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Multi-objective optimization of vibrating particle systems applied to engineering system design

Sistemas de partículas vibrantes e otimização multi-objetivo aplicado ao projeto de sistemas de engenharia

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ABSTRACT

The design of systems constitutes one of the most important pillars in engineering, in which it is desired to reduce costs, minimize time and resources, and guarantee the quality, efficiency, and safety of products. In recent decades, numerous population-based algorithms have been employed for this purpose. Among these, we can cite a new heuristic approach, the Vibrating Particles System (VPS) algorithm. This is based on simulating the free vibration of systems with one degree of freedom and viscous damping. The present work aims to extend the VPS to a multi-objective context. The results obtained considering two engineering design problems demonstrate that the proposed methodology was able to obtain good performance in terms of convergence and computational cost.

Keywords: Engineering system design; Vibrating particles system algorithm; Heuristic optimization; Multi-objective optimization

RESUMO

O projeto de sistemas configura um dos pilares mais importantes em engenharia, em que se deseja reduzir custos, economizar tempo e recursos, e garantir a qualidade, eficiência e segurança dos produtos. Nas últimas décadas, inúmeros algoritmos baseados em população têm sido empregados para essa finalidade. Dentre estes, destaca-se uma nova abordagem heurística, a saber, o Algoritmo de Partículas Vibrantes (APV). Este é fundamentado na simulação da vibração livre de sistemas com um grau de liberdade e que apresentam amortecimento viscoso. O presente trabalho tem por objetivo estender o APV para o contexto multi-objetivo. Os resultados obtidos considerando dois problemas de projeto em engenharia demonstram que a metodologia proposta foi capaz de obter bom desempenho em termos de convergência e custo computacional.

Palavras-chave: Projeto de sistemas de engenharia; Algoritmo de partículas vibrantes; Otimização heurística; Otimização multi-objetivo

1 INTRODUCTION

Optimization is an indispensable mathematical tool during engineering systems design. In this case, it is desirable to obtain products with greater efficiency and lower cost, without testing all the possibilities involved. For this purpose, the optimization problem must be well formulated, i.e.; all constraints that define this problem must be well specified (Ravindran et al., 2009).

From a mathematical point of view, the methods for solving the optimization problem can be classified as (Deb, 2001): classical (or deterministic) and heuristics (or non-deterministic). The first class consists of the use of information about the gradient of objective function and constraints to update a candidate solution to the optimization problem. On the other hand, the second, in general, consists of using a population of candidates, whose information about the individuals (objective function and constraints) is used to update the population along the generations. It is worth noting that in this case no information about the gradient of objective function and constraints is used to direct the search process (Vanderplaats, 1999).

As mentioned by Deb (2001), the main advantage of population-based algorithms is their ability to escape from local optima. In addition, it is important to mention that these algorithms are capable of achieving good approximations for optimal solutions in both mono and multi-objective optimization problems. On the other hand, there is no guarantee that a given optimization strategy can solve any type of problem (Wolpert & Macready, 1997). This means that new strategies must be proposed or improved to solve a wider range of optimization problems.

Due to the success of traditional population-based optimization techniques (Genetic Algorithms, Particle Swarm, Differential Evolution, among others) in applications in different fields of science and engineering, new strategies have been proposed. Among these, the Vibrating Particles System - VPS, proposed by Kaveh and Ghazaan (2017) for single-objective design problems, can be mentioned as a promising approach for solving optimization problems. VPS has been used to solve problems with different complexities, such as truss optimization with multiple natural frequency constraints (Kaveh & Ghazaan, 2017), tower cranes and supply points locating problems (Kaveh & Vazirinia, 2017), optimal design of reinforced concrete cantilever retaining walls (Kaveh & Jafarpour, 2017), and engineering system design (Andrade & Lobato, 2023).

This work aims to propose the extension of VPS to the multi-objective context, as well as compare it with other optimization strategies. This work is organized as follows. In Section 2, the VPS is briefly presented. The proposed methodology is described in Section 3. The results of two applications are presented in Section 4. Finally, in the last section, the conclusions are drawn.

