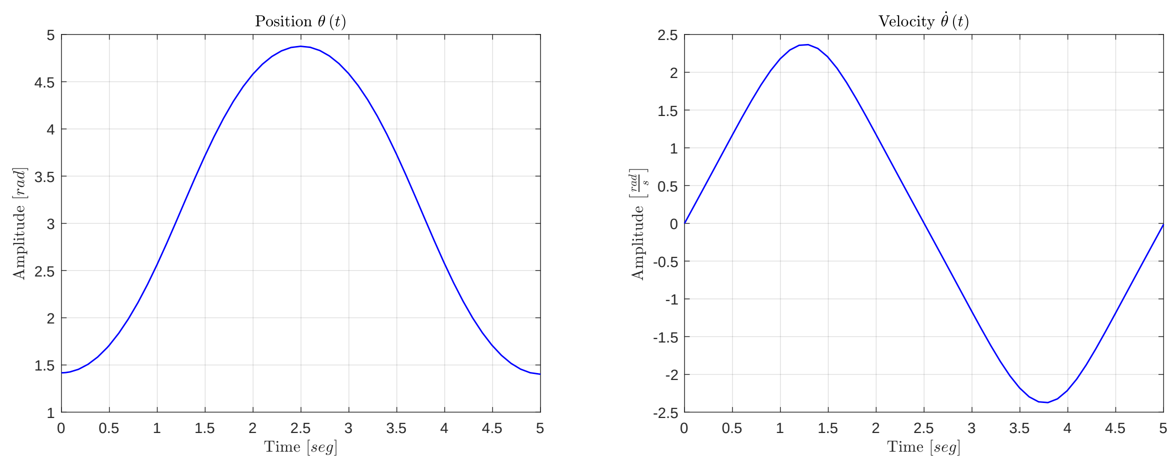
Figure 4 shows the Matlab®/Simulink™ simulation diagram of the nonlinear ordinary differential equations described in (17) and (18). Some modifications were made to the simulation diagram, such as: $m_r = m_1$; $m_B = m_2$; $J_B = I$ and $\theta_B = \theta$. The friction in the main body of the TWSBC was considered negligible, i.e., $f_B \cong 0$.

Figure 3 – Non-linear open-lop of the TWSBC system simulation Matlab®/Simulink™



Source: Matlab®/Simulink™

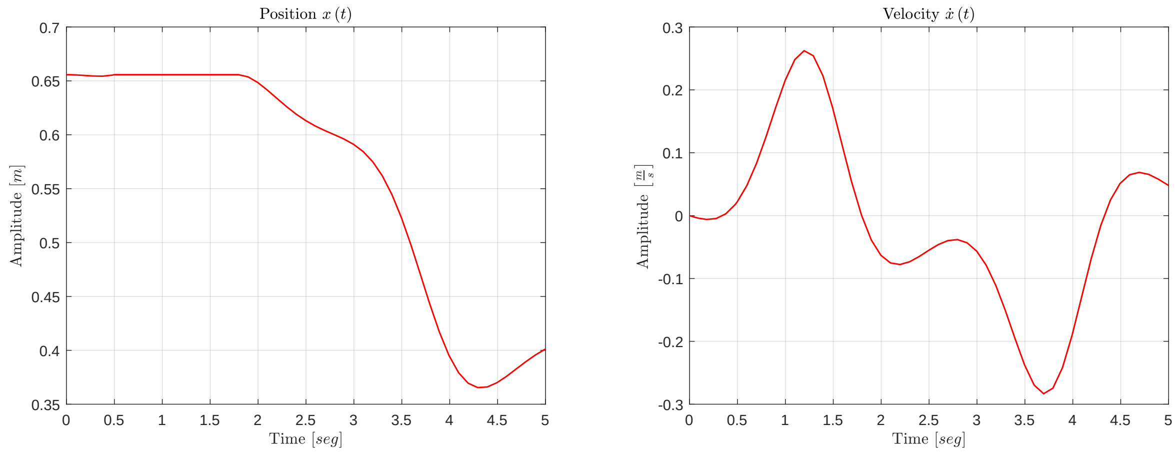
Figure 4 – Non-linear open-loop model of the TWSBC system (Matlab®/Simulink™) with (a) Position $\theta(t)$ e (b) Velocity $\dot{\theta}(t)$



