CIÊNCIA e NATURA

ISSN 2179-460X

OPEN ACCESS

Ci. e Nat., Santa Maria, v. 46, e86737, 2024 • https://doi.org/10.5902/2179460X86737

Submitted: 19/02/24 • Approved: 11/06/24 • Published: 29/11/24

Mathematics

Analysis of the interdependence between species in reforestation projects in the Amazon

Análise da interdependência entre espécies em projetos de reflorestamento na Amazônia

Nathan Souza da Silva [I], **João Frederico da Costa Azevedo Meyer** [II], **Marcos Marreiro Salvatierra** [III], **Agbata Benedict Celestine** [IV], **Carlos Frank Lima dos Santos** [III]

[I] Baptist Superior School of Amazonas, Manaus, AM, Brazil
[II] State University of Campinas, Campinas,SP, Brazil
[III] University of Amazonas State, Manaus, AM, Brazil
[IV] Confluence University of Science and Technology, Osara, KO, Nigeria

ABSTRACT

This study employs mathematical and computational approaches to model the interaction between four tree species in a reforestation area. The modeling integrates the diffusion-advection equation with the classical Lotka-Volterra model, which results in a complex system of non-linear partial differential equations.To tackle the complexity of the system, numerical methods, such as the Central Finite Difference Method for the discretization of space and the Crank-Nicolson Method for the discretization of time are employed. The researchers implement these methods using MATLAB software to create an algorithm that generates realistic scenarios over a specified period, and the parameters were estimated based on results extracted from the literature. The generated results provide valuable insights, represented through graphical depictions of approximate solutions in hypothetical scenarios. These visuals allow for the exploration of potential ecological succession trajectories within the environment.

Keywords: Biomathematics; Mathematical ecology; Partial differential equations; Central finite difference method; Crank-Nicolson method

RESUMO

Este estudo emprega abordagens matemáticas e computacionais para modelar a interação entre quatro espécies de árvores em uma área de reflorestamento. A modelagem é feita combinando a equação de difusão-advecção com o modelo clássico de Lotka-Volterra, o que resulta em um sistema de equações diferenciais parciais não-lineares. Devido à complexidade do sistema, recorremos a métodos

Published by Ciência e Natura under a CC BY-NC-SA 4.0 license.

numéricos, como o Método das Diferenças Finitas Centrais para a discretização do espaço e o Método de Crank-Nicolson para a discretização do tempo. Utilizamos o software MATLAB para criar um algoritmo que gera cenários realistas ao longo do período de tempo considerado nas simulações, sendo que os parâmetros foram estimados com base em resultados extraídos da literatura. Com base nesses resultados, conseguimos gerar representações gráficas das soluções aproximadas em cenários hipotéticos, permitindo-nos explorar as possíveis trajetórias de sucessão ecológica no ambiente.

Palavras-chave: Biomatemática; Ecologia matemática; Equações diferenciais parciais; Método das diferenças finitas centrais; Método de Crank-Nicolson

1 INTRODUCTION

Man has caused drastic ecological damage to the vegetation cover of the Brazilian Amazon. This problem is more severe and lasting than that caused by natural clearings, as it affects not only the cut area, but also all species that live in the region, directly and indirectly compromising local biodiversity and ecological balance.

Therefore, it is urgent and necessary to look for ways to restore the Amazon Forest and protect the species that inhabit it. In this article, we present a hypothetical case study of a reforestation project in the Amazon, which analyzes the interdependence between species and population dynamics. Our objective is to contribute to scientific knowledge about the ecological processes involved in the recovery of degraded ecosystems.

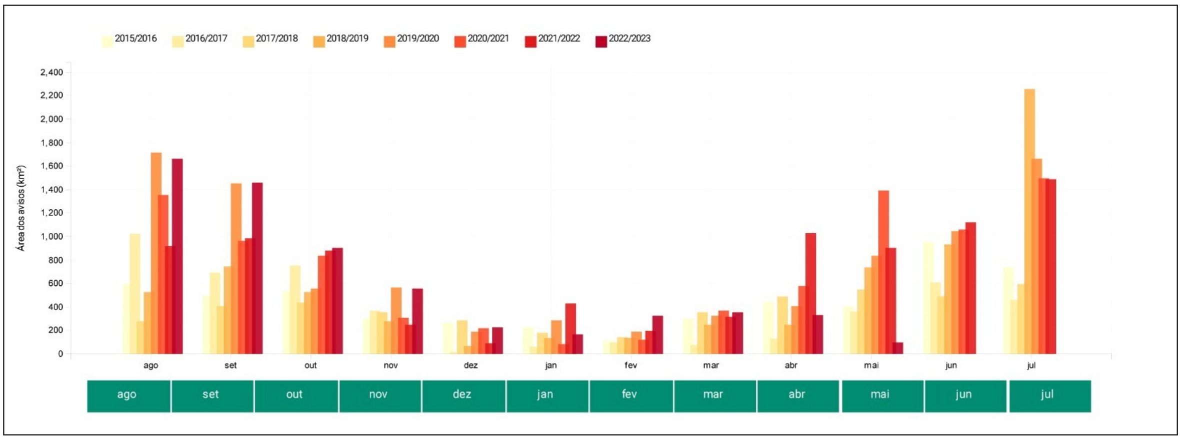
1.1 Environmental challenges in the Amazon

The intense degradation of the Amazon Forest in Brazil is driven by several human activities, such as livestock, agriculture, infrastructure and illegal exploitation (Benatti, 2005). Two major phases of colonization in the Amazon are highlighted, initially encouraged by fiscal (Carvalho et al., 2001) and colonization (Laurance et al., 2004) policies, and later driven by the profitability of logging, agribusiness and livestock farming (Alencar et al., 2004).

Analysis by the National Institute for Space Research (INPE) and the Real-Time Deforestation Detection System (DETER) reveal an alarming degradation, evidenced by satellite images from TerraBrasilis (2022). The data shows a large variation in the increase in deforestation over the years (see Fig. 1).

DETER identifies the main causes of deforestation, highlighting more severe and lasting negative environmental impacts caused by human activities. This highlights the need for studies to assess impacts in the short, medium and long terms, as well as efficient actions to recover deforested areas.

Figure 1 – Monthly variation in the DETER project area: Deforestation notices



Source: DETER

Proposals include participatory environmental planning exercises (Alencar et al., 2004), promoting the sustainable use of the forest and encouraging conservation through environmental certification (Nepstad et al., 2006). Biome recovery suggests the use of local species to restore the soil, especially in advanced stages of degradation, which may require reforestation with adapted pioneer species.

This study aims to understand the interaction between species and the relevant impacts on natural regeneration and ecological restoration, providing valuable information for sustainable practices in the region.

1.2 Related works

For at least five decades, mathematical modeling has been used to evaluate reforestation scenarios in degraded areas. Johnson (Johnson, 1977) constructed a linear, constant-coefficient compartment model to simulate temporal changes in the areal extent of major forest types in North Caroline Piedmont. The recovery of areas affected by fires was the subject of study by Usó-Domenéch et al. (Usó-Doménech et al., 2018), who developed a non-linear, stochatic deterministic and compartmental model – among other characteristics – capable of predicting the effect that the climatic changes of several variables produce on others, validating the model with data from the Benlloch region, in Spain. More recently, Altamirano-Fernández et al. (Altamirano-Fernández et al., 2024) formulate and numerically solve an optimal control problem based on a system of ordinary differential equations in order to simulate the growth of forest plantations in areas affected by fire, validating the model with data from the exotic species Pinus radiata, in the central region of Chile.

1.3 Our results

We used a mathematical model that describes the population dynamics of species that interact and disperse in a heterogeneous environment with interdependence. This model takes into account factors such as birth rate, mortality rate, carrying capacity, intra- and interspecific competition, spatial dispersion and environmental variation.

To simulate the reforestation, we resorted to mathematics where a system of non-linear Partial Differential Equations (PDEs) was used, which are capable of describing situations of this type. Such equations consider population dispersion phenomena, vital dynamics, inter and intraspecific competitions in the sense of classical Lotka-Volterra modeling (Gotelli, 1996).

Our approach involves mathematical modeling of the coexistence between four tree species – Andiroba (Carapa guianensis), Goiaba-de-anta (Bellucia grossularioides), Lacre (Vismia guianensis) and Ucuúba (Virola surinamensis). These species compete for available natural resources, such as water, sunlight and soil nutrients. Our objective is to investigate which species will have supremacy and shape the future local ecological scenario, considering the sustainable exploitation of the forest.

The model and domain were discretized with the objective of obtaining a numerically approximate solution, using the Crank-Nicolson Method for time (Santos, 2022) and Central Finite Differences for the spatial dimension (Prestes, 2011), together with Robin (Santos, 2013) boundary conditions.

The initial results obtained confirm that the mathematical model developed to simulate vegetation recovery in a hypothetical area of degraded Amazon Forest is aligned with the phenomena considered. The simulation allowed the observation of evolutionary behavior in the reforestation of the area, suggesting the emergence of a new local ecological landscape.

These results highlight the relevance of studies and initiatives aimed at the ecological recovery of degraded areas in an effective and sustainable manner.