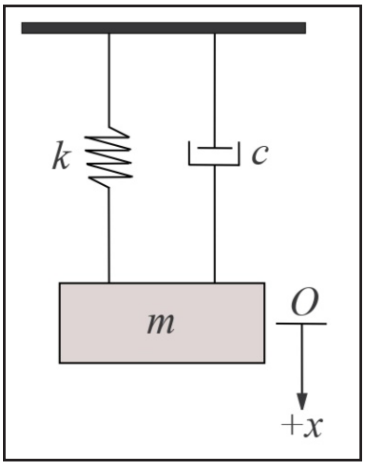
2 VIBRATING PARTICLES SYSTEM ALGORITHM

2.1 Conceptual Conception

The VPS is an optimization approach proposed by Kaveh and Ghazaan (2017) for trusses design. This recent optimization strategy consists of simulating the free vibration of systems with one degree of freedom and that present viscous damping. In this case, particles are considered as solution candidates of a system that gradually tends toward its equilibrium position. The vibrations can be classified into free (motion is maintained only by conservative forces) and forced (a periodic force is applied to the system) (Kaveh & Ghazaan, 2017).

Based on these concepts, the study object on which the VPS is founded refers to the free vibration of systems which can be represented as shown in Figure 1.

Figure 1 – Free vibration of a system with one degree of freedom with damping



Source: Adapted from Kaveh & Ghazaan (2017)

In this figure, it is possible to observe a block of mass m, a spring of spring constant k and a damper of damping coefficient c. By displacing the block at a distance x in relation to the equilibrium position (indicated by O), the block's motion equation is given by Eq. (1):

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{1}$$

The critical damping coefficient (c_c) is defined from the natural frequency (ω_n) of the system (Eq. (2) and Eq. (3)):

$$c_c = 2m\omega_n \tag{2}$$

$$\omega_n = \sqrt{k/m} \tag{3}$$

According to Kaveh and Ghazaan (2017), the value of the critical damping coefficient differentiates three types of systems: overdamped, critically damped, and underdamped. However, in VPS only the underdamped system is considered as the solution (the other two types refer to a non-vibratory movement). Thus, the solution of the differential equation (Eq. (1)) is given by Eq. (4):

$$x(t) = \rho e^{-\xi\omega_n t}\sin(\omega_D t + \phi) \tag{4}$$

$$\omega_D = \omega_n\sqrt{1 - \xi^2} \tag{5}$$

$$\xi = c/c_c \tag{6}$$

where the ω_D is the damped natural frequency and ξ is the damping rate.

The solution $x(t)$ requires the determination of constants ρ and ϕ, obtained by the definition of the initial conditions of the problem.

2.2 Basic Steps

The basic steps considered in VPS are presented as follows (Kaveh, 2017):

Step 1 – Initialization: The objective function, design variables (design space), constraints, and VPS parameters are defined. In order to initialize the optimization algorithm, the initial positions of particles are defined (randomly) inside the design space.

Step2 – Evaluation of the candidates' solutions: Each potential candidate (particle) is evaluated according to the objective function defined by the user.

Step 3 – Update of the particle positions: For each particle, three equilibrium positions considering different weights are defined: *i*) the best solution found until the current generation (HB); *ii*) a good particle (GP); and *iii*) a bad particle (BP). To select and, a current population is ordered according to the values of the objective function. After this ordering, these values are randomly chosen from the first and second halves, respectively. To represent the effect of damping, a descending function, proportional to the number of generations, is proposed:

$$D = \left(\frac{q}{q_{max}}\right)^{-\alpha} \tag{7}$$

where q is the actual generation, q_{max} is the total number of generations and is a constant defined by the user.