(a) Position $\theta(t)$ (b) Velocity $\dot{\theta}(t)$

Source: Elaborated by the authors (2022)

Figure 5 – Non-linear open-loop model of the TWSBC system (Matlab®/Simulink™) with (a) Position $x(t)$ e (b) Velocity $\dot{x}(t)$



(a) Position $x(t)$ (b) Velocity $\dot{x}(t)$

Source: Elaborated by the authors (2022)

Random positive values of the parameters were used and several simulations were performed. An example of the simulations performed is shown in Figure 4 and Figure 5.

3.1 Linearization of the TWSBC sytem model

The motion equations (17) and (18) are linearized around the unstable equilibrium point of the system at $\theta_B = \dot{\theta}_B = 0$, since the desired operating point is considered when the TWSBC system is in a stable vertical position, with angle zero tilt. In this sense, for an inclination angle θ_B zero, it implies making the following approximations for linearization: $\sin(\theta_B) \cong \theta_B$; $\cos(\theta_B) \cong -1$; $\left(\dot{\theta}_B\right)^2 \cong 0$.

Using the previous approaches to linearize (17) and (18) are making negligible the friction in the main body of TWSBC $f_B \cong 0$, we have:

$$\ddot{x} = \frac{m_B l}{(m_B + m_r)}\ddot{\theta}_B + \frac{1}{(m_B + m_r)}\left(T_W - f_R \cdot \dot{x}\right) \quad (19)$$

$$\ddot{\theta}_B = \frac{m_B l}{(J_B + m_B l^2)}\,\ddot{x} + \frac{m_B l g}{(J_B + m_B l^2)}\,\theta_B\,. \quad (20)$$

Reorganizing the equations (19) and (20), we have:

$$(m_B + m_r)\ddot{x} \ = \ m_B l\ddot{\theta}_B \ + \ (T_W - f_R \cdot \dot{x}) \tag{21}$$

$$(J_B + m_B l^2)\ddot{\theta}_B \ = \ m_B l\ddot{x} \ + \ m_B lg\theta_B \ . \tag{22}$$

Now, apply the Laplace Transform to the equations (21) and (22):

$$(m_B + m_r)s^2 \, X(s) \ + \ f_R sX(s) \ - \ m_B ls^2 \, \Theta_B(s) \ = \ T_W(s) \tag{23}$$

$$(J_B + m_B l^2)s^2 \, \Theta_B(s) \ - \ m_B lg\Theta_B(s) \ = \ m_B ls^2 \, X(s). \tag{24}$$

Isolating $X(s)$ from (24), we have:

$$X(s) \ = \ \left(\frac{(J_B + m_B l^2)}{m_B l} \ - \ \frac{g}{s^2}\right)\Theta_B(s). \tag{25}$$

Replacing (25) in (23) we obtain the transfer function of the system that relates the output of angular position $\Theta_B(s)$ with respect to the input torque $TW(s)$:

$$\frac{\Theta_B(s)}{T_W(s)} \ = \ \frac{\dfrac{m_B ls^2}{p}}{s^4 \ + \ \dfrac{f_R(J_B + m_B l)s^3 \ - \ m_B lg(m_B \ + \ m_r)s^2 - f_R m_B lg \ s}{p}}, \tag{26}$$

with $p = j_B(m_B \ + \ m_r) \ + \ m_B m_r l^2$.