1.4 Text organization

In the next Section, the modeling of the diffusive and advective behavior of the species under study is presented, followed by the introduction of the effect of interspecific competition in the model, in Section 3. In Section 4, the results of the computer simulations are presented and discussed and finally, in Section 5, conclusions are drawn and some perspectives for future research on the topic are outlined.

2 MODELING FOR RECOVERY OF DEGRADED AREA VIA DIFFUSION-ADVECTION EQUATION

The PDE that describes the density of species is represented by $E(x, y, t)$ which disperses via diffusion and undergoes transport or migration effects (advection). Its modeling will present characteristics of the classic diffusion-advection equation applied in studies focused on mathematical ecology (Miyaoka, 2015), that is:

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial x}\left(\alpha \frac{\partial E}{\partial x}\right) - \frac{\partial}{\partial x}(VE)$$

In the above equation, we have:

- $\frac{\partial E}{\partial t} \to$ Temporal variation.
- $\frac{\partial}{\partial x}\left(\alpha \frac{\partial E}{\partial x}\right) \to$ Diffusion.
- $\frac{\partial}{\partial x}(VE) \to$ Advection.

Where $\alpha(x)$ is the diffusion coefficient, $V(x)$ is the advective velocity.

We will consider a function for the variation in vegetation density in the domain is $\Omega \subset \mathbb{R}^2$, open, non-empty, limited and acceptably regular boundary.

Considering the vegetation density $E_i = E_i(x, y, t)$ in the area under study at each point (x, y) of the domain Ω at the time $t \in (0, T]$ at point, we have the diffusion–advection equation, for each species E_i:

$$\frac{\partial E_i}{\partial t} = div(\alpha_i \nabla E_i) - div(E_i.V_i) - \mu_i E_i + f_i. \tag{1}$$

The diffusive-advective model contemplates the following phenomena:

- Diffusion represents the spread of vegetation in the deforested environment.
- Advection describes the effect caused by movement in the environment itself; (in the case under study, reforestation due to human interference, planting of seedlings, and/or wind depending on its direction may be a field considered).
- Decay, the combination of all events that reduce vegetation density over time.
- Source term, which can be influenced by the intrinsic growth rate of the species as well as being limited by environmental factors.

Adjusting the equation 1 to better fit the study in question, we will consider α_i constant and V_i, being a solenoidal field, that is, $div(V_i) = 0$.

Reforestation (increase in vegetation density) will be represented by classic dynamics of the Verhulst type (Gotelli, 1996), in which K represents a maximum density, considered in this modeling as a carrying capacity of the environment. Considering $(x, y) \in \Omega$ and $t \in (0, T]$, the model that represents the phenomenon under study is given by:

$$\frac{\partial E_i}{\partial t} = div(\alpha_i \nabla E_i) - div(E_i.V_i) - \mu_i E_i + \lambda_i E_i \left(1 - \frac{\sum_i E_i}{K}\right). \tag{2}$$

Where:

- E_i are the plant species considered in this model for recovery of the degraded area under study.
- $V_i = (u_i, v_i)$ characterizes the advective velocity field, describing a possible effect of advection to the medium, with $u_i = u_i(x, y)$ and $v_i = v_i(x, y)$.
- $\alpha_i = \alpha_i(x, y, t)$ are the diffusion parameters of each species E_i in the deforested environment.

- K, is the carrying capacity of the environment, in this case the maximum density of vegetation that the region under study supports.

- $\mu_i = \mu_i(x, y, t)$ are the natural mortality rates of each species E_i.
- λ_i are the growth rates of E_i.

We will resort to the homogeneous Robin conditions (Santos, 2013), (Miyaoka, 2015), which considers a variation of the species E_i on the boundary depending on the species itself:

$$-\alpha_i \left.\frac{\partial E_i}{\partial \eta}\right|_{\partial\Omega_i} = c_i E_i|_{\partial\Omega} \, \forall t \in (0, T]$$

Rewriting the equation 2 with the boundary and initial conditions, we have:

$$\begin{cases} \dfrac{\partial E_i}{\partial t} = div(\alpha_i \nabla E_i) - div(E_i.V_i) - \mu_i E_i + \lambda_i E_i \left(1 - \dfrac{\sum_i E_i}{K}\right) \\ E_i(x, y, 0) = E_{i0}(x, y), \forall (x, y) \in \Omega \\ -\alpha_i \left.\dfrac{\partial E_i}{\partial \eta}\right|_{\partial\Omega} = c_i E_i|_{\partial\Omega} \, \forall t \in (0, T] \end{cases} \tag{3}$$

3 INTERSPECIFIC POPULATION DYNAMICS

Let's consider the cases of interaction between four plant species that compete with each other. These competitions are classified as interspecific (elements from different species) and intraspecific (elements from the same species) for space and nutrients.

Intraspecific competition is modeled according to Verhulst (which, unlike Malthus' modeling, limits population growth to a carrying capacity) and interspecific competition according to Lotka-Volterra, which considers that in this case, for there to be interaction between species, it is necessary that species use the same resources.

Using this brief knowledge as a basis, the proposal to build a model to describe the variation in vegetation density resulting from competitions between four plant species, in a degraded area in order to define the future local ecological landscape.

Considering the four species studied – Goiaba-de-anta (A), Lacre (B), Ucuúba (C) and Andiroba (D) – with constant diffusion we have:

$$
\begin{cases}
\dfrac{\partial A}{\partial t} = \alpha_A \left(\dfrac{\partial^2 A}{\partial x^2} + \dfrac{\partial^2 A}{\partial y^2} \right) - u_A \dfrac{\partial A}{\partial x} - v_A \dfrac{\partial A}{\partial y} - \mu_A A + \lambda_A A \left(1 - \dfrac{A + \gamma_A B + \rho_A C + \delta_A D}{K} \right) \\[2ex]
\dfrac{\partial B}{\partial t} = \alpha_B \left(\dfrac{\partial^2 B}{\partial x^2} + \dfrac{\partial^2 B}{\partial y^2} \right) - u_B \dfrac{\partial B}{\partial x} - v_B \dfrac{\partial B}{\partial y} - \mu_B B + \lambda_B B \left(1 - \dfrac{B + \gamma_B A + \rho_B C + \delta_B D}{K} \right) \\[2ex]
\dfrac{\partial C}{\partial t} = \alpha_C \left(\dfrac{\partial^2 C}{\partial x^2} + \dfrac{\partial^2 C}{\partial y^2} \right) - u_C \dfrac{\partial C}{\partial x} - v_C \dfrac{\partial C}{\partial y} - \mu_C C + \lambda_C C \left(1 - \dfrac{C + \gamma_C B + \rho_C A + \delta_C D}{K} \right) \\[2ex]
\dfrac{\partial D}{\partial t} = \alpha_D \left(\dfrac{\partial^2 D}{\partial x^2} + \dfrac{\partial^2 D}{\partial y^2} \right) - u_D \dfrac{\partial D}{\partial x} - v_D \dfrac{\partial D}{\partial y} - \mu_D D + \lambda_D D \left(1 - \dfrac{D}{K + \gamma_D A + \rho_D B + \delta_D C} \right) \\[2ex]
A(x,y,0) = A_0(x,y),\ B(x,y,0) = B_0(x,y),\ C(x,y,0) = C_0(x,y),\ D(x,y,0) = D_0(x,y),\ \forall (x,y) \in \Omega \\[2ex]
-\alpha_A \left. \dfrac{\partial A}{\partial \eta} \right|_{\partial\Omega} = c_A A|_{\partial\Omega}\, \forall t \in (0,T] \\[2ex]
-\alpha_B \left. \dfrac{\partial B}{\partial \eta} \right|_{\partial\Omega} = c_B B|_{\partial\Omega}\, \forall t \in (0,T] \\[2ex]
-\alpha_C \left. \dfrac{\partial C}{\partial \eta} \right|_{\partial\Omega} = c_C C|_{\partial\Omega}\, \forall t \in (0,T] \\[2ex]
-\alpha_D \left. \dfrac{\partial D}{\partial \eta} \right|_{\partial\Omega} = c_D D|_{\partial\Omega}\, \forall t \in (0,T]
\end{cases}
\tag{4}
$$

where the coefficients γ_σ, ρ_σ and δ_σ represent the rates of interspecific competition between species, for $\sigma = A, B, C, D$.

It can be noted that the source term for species D (Andiroba) differs from those of other species. This is due to the fact that this species is more resilient to conditions of low soil fertility, and can grow both in shady areas and under light. In this way, the rates of interspecific competition between this species and others will contribute to increasing the carrying capacity.

4 RESULTS AND DISCUSSION

The models and the domain were discretized aiming for their solution by numerical approximation by central finite differences in the spatial dimension (Santos, 2013), and by the Crank-Nicolson finite difference method in time (Santos, 2022). The growth rates of the species Goiaba-de-anta, Ucuúba and Lacre were obtained from Macedo (2008). The growth rate of the Andiroba species was extracted from Carvalho (2014).

Considering the information provided on the species Goiaba-de-anta, Ucuúba, Lacre and Andiroba, it is possible to infer that the rate of diffusion and advection can be affected by several environmental factors, such as the availability of resources, the type of soil, the climate and the presence of other species in the region, making them specific characteristics of each species. Although the growth rates of the species were obtained from different sources, there is no concrete information about the diffusion and advection rate of each of them.