Thus, the particle positions are updated as:

$$x_i^j(q+1) = w_1\left[DAr_1 + HB^j\right] + w_2\left[DAr_2 + GP^j(q)\right] + w_3\left[DAr_3 + BP^j(q)\right] \tag{8}$$

$$A = \left[w_1\left(HB^j - x_i^j(q)\right)\right] + \left[w_2\left(GP^j(q) - x_i^j(q)\right)\right] + \left[w_3\left(BP^j(q) - x_i^j(q)\right)\right] \tag{9}$$

$$w_1 + w_2 + w_3 = 1 \qquad (10)$$

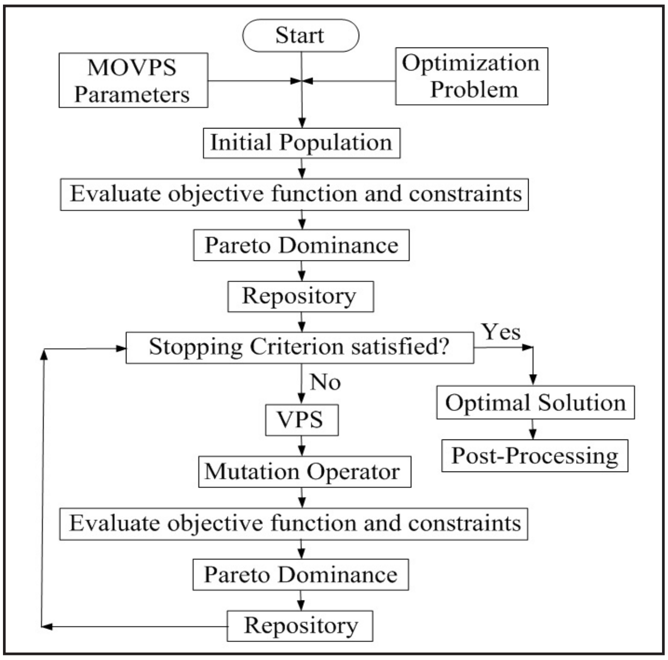
where w_1, w_2 and w_3 are parameters that represent the relative importance of HB, GP and BP, respectively, r_1, r_2 and r_3 are random numbers in the interval [0,1]; and x_i^j is the j-th variable of the i-th particle.

Parameters A and D (see Eq. (8)) represent the effect of ρ and $e^{-\xi\omega_n t}$ (see Eq. (4)), while the value of $\sin(\omega_D t + \phi)$ is considered equal to unity, for simplicity. The effect of a bad particle (BP) is not always considered during the optimization process in an attempt to improve the convergence of the algorithm. For this purpose, the parameter p (between 0 and 1) is defined and, for each particle, its value is compared with a random number ($rand$) in the range [0, 1]. If $p < rand$, then the weight of BP is equal to zero ($w_3 = 0$). As a consequence, $w_2 = 1 - w_1$

Step 4 – Stopping Criterion: Steps 2 and 3 are repeated until the established stopping criterion is satisfied and the optimal solution is presented.

3 MULTI-OBJECTIVE OPTIMIZATION VIBRATING PARTICLES SYSTEM

Figure 2 – MOVPS algorithm flowchart



Source: Authors (2023)

Due to the success of VPS in applications considering only a single objective, its extension to the multi-objective context is a natural step. In this case, the VPS for multi-objective problems, called here MOVPS (Multi-objective Optimization Vibrating Particles System), is structured as shown in Figure 2, with the main steps described below.

Step 1 – Initialization: The problem of interest (objective function, design space and constraints) and the MOVPS parameters (population size -NP- and mutation probability -p_m, in addition to q_{max}, w_1, w_2, p and α) are defined. Furthermore, the initial positions of all particles are determined randomly within the defined design space.

Step 2 – Evaluation of candidate solutions: Each potential candidate (particle) is evaluated according to the objective vector.

Step 3 – Pareto dominance criterion: This criterion is applied to the current population by comparing each solution i with all the others to check whether it is dominated by any other solution within the population. If so, there is at least one candidate that is better than i considering all the objectives and, therefore, the particle i cannot belong to the non-dominated set. On the other hand, if no solution dominates i, then this will constitute the non-dominated set.