After the cancellation of the pole and the zero at the origin and the term p of the denominator we obtain (27):

$$\frac{\Theta_B(s)}{T_W(s)} \ = \ \frac{m_B ls}{ps^3 \ + \ f_R(J_B + m_B l)s^2 \ - \ m_B lg(m_B \ + \ m_r)s \ - \ f_R m_B lg}. \tag{27}$$

Now, assuming that the used DC motor has a constant output torque, it can be considered that what is going to be controlled is the angular velocity of the DC motor output shaft that corresponds to the rotation velocity of each wheel $\dot{\Theta}_W$, which is related to the input motor torque according to the relationship:

$$r\,\dot{\Theta}_W = u(t) = T_W(t). \tag{28}$$

Replacing the term $T_W s = s\, r\dot{\Theta}_W$ of the relation (28) and if the friction of the assembled rotation system $f_R \cong 0$ is neglected, we obtain the simplified linear transfer function around the point of operation of the TWSBC system:

$$\frac{\Theta_B(s)}{\Theta_W(s)} = \frac{r m_B l s}{p s^2 - m_B l g (m_B + m_r)}. \tag{29}$$

$$\frac{\Theta_B(s)}{\Theta_W(s)} = \frac{K_T s}{\dfrac{s^2}{A_T} - 1}. \tag{30}$$

Equation (30) indicates that with proper adjustment of the angular velocity of the DC motor output shaft $\dot{\Theta}_W$, its is possible to keep $\dot{\Theta}_B$ close to zero degrees, that is, it places the system in vertical position. Replacing the p term gives the location of the actual pair of A_T poles of TWSBC system. One pole is located in the left half plane (stable) and the other in the right half plane (unstable) of the complex S-plane.

$$K_T = \frac{r}{g(m_B + m_r)}. \tag{31}$$

$$A_T = \pm\sqrt{\frac{(m_B + m_r) m_r g l}{(m_B + m_r)(J_B + m_B l^2) - (m_B l)^2}}. \tag{32}$$

4 DESIGN OF DIGITAL CONTROLLER - FUTURE WORK

The TWSBC system is not stable in open loop since, when a bounded input stimulus, the output of the inclination angle Θ_B is not bounded, that is, it does not meet the superposition principle and has an unstable response.

As future work, a controller design will be proposed to be implemented digitally whose main objective will be to stabilize the system and keep it in the main body of the TWSBC in vertical position at (desired operating point) where the tilt angle is $\Theta_B = 0$, regardless of the disturbances.

A classic PID controller will be proposed, simulations using Matlab®/Simulink™ and implementation of digital algorithms on microcontrollers in the Arduino ecosystem. The PID control architecture is feedback where the input to the controller is the system error signal $e(t)$. The error is the deviation between the desired state of the tilt angle $\Theta_B = 0$ and the state of the actual tilt angle measured by the IMU of the TWSBC sensing module.

The PID controller calculates the control signal value $u(t)$ that provides energy to the DC motors of the TWSBC system through PWM signals. The objective of the project is to calculate the gains from control actions through the control law delivered by the PID Controller:

$$u(t) \;=\; k_p e(t) \;+\; \frac{k_p}{t_i}\int e(t)dt \;+\; k_p\, t_d \frac{d}{dt}e(t) \tag{33}$$

where the error $e(t) \;=$ set-point $-$ input.

To implement in a microcontroller (For example, microcontrollers from the Arduino ecosystem), it is necessary to discretize the continuous equation above.

A trapezoidal sum was used to approximate the integral and for the derivative term we will use finite differences (regressive).

$$\int e(t)dt \;=\; \sum\left(\frac{e(k)-e(k-1)}{2}\right)T_s, \tag{34}$$

$$\frac{d}{dt}e(t) = \frac{e(k)-e(k-1)}{T_s}. \tag{35}$$

Where T_s is the sampling time. Choosing the correct sampling time is very important for digital systems. A good choice can be made using the method proposed by Zigle-Nichols.

$$T_s < \frac{\theta}{4}, \tag{36}$$

$$.\frac{\tau}{10} \leq T_s \leq \frac{\tau}{20}. \tag{37}$$

Thus, the PID digital transfer function can be written as:

$$C(z^{-1}) = \frac{u(k)}{e(k)} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}}, \tag{38}$$

where,

$$q_0 = k_p\left(1 + \frac{T_s}{2t_i} + \frac{t_d}{T_s}\right),$$
$$q_1 = -k_p\left(1 - \frac{T_s}{2t_i} + \frac{2t_d}{T_s}\right),$$
$$q_2 = \frac{k_p t_d}{T_s}.$$