Therefore, it is possible to state that the diffusion and advection rate of the studied species can be considered as heroic characteristics, as they depend on multiple factors that vary according to environmental conditions.

All simulations were carried out with a view to recovering a degraded area – thus, the remaining parameters were determined aiming at this optimistic scenario – and the values used for each species are presented in Table 1. In all simulations, we consider $c_\sigma = 0$ for all $\sigma = A, B, C, D$.

Table 1 – Values of computational parameters referring to the model 4

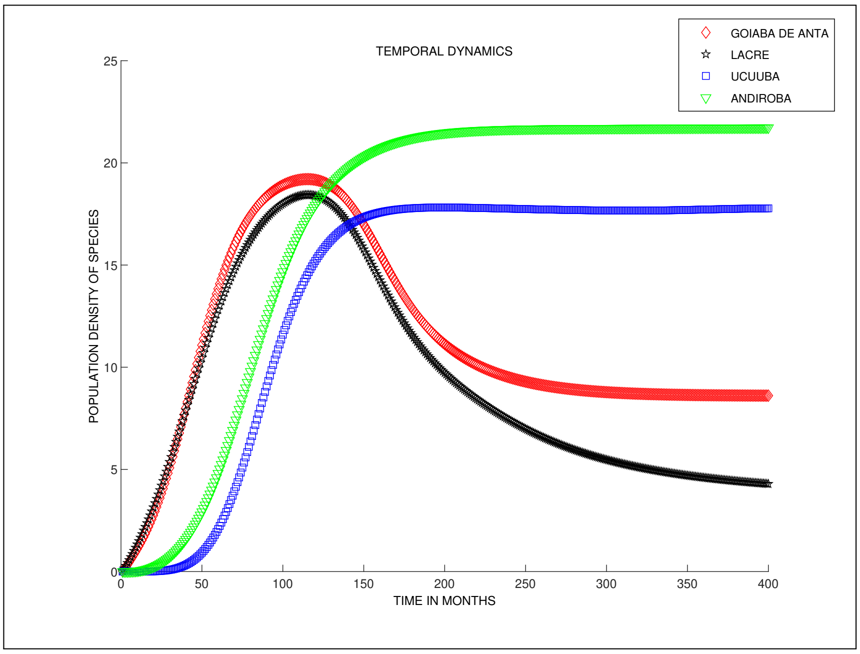
Par.	Values	Par.	Values	Par.	Values	Par.	Values	Units
α_A	0,025	α_B	0,035	α_C	0,037	α_D	0,03	area/time
u_A	0,002	u_B	0,001	u_C	0,01	u_D	0,01	Km/month
v_A	0,04	v_B	0,04	v_C	0,03	v_D	0,03	Km/month
μ_A	0,0	μ_B	0,0	μ_C	0,00	μ_D	0,00	h^{-1}
λ_A	0,7	λ_B	0,6	λ_C	0,7	λ_D	0,6	area/ind.t^2
γ_A	0,05	γ_B	0,05	γ_C	0,04	γ_D	0,03	area/ind.t^2
ρ_A	0,029	ρ_B	0,16	ρ_C	0,06	ρ_D	0,01	area/ind.t^2
δ_A	0,05	δ_B	0,06	δ_C	0,07	δ_D	0,08	area/ind.t^2
K	20	nx	32	ny	25	dx	0,1875	real no.
dy	0,12	dt	0,099					real no.

Source: the authors (2024)

4.1 Temporal dynamics

Let's look at the results of the simulations with the 4 species in temporal dynamics, initially considering their decay rate to be zero. The graph below represents the temporal dynamics of the model 4 of the four vegetation species, it is noted that Andiroba (species D) predominates in relation to the other species.

Figure 2 – Simulation of the temporal dynamics of the species under study



Source: the authors (2024)

When we observe the initial conditions used in the simulation, we realize that the species studied grow over time, but at a certain point they reach the maximum carrying capacity of the environment in which they are inserted.

It is noteworthy that species A stood out from the beginning of the simulation, maintaining a significant increase in its density compared to the other species and over time we observed its population decline, this is because its lifespan is between 30 and 40 years old, another important piece of information is that its slight population growth stands out. According to EMBRAPA (2022), in favorable climate and soil conditions, the Goiaba-de-anta tree can start producing fruit from 3 years old age, which can extend up to 6 years. However, it is important to highlight that the exact period at which fruit production begins may vary depending on the specific conditions of each plant and the environment in which it is found, meaning seed dispersion becomes wider due to the animals that feed on it of its fruits.

A certain instability is observed in species C (Ucuúba). This species is common in flooded areas and has a high survival rate in full sun. However, in areas of dry land, their survival depends on the shade of other species. Thus, when other species increase their density, Ucuúba develops better in the environment.

4.2 Spatial dynamics

In spatial dynamics we will simulate the distribution of species in space under different initial conditions, highlighting:

- Distribution of species in the form of concentric circles.
- Distribution of species alternately.
- Distribution of species in a circular shape, arranged in quadrants, taking the center of the mesh as a reference.

We highlight that for the following computer simulation, the model 4 was used and the parameters used are in accordance with Table 1.

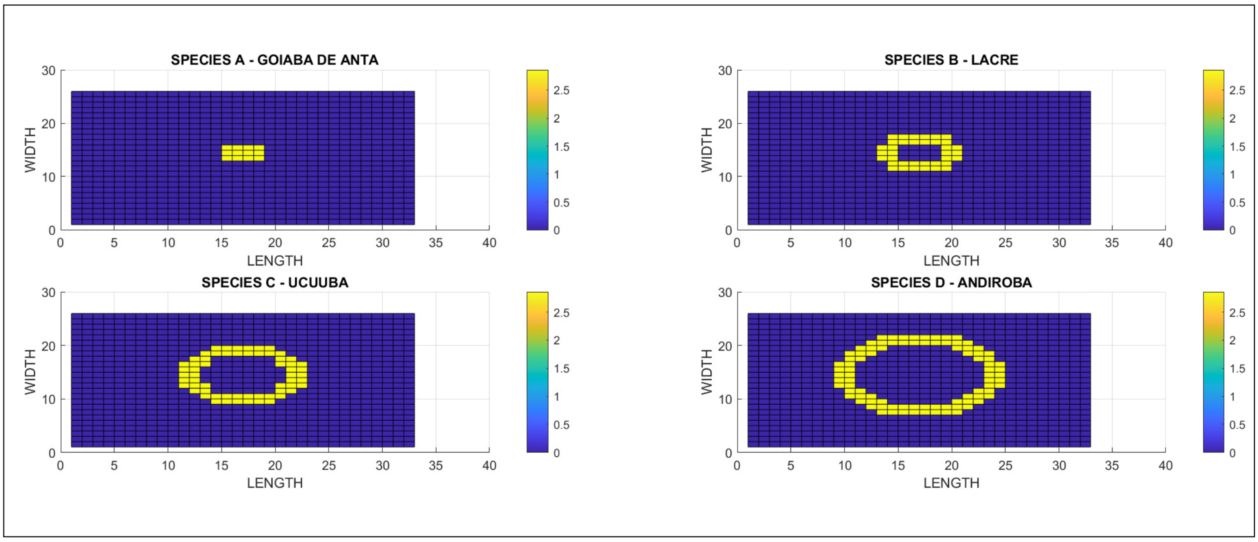
4.2.1 Distribution of species in the form of concentric circles

The technique of distributing species in concentric circles is a valuable tool for analyzing the behavior of species in different initial conditions, which is why we will simulate the study using this technique. Additionally, it can be used to guide conservation efforts and ensure the protection of important species and habitats.

This technique is widely used in studies of ecology, evolutionary biology and biodiversity conservation. For example, a study carried out in the Amazon used the concentric circle technique to analyze the distribution of tree species in different types of soil (Barbosa et al., 2018).

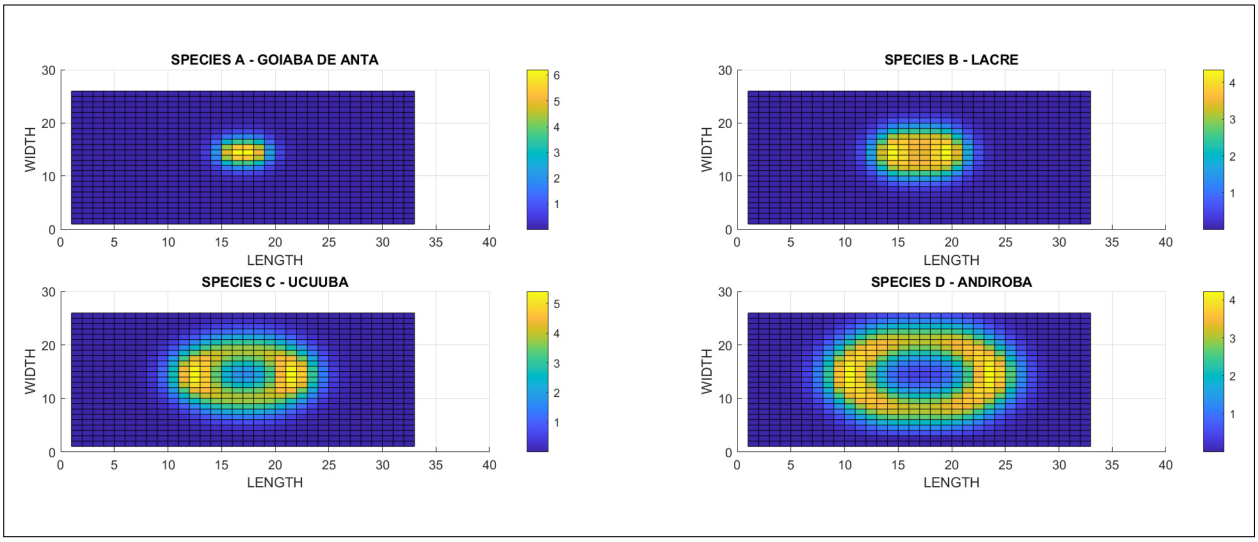
Let's observe how the species are distributed in the mesh, applying the species distribution technique in concentric circles.