Step 4 – Registration of non-dominated solutions in the Repository: The non-dominated solutions are stored in the Repository file. If the number of individuals in this file is greater than the size defined by the user, it is truncated according to a criterion called crowding distance (Deb, 2001). This operator describes the density of individuals present in the current solution so that the closest points are eliminated and the extreme points are always preserved.

Step 5 – Updating particle positions: In the multi-objective context, when using the main VPS operator, it does not make sense to order the population in terms of the values of the objective functions for the selection of HB, GP and BP, since the current optimal solution is composed of non-dominated candidates. Therefore, to order the population, the Euclidean distance computed between each point of the current solution and the origin of the objective space will be considered. Thus, GP and BP are chosen, randomly, from the first and second halves of the current ordered population, respectively. HB is determined through a random choice within the set of non-dominated solutions that constitute the population stored in the Repository. This choice is made after using the roulette method to select the cuboid that will contain the particle to represent HB (Coello, Pulido & Lechuga, 2004).

Step 6 – Application of the mutation operator: In order to avoid premature convergence to local solutions, the exploration of candidate neighborhoods is carried out by applying this mutation operator after dividing the population into three parts: *i*) the first is not modified; *ii*) the second is subjected to a uniform mutation over generations according to a defined probability; *iii*) and the third is modified by a non-uniform mutation over generations. In the latter case, the decay function (P_f) to quantify the percentage of individuals that are affected by this type of refinement is defined as (Coello, Pulido & Lechuga, 2004):

$$P_f = 1 - \left(\frac{q}{q_{max}}\right)^{(1/p_m)} \tag{11}$$

Thus, at the beginning of the evolutionary process, all particles in this population group are affected by the mutation operator and, as the evolutionary process progresses, it stops influencing this portion of the current population.

Step 7 – Repository Update: The new candidates generated are evaluated according to the vector of objective functions and grouped with the solutions present in the Repository. Then, the Pareto dominance criterion and, if necessary, the crowd distance operator are applied, so that only non-dominated candidates remain in Repository.

Step 8 – Stop Criteria: Steps 5 to 7 are repeated until the established stopping criterion is satisfied and then the optimal solution found by the algorithm, represented by the current population of the Repository, can be presented and submitted to the post-processing.

4 RESULTS AND DISCUSSION

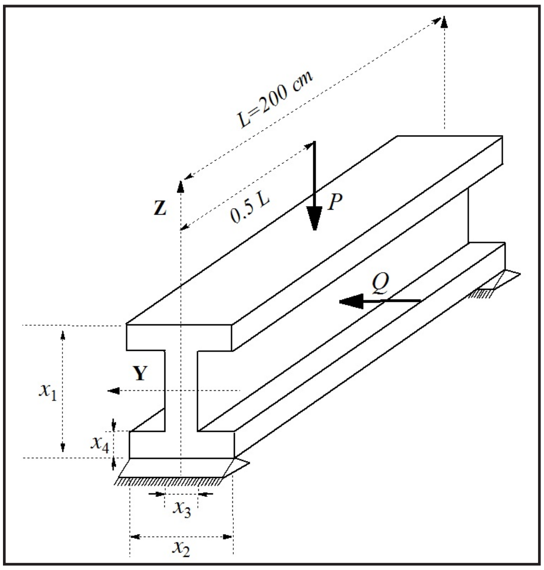
In order to evaluate the quality of obtained results by MOVPS, two case studies are considered (I-beam design and welded beam design). The population size (NP), the maximum number of generations (q_{max}) and the mutation probability (p_m) vary in these problems (these were defined after preliminary simulations). The number of particles in the repository was considered equivalent to NP and the other parameters used were: $w_1 = 0.3$, $w_2 = 0.3$, $p = 0.1$ and $\alpha = 0.05$.

To evaluate the inequality constraints, the External Penalty Method (Vanderplaats, 1999) was used (with a constant penalty factor equal to 10^6). The stopping criterion considered in all applications was the maximum number of generations. To compare the obtained results by MOVPS, the PMOGA (Pareto Multi-Objective Genetic Algorithm) (Castro, 2001) and MODE (Multi-objective Optimization Differential Evolution) (Lobato, 2008) algorithms are considered.