With this, we can manipulate the 18 equation to obtain the control law that will be implemented.

$$u(k)(1 - z^{-1}) = q_0 e(k) + q_1 z^{-1} e(k) + q_2 z^{-2} e(k),$$
$$u(k) - u(k) z^{-1} = q_0 e(k) + q_1 z^{-1} e(k) + q_2 z^{-2} e(k),$$
$$u(k) = u(k) z^{-1} + q_0 e(k) + q_1 z^{-1} e(k) + q_2 z^{-2} e(k).$$

Now we apply the inverse z-transform to obtain the difference equation (which will be implemented in the microcontroller).

$$u(k) = u(k-1) + q_0 e(k) + q_1 e(k-1) + q_2 e(k-2). \tag{39}$$

5 CONCLUSIONS

In this work a complete modeling of the two-wheeled self-balancing car was presented using the Lagrange mechanical dynamics equations using the simile with the dynamic behavior of the inverted pendulum. A simulation of the nonlinear ordinary differential equations of the two-wheeled self-balancing car is presented using Matlab®/Simulink™. A linearization around an equilibrium point is presented in the form of a transfer function. As future work, classical control tools will be used to design a classic PID controller. Control identification techniques will be addressed, and a comparison between the identified models with the real system will be evaluated.

This work was a first contact of undergraduate students with the application of feedback control, as well as the first article written in English.

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Author contributions

1 – Pedro Henrique Rodrigues da Rosa
Electrical Engineering student at Universidade Federal de Santa Maria, campus Cachoeira do Sul (UFSM/CS)
https://orcid.org/0009-0000-5391-8104 • pedrohrosa10@hotmail.com
Contribution: Conceptualization, Methodology, Writing - original draft, Preparation, Formal analysis, Investigation

2 – Vandrei Rubin Steffanello
Electrical Engineering student at Universidade Federal de Santa Maria, campus Cachoeira do Sul (UFSM/CS)
https://orcid.org/0009-0004-4516-4567 • vandrei.rubin@acad.ufsm.br
Contribution: Conceptualization, Methodology, Writing - original draft, Preparation, Formal analysis, Investigation

3 – André Francisco Caldeira
PhD in Automation and Systems at Universidade Federal de Santa Catarina (UFSC). Professor at Universidade Federal de Santa Maria, campus Cachoeira do Sul (UFSM/CS)
https://orcid.org/0000-0002-4939-2709 • andre.caldeira@ufsm.br
Contribution: Conceptualization, Methodology, Writing - original draft, Preparation, Formal analysis, Investigation

4 – Charles Rech
PhD in Mechanical Engineering. Professor of Mechanical Engineering, Universidade Federal de Santa Maria, campus Cachoeira do Sul (UFSM/CS)
https://orcid.org/0000-0001-8523-6300 • andre.caldeira@ufsm.br
Contribution: Conceptualization, Methodology, Writing - original draft, Preparation, Formal analysis, Investigation

5 – Cristiano Frandalozo Maidana
PhD in Mechanical Engineering. Professor of Mechanical Engineering, Universidade Federal de Santa Maria, campus Cachoeira do Sul (UFSM/CS)
https://orcid.org/0000-0003-3137-6177 • cristiano.maidana@ufsm.br
Contribution: Conceptualization, Methodology, Writing – Original draft, Preparation

6 – Simone Ferigolo Venturini
Master in Mechanical Engineering. Professor of Mechanical Engineering, Universidade Federal de Santa Maria, campus Cachoeira do Sul (UFSM/CS)
https://orcid.org/0000-0002-9439-0008 • sfventurini@gmail.com
Contribution: Conceptualization, Methodology, Writing – Original draft, Preparation

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