Figure 3 – Initial spatial distribution of species in the form of concentric circles



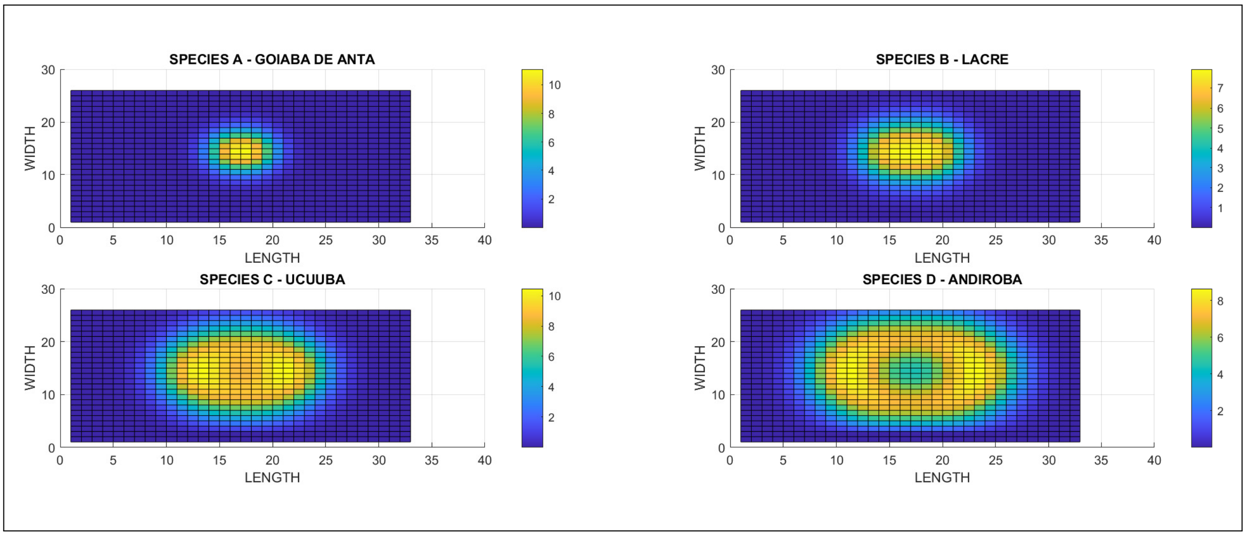
Source: the authors (2024)

Figure 4 – Spatial dynamics distribution of species in the form of concentric circles, for $t = 2$ years



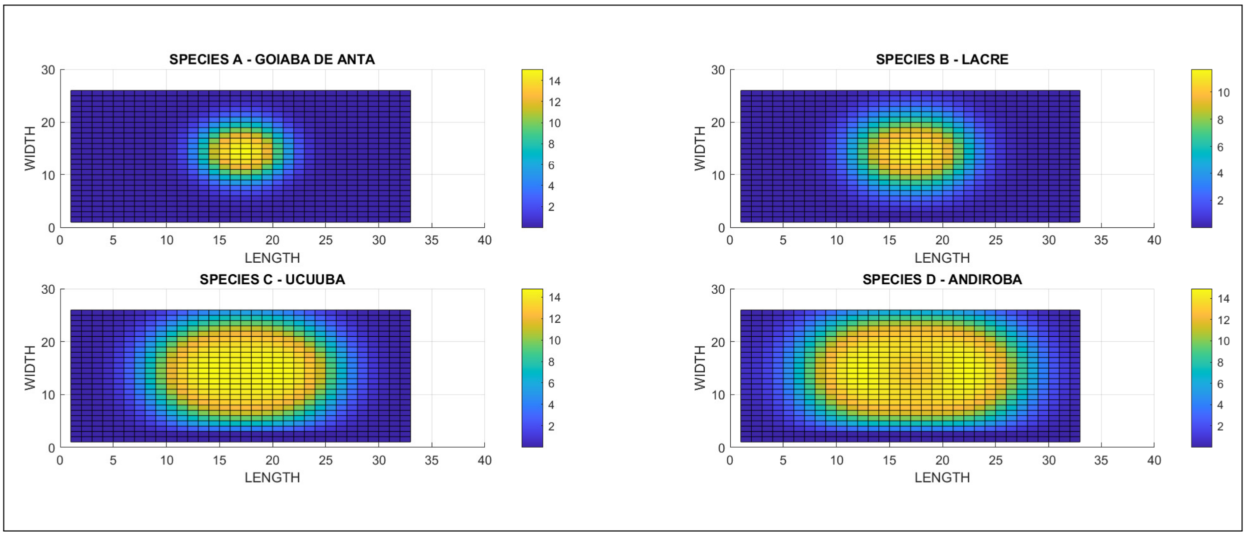
Source: the authors (2024)

Figure 5 – Spatial dynamics distribution of species in the form of concentric circles, for $t = 4$ years



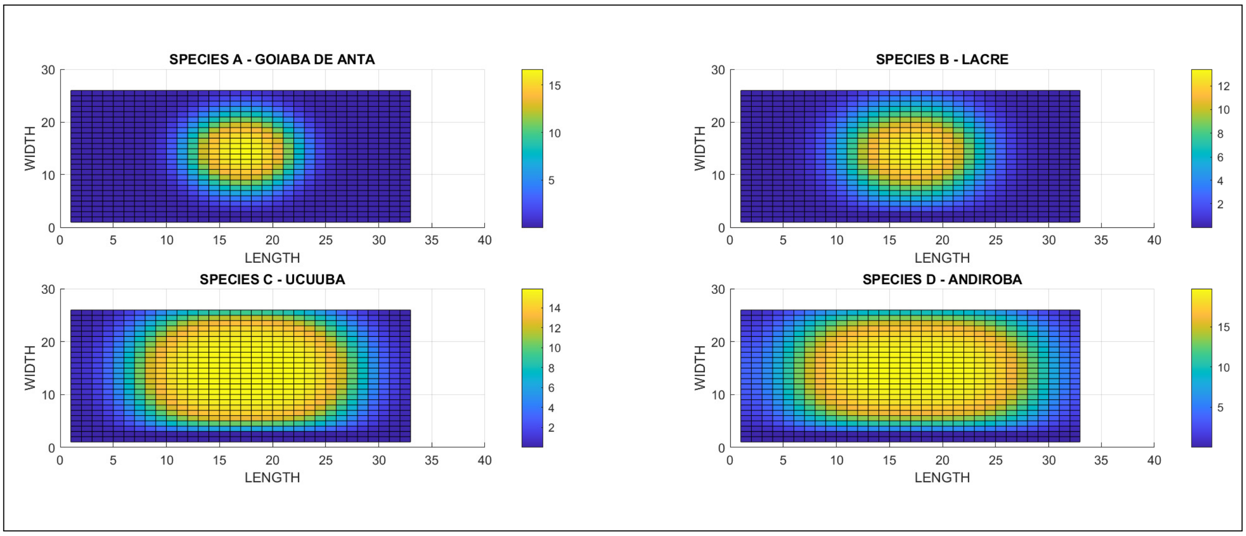
Source: the authors (2024)

Figure 6 – Spatial dynamics distribution of species in the form of concentric circles, for $t = 6$ years



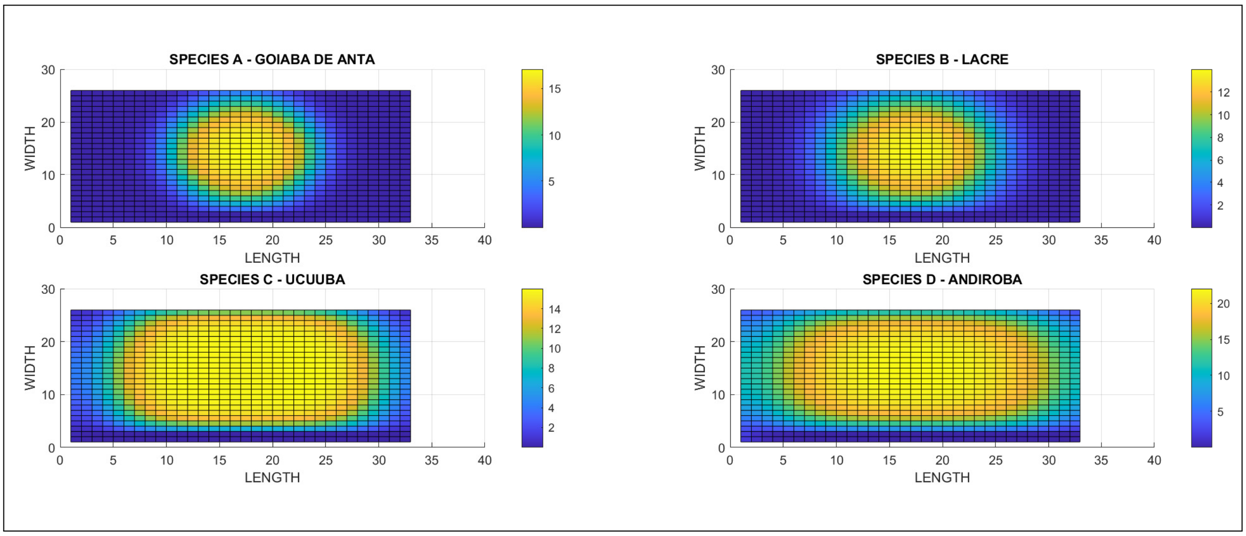
Source: the authors (2024)