4.1 Beam Design

The first design problem considers a beam with an I-beam cross-section (Castro, 2001), whose dimensions are the design variables x_1, x_2, x_3 and x_4, as observed in Figure 3.

Figure 3 – Schematic representation of an I-beam



Source: Adapted from Castro (2001)

In this optimization problem, it is desired to minimize the cross-sectional area (cm²) and the maximum static displacement (cm), given respectively by:

$$\min f_1 = 2x_2x_4 + x_3(x_1 - 2x_4) \tag{12}$$

$$\min f_2 = \frac{PL^3}{48EI} \tag{13}$$

where the moment of inertia I is calculated as:

$$I = \frac{x_3(x_1 - 2x_4)^3 + 2x_2x_4\left(4x_4^2 + 3x_1(x_1 - 2x_4)\right)}{12} \tag{14}$$

and E is Young's Modulus (2×10⁴ kN/cm²), σ is the beam design stress (16 kN/cm²), P and Q are the vertical (600 kN) and horizontal (50 kN) loads, applied at the midpoint of the beam, respectively.

Regarding lateral constraints of the design variables, the following ranges are defined: 10 cm ≤ x_1 ≤ 80 cm, 10 cm ≤ x_2 ≤ 50 cm, 0.9 cm ≤ x_3 ≤ 5 cm and 0.9 cm ≤ x_4 ≤ 5 cm. Additionally, the following design constraint is considered:

$$g(x) = \frac{M_Y}{W_Y} + \frac{M_Z}{W_Z} \leq \sigma \tag{15}$$

where M_Y (30000 kN.cm) and M_Z (25000 kN.cm) are the maximum moments in the Y and Z directions; and and are the resistance modules in the W_Y and W_Z directions. The resistance modules are calculated by the following expressions:

$$W_Y = \frac{x_3(x_1 - 2x_4)^3 + 2x_2x_4\left(4x_4^2 + 3x_1(x_1 - 2x_4)\right)}{6x_1} \tag{16}$$

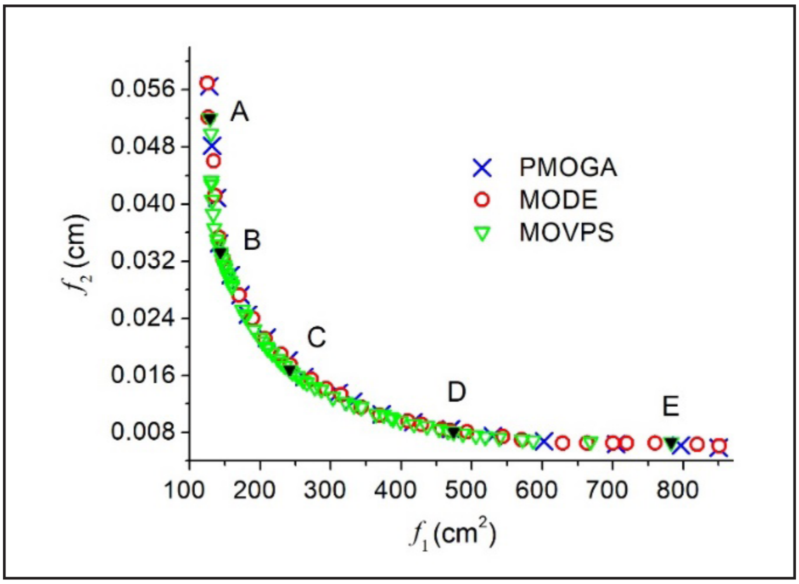
$$W_Z = \frac{(x_1 - 2x_4)x_3^3 + 2x_4x_2^3}{6x_2} \tag{17}$$

The parameters considered by MOVPS are: $p_m = 0.8$; $NP = 60$; $q_{max} = 200$ (these parameters represent 12060 objective function evaluations). Figure 4 presents the solutions obtained by MOVPS, PMOGA (25050 evaluations) and MODE (15030 evaluations). In this figure, it is possible to observe that the proposed methodology was able to obtain a good approximation of the Pareto curve concerning PMOGA and MODE algorithms. The observed differences are due to the scattering quality of the MOVPS in the extreme solutions found and a subtle dominance of the MOVPS points located in the neighborhood of point C in relation to other algorithms.