Figure 7 – Spatial dynamics distribution of species in the form of concentric circles, for $t = 8$ years



Source: the authors (2024)

Figure 8 – Spatial dynamics distribution of species in the form of concentric circles, for $t = 10$ years



Source: the authors (2024)

When analyzing the graphs in Fig. 3 – 8, we notice that the species develop differently in each concentric circle. This indicates that the distribution of species in concentric circles can significantly influence their development.

We can observe, for example, that species A showed greater growth in the innermost circles, while species B had greater development in the outermost circles, but maintaining development in the center of the distribution, also present where we have species A. Species C showed uniform growth in all the innermost circles. As for species D, in the innermost circles, its presence is almost zero, standing out more towards the outer area. These differences can be explained by the different environmental conditions present in each circle, such as the availability of nutrients, humidity, light, among other factors.

4.2.2 Distribution of species alternately

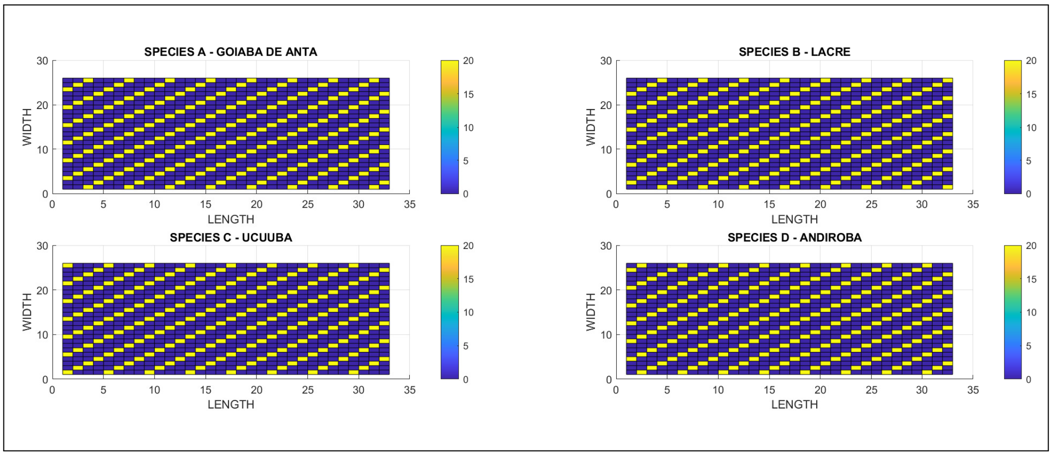
The alternating distribution of four tree species can be an interesting strategy to investigate the ecological interactions between species in a reforestation process.

A study carried out in the Brazilian tropical Amazon highlighted that the alternating distribution of species can affect the diversity of fauna associated with the ecosystem and influence the dynamics of the plant community (Santos et al., 2019).

Therefore, it is important to observe how species are distributed in the network, applying the distribution alternately, to assess whether this strategy can promote coexistence and ecosystem balance.

Let's observe how the species are distributed in the mesh, applying the distribution alternately.

Figure 9 – Spatial dynamics distribution of species in the form of concentric circles. For $t = 0$ years



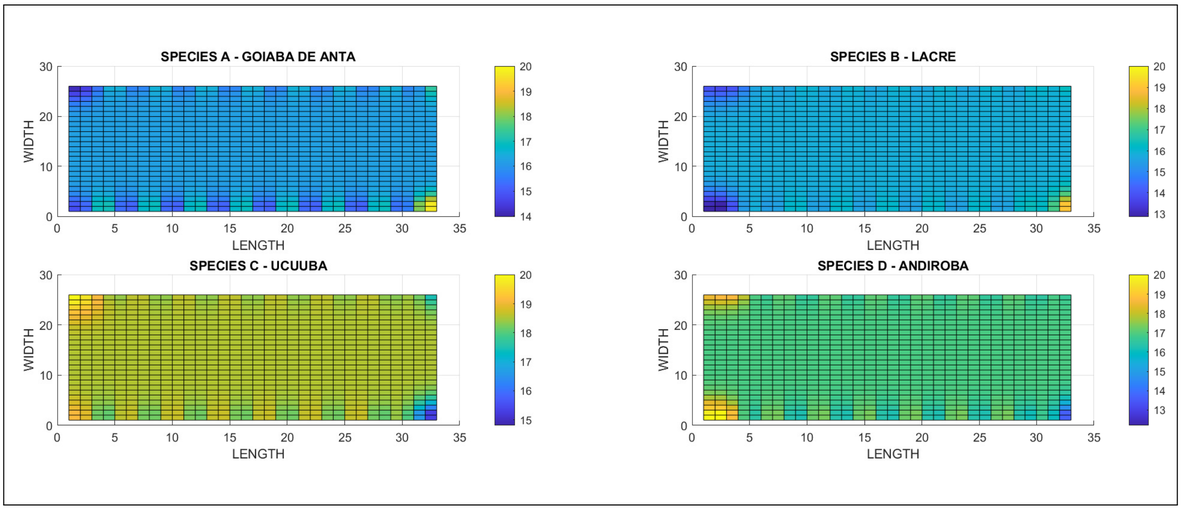
Source: the authors (2024)

When distributing the species alternately, it was observed that species C obtained the best competitive performance in the space under study. This discovery is in line with the ecological reality of the species, which requires shady areas to develop.

These results highlight the importance of the alternating spatial distribution of species in an ecosystem and its relationship with the competitive performance and survival of species.

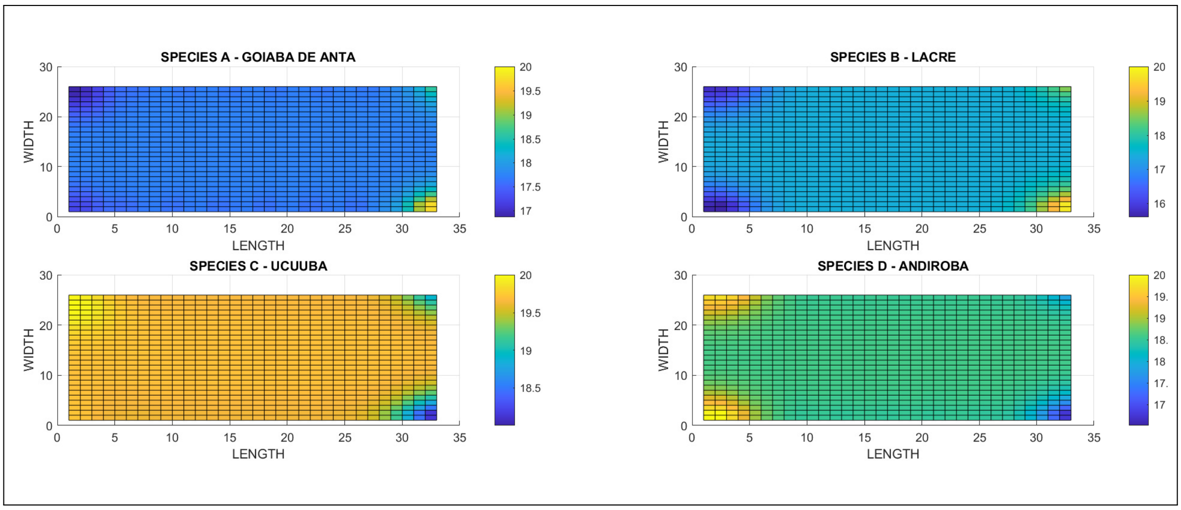
We highlight that the alternating planting showed that the population density of the species is very similar between them and that the forest homogenizes more quickly.

Figure 10 – Spatial dynamics distribution of species in the form of concentric circles For $t = 2$ years



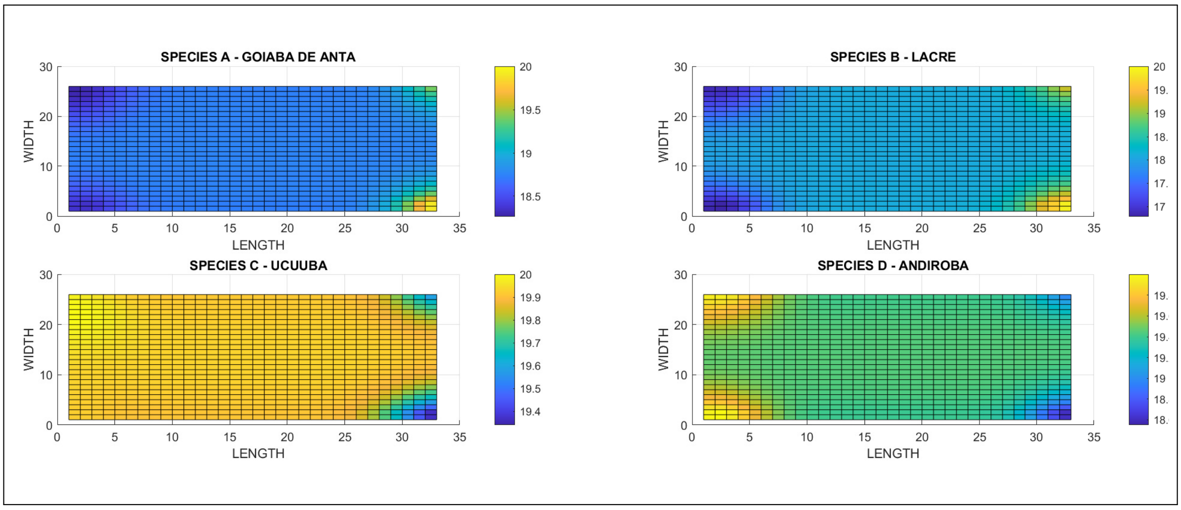
Source: the authors (2024)

Figure 11 – Spatial dynamics distribution of species in the form of concentric circles. For $t = 4$ years



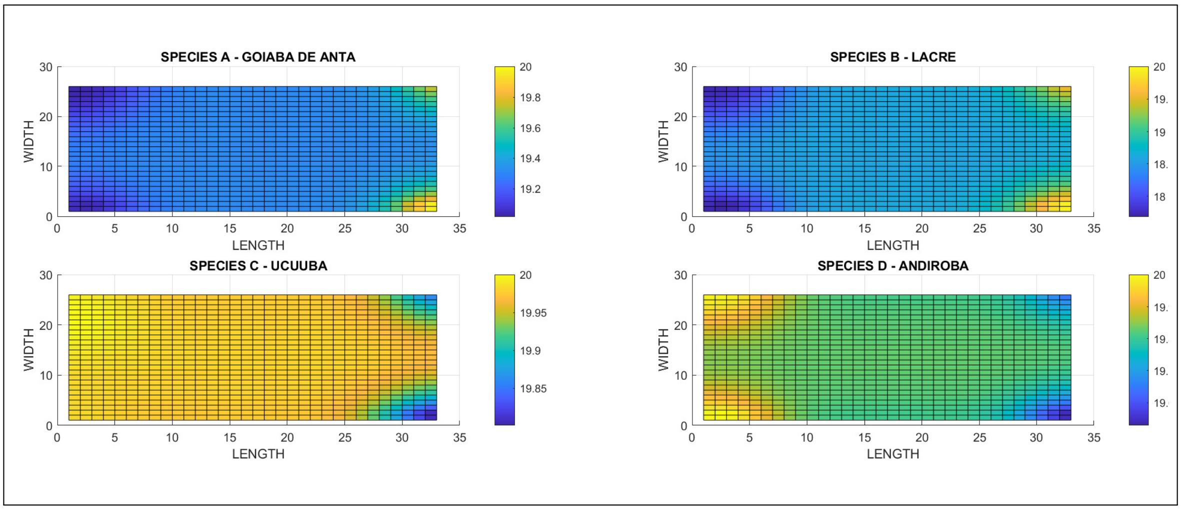
Source: the authors (2024)

Figure 12 – Spatial dynamics distribution of species in the form of concentric circles. For $t = 6$ years



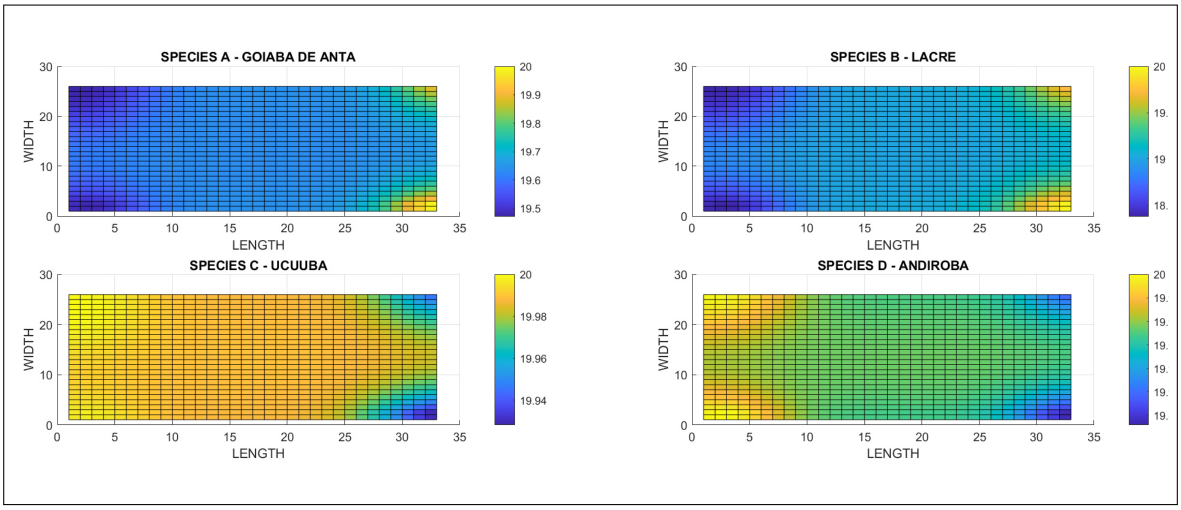
Source: the authors (2024)

Figure 13 – Spatial dynamics distribution of species in the form of concentric circles. For $t = 8$ years



Source: the authors (2024)

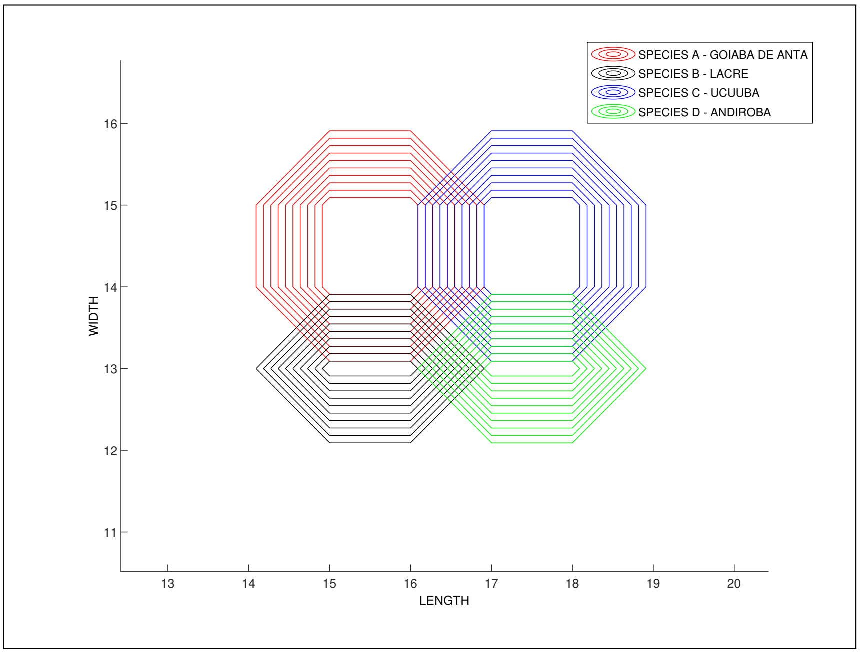
Figure 14 – Spatial dynamics distribution of species in the form of concentric circles. For $t = 10$ years



Source: the authors (2024)

4.2.3 Distribution of species in a circular shape, arranged in quadrants, taking the center of the mesh as a reference

Figure 15 – Spatial dynamics distribution of species in the form of concentric circles. For $t = 0$ years

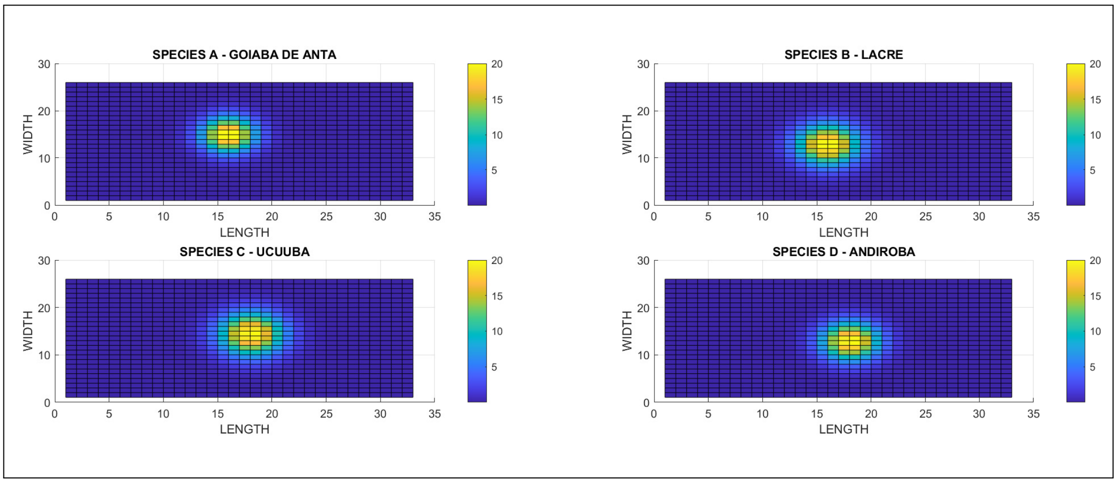


Source: the authors (2024)

Again we detail the distribution of species in a circular way, but now we determine a distribution in quadrants, where species A interacts with species B and C, species B interacts with A and D, species C interacts with A and D and finally species D interacts with C and B. We highlight that species interactions occur through alternating planting.

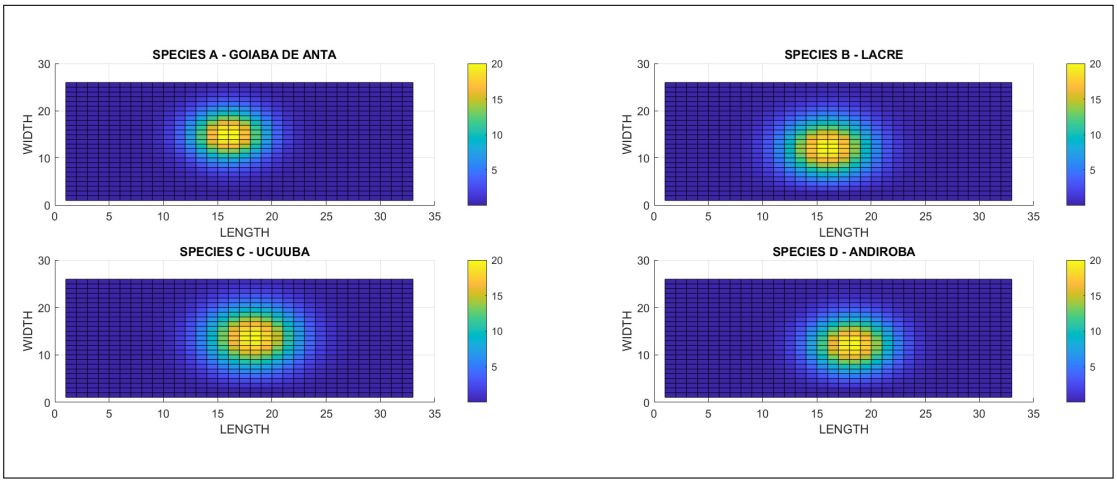
Taking into account the two distribution techniques, circular and alternating, when carrying out the simulation with the same parameters used in previous simulations, we obtained the following situation.

Figure 16 – Spatial dynamics distribution of species in the form of concentric circles. For $t = 2$ years



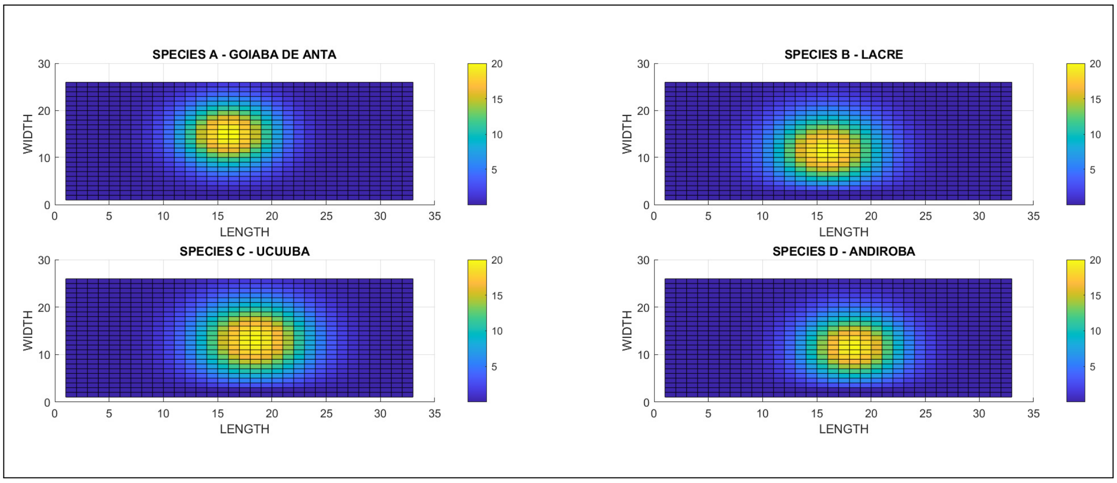
Source: the authors (2024)

Figure 17 – Spatial dynamics distribution of species in the form of concentric circles. For $t = 4$ years



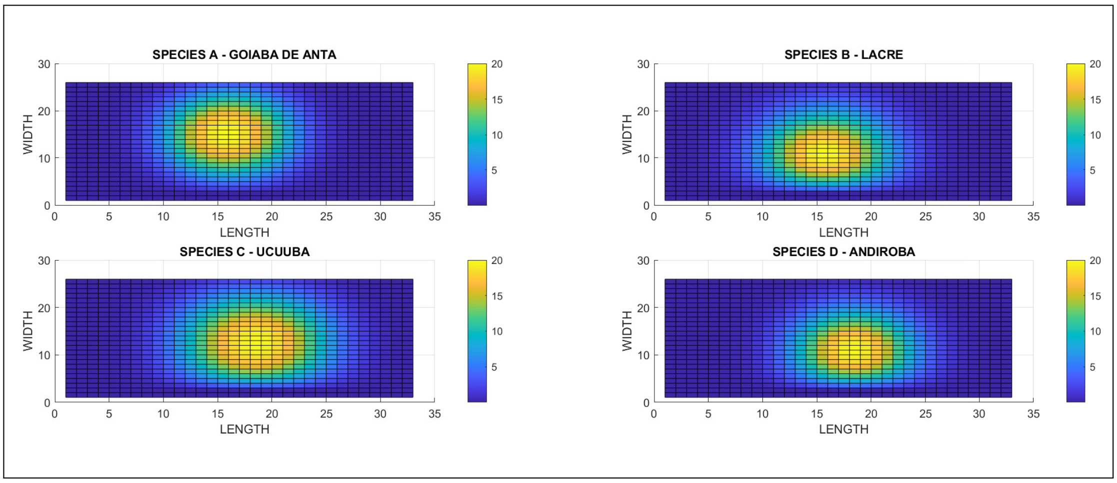
Source: the authors (2024)

Figure 18 – Spatial dynamics distribution of species in the form of concentric circles. For $t = 6$ years



Source: the authors (2024)

Figure 19 – Spatial dynamics distribution of species in the form of concentric circles. For $t = 8$ years

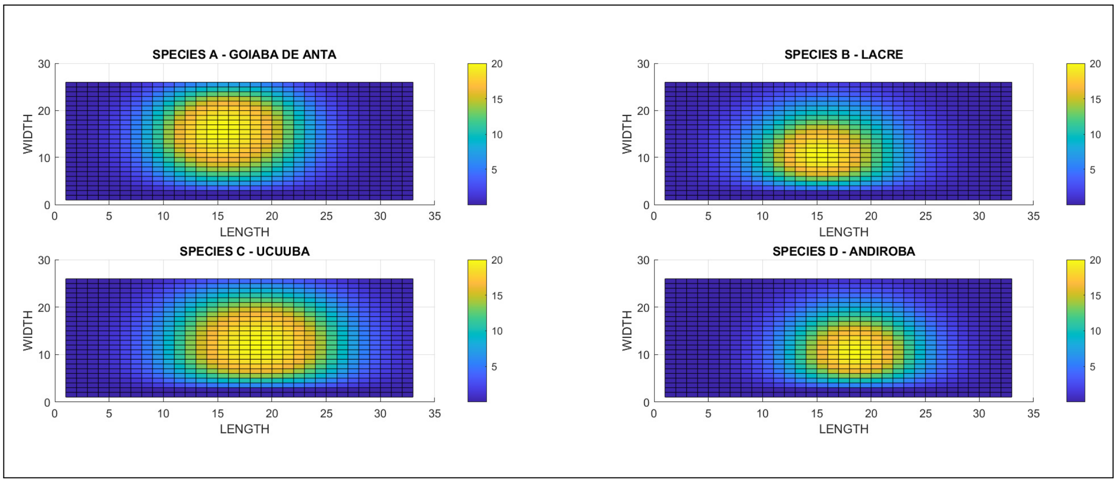


Source: the authors (2024)

We can highlight that the species C (Ucuúba) presented a behavior similar to that of the alternating distribution and obtained the best competition strategy in the space under study, highlighting the importance of shade for its development.

The remaining species developed according to the concentric circle technique, where each one developed differently in each circle, highlighting the intraspecific competition of each species.

Figure 20 – Spatial dynamics distribution of species in the form of concentric circles. For $t = 10$ years



Source: the authors (2024)

5 CONCLUSIONS

In this groundbreaking research, a comprehensive mathematical model was developed to simulate the recovery of deforested areas in the Amazon, with a specific focus on the interdependence and competition among four tree species crucial for reforestation efforts. The model, implemented through numerical solutions stands out as a valuable tool for predicting the intricate ecological succession within a given environment.

The mathematical approach leverages the central finite differential methods for spatial discretization and the Crank-Nicolson method for temporal discretization. These methods, applied using MATLAB software, contribute to the development of algorithms – availabe at https://github.com/marcosmsuea/reforestation – capable of generating realistic scenarios over a specified period. By incorporating estimated parameters, the researchers provide a nuanced understanding of how different tree species interact and compete during the reforestation process.

The research underscores the significance of mathematical modeling the interdependence of species during reforestation projects in the Amazon. The numerical solutions offer valuable insights into the ecological succession dynamics, guiding future conservation efforts and optimizing resource allocation.

The finding of this study hold immense practical implications for real-life reforestation project in the Amazon. By utilizing the developed model, conservationists and environmentalists can make informed decisions on the selection and distribution of tree species, ensuring a more resilient and diverse ecosystem. this research contributes to the ongoing global efforts to combat deforestation and promote sustainable reforestation practices.

The adoption of numerical methods specifically central finite differential methods and the Crank-Nicholson method, provides a robust foundation for simulating complex ecological processes. These methods allow researchers to computationally analyze and visualize the dynamics of species interdependence over time, offering more practical and efficient alternative to solely relying on theoretical approaches.

The study explored computer simulations to evaluate different reforestation strategies in the Amazon, using different types of planting arrangements. The proposals analyzed included cultivation in the form of concentric circles, the alternating distribution of species, and circular planting organized in quadrants. The simulation with concentric circles demonstrated efficiency in maximizing biodiversity and facilitating forest management, while the alternating distribution of species was effective in promoting beneficial ecological interactions, resulting in greater ecosystem resilience. On the other hand, the circular arrangement in quadrants proved to be advantageous for optimizing the use of space and improving accessibility for human interventions. Each strategy presented specific advantages, suggesting that combining these approaches could be the key to more efficient and sustainable reforestation in the Amazon region.

In essence, the integration of numerical methods into ecological modeling, as demonstrated in this study, presents a powerful tool for advancing our understanding for reforestation dynamics and fostering sustainable practices in one of the world's most vital ecosystems, the Amazon rainforest. One possibility for future research is analyzing the sensitivity of the parameters involved in modeling, as well as incorporating geospatial data and specific characteristics data from other regions into the model. Another possibility is to use the finite element method in the spatial discretization of the model, combined with another implicit or semi-implicit scheme in the temporal discretization. These improvements could provide more accurate and comprehensive insights into the recovery of deforested areas in the Amazon.

REFERENCES

Alencar, A., Nepstad, D., McGrath, D., Moutinho, P., Pacheco, P., Diaz, M. V., Soares Filho, B., et al. (2004). *Desmatamento na Amazônia: indo além da "emergência crônica".* Belém: IPAM.

Altamirano-Fernández, A., Rojas-Palma, A., & Espinoza-Meza, S. (2024). Optimal rotation age in fast growing plantations: a dynamical optimization problem. *Bulletin of Mathematical Biology*, *86*(51).

Barbosa, F. M., Dantas, M. V. d. O., Sales, M. d. G. S., Maia, G. N., & Santos, J. d. (2018). Distribuição de espécies arbóreas em função de variáveis edáficas em um trecho de floresta na Amazônia Central. *Acta Amazonica, 48*(3), 228-238.

Benatti, José Heder & Rodrigues, L. (2005). Privately-owned forests and deforestation reduction: an overview of policy and legal issues. In MOUTINHO, P. SCHWARTZMAN, S. *Tropical Deforestation and Climate Change.* Brasília: IPAM.

Carvalho, G., Cristina Barros, A., Moutinho, P., & Nepstad, D. (2001). Sensitive development could protect amazonia instead of destroying it. *Nature*, *409*(131).

Carvalho, P. E. R. (2014). *Espécies arbóreas brasileiras: volume 5*. Colombo: Embrapa Florestas.

EMBRAPA (2022). *Goiabeira-serrana: cultivar a fruta in natura e em produtos*. Retrieved from https://www.embrapa.br/busca-de-noticias/-/noticia/30257281/goiabeira-serrana-cultivar-a-fruta-in-natura-e-em-produtos.

Gotelli, Nicholas J & Graves, G. R. (1996). *Null models in ecology*. Washington, D.C.: Smithsonian Institution Press.

Johnson, W. C. (1977). A mathematical model of forest succession and land use for the North Carolina Piedmont. *Bulletin of the Torrey Botanical Club, 104*(4), 334-346.

Laurance, W. F., Albernaz, A. K., Fearnside, P. M., Vasconcelos, H. L., & Ferreira, L. V. (2004). Deforestation in amazonia. *Science*, pages *304*(5674), 1109–1111.

Macedo, S. T. (2008). *Crescimento inicial de espécies pioneiras e clímax em resposta a aplicação de calcário e formulações de npk em plantios para recuperação de áreas degradadas na região do Rio Urucu, Coari - AM* (Dissertação de Mestrado) - Instituto Nacional de Pesquisas da Amazônia, Manaus, AM, Brasil.

Miyaoka, Tiago Y & Meyer, J. (2015). Impacto ambiental e populaçoes que interagem: uma modelagem inovadora, aproximaçao e simulaçoes computacionais. *Revista Biomatemática (IMECC-Unicamp), 25*, 109-130.

Nepstad, D. C., Stickler, C. M., & Almeida, O. T. (2006). Globalization of the amazon soy and beef industries: opportunities for conservation. *Conservation biology*, pages *20*(6), 1595–1603.

Prestes, M. F. B. (2011). *Dispersão de material impactante em meo aquático: Modelo matemático, aproximação numérica e simulação computacional-Lagoa do Taquaral* (Dissertação de Mestrado) - Universidade Estadual de Campinas, Instituto de Matemática, Estaística e Computação Científica, Campinas, SP, Brasil.

Santos, C. F. L. (2013). *Modelagem matemática do aumento de densidade de vegetação na Amazônia e dinâmica populacional com competição intra e interespecífica* (Dissertação de Mestrado) - Universidade Estadual de Campinas, Instituto de Matemática, Estatística e Computação Científica, Campinas, SP, Brasil.

Santos, C. F. L. (2022). *Modelagem matemática para descrever a recuperação de áreas degradadas na Amazônia* (Tese de Doutorado) - Universidade Estadual Paulista, Instituto de Biociências, Botucatu, SP, Brasil.

Santos, J. P., Silva, L. A., Lima, J. S., Barbosa, R. I., Miranda, I. S., & Moura, M. V. S. (2019). Alternating species distribution on a grid: an analysis of the spatial distribution of three tree species in a tropical forest in the Brazilian Amazon. *Ecological Modelling, 407*, 1-9.

TerraBrasilis (2022). *Avisos de desmatamento – Amazônia legal*. Retrieved from http://terrabrasilis.dpi.iFsantosnpe.br/app/map/alerts?hl=pt-br.

Usó-Doménech, J.-L., Nescolarde-Selva, J.-A., Lloret-Climent, M., & González-Franco, L. (2018). Behavior of pyrophite shrubs in mediterranean terrestrial ecosystems (i): Population and reproductive model. *Mathematical Biosciences, 297*, 58-77.

Author contributions

1 – Nathan Souza da Silva (Corresponding Author)
Master in Applied Mathematics (Biomathematics) from the State University of Campinas
https://orcid.org/0009-0008-7469-454X • 19nsilva87@gmail.com
Contribution: Investigation, Visualization, Writing - original draft

2 – João Frederico da Costa Azevedo Meyer
Master and PhD in Applied Mathematics (Biomathematics) from the State University of Campinas
https://orcid.org/0000-0001-8002-7075 • jmeyer@unicamp.br
Contribution: Conceptualization, Project administration

3 – Marcos Marreiro Salvatierra
Master in Applied Mathematics (Biomathematics) from the State University of Campinas, PhD in Informatics (Computer Science) from the Federal University of Amazonas
https://orcid.org/0000-0002-6680-4023 • msalvatierra@uea.edu.br
Contribution: Formal analysis, Methodology, Validation, Writing - review & editing

4 – Agbata Benedict Celestine
Master in Mathematics from the Confluence University of Science and Technology, Nigeria
https://orcid.org/0000-0003-3217-7612 • agbatacelestine92@gmail.com
Contribution: Writing - review & editing

5 – Carlos Frank Lima dos Santos
Master in Applied Mathematics (Biomathematics) from the State University of Campinas, PhD in Biometrics from the São Paulo State University Júlio de Mesquita Filho
https://orcid.org/0000-0002-9163-8135 • cfsantos@uea.edu.br
Contribution: Writing - original draft

How to cite this article

Silva, N. S. da, Meyer, J. F. da C. A., Salvatierra, M. M., Celestine, A. B., & Santos, C. F. L. dos (2024). Analysis of the interdependence between species in reforestation projects in the Amazon. *Ciência e Natura*, Santa Maria, v.46, e86737. https://doi.org/10.5902/2179460X86737