In this case, the reduction in the cross-sectional area of the I-beam (f_1) implies an increase in the maximum static displacement (f_2), and vice versa. This demonstrates the conflicting nature of the objectives. In Table 1 some points belonging to the optimal solution obtained by MOVPS are presented.

The extreme points obtained by MOVPS prioritize a single objective, allowing either a minimum deflection with an area of 782.4778 cm^2 (point E) or a minimum area with a deflection of 0.0520 cm (point A). The intermediate points present two design variables with common results (x_1 assuming its maximum value and x_3 assuming its minimum value). Points B and D present a greater specification by minimizing f_1 and f_2, respectively, while point C is a point with a good compromise between the objectives, i.e.; the minimization of any objective is not privileged.

Figure 4 – Pareto curve for the I-beam design



Source: Authors (2023)

Table 1 – Some points on the Pareto curve obtained by MOVPS for the I-beam design problem

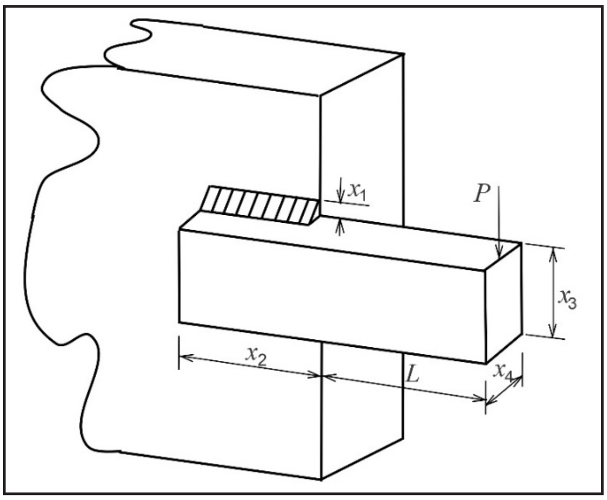
	A	B	C	D	E
x_1 (cm)	66.5244	80.0000	80.0000	80.0000	79.7800
x_2 (cm)	39.0870	38.5006	43.2309	50.0000	44.3796
x_3 (cm)	0.9000	0.9000	0.9000	0.9000	4.9675
x_4 (cm)	0.9000	0.9527	2.0082	4.0939	4.8991
f_1 (cm^2)	128.6086	143.6477	242.0177	474.0221	782.4778
f_2 (cm)	0.0520	0.0333	0.0168	0.0081	0.0066

Organized by the authors (2023)

4.2 Welded Beam Design Problem

The last application considers the welded beam design problem to minimize both fabricating cost (Eq. (18)) and displacement of the free end of the beam (Eq. (19)), subject to constraints on shear stress (τ), bending stress (σ), buckling load (P_c), end deflection (δ), and side constraints. The design variables (geometric characteristics of the weld and beam) are indicated by x_1, x_2 , x_3 and x_4, according to Figure 5.

Figure 5 – Schematic representation of a welded beam design



Source: Adapted from Castro (2001)

Mathematically, this problem can be represented by the following expressions (Castro, 2001):

$$\min f_1 = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (L + x_2) \tag{18}$$

$$\min f_2 = \frac{4FL^3}{x_3^3 x_4 E} \tag{19}$$

$$\tau - \tau_{max} \leq 0 \tag{20}$$

$$\sigma - \sigma_{max} \leq 0 \tag{21}$$

$$F - P_C \leq 0 \tag{22}$$

$$\frac{4FL^3}{x_3^3 x_4 E} - u_{max} \leq 0 \tag{23}$$

$$x_1 - x_4 \leq 0 \tag{24}$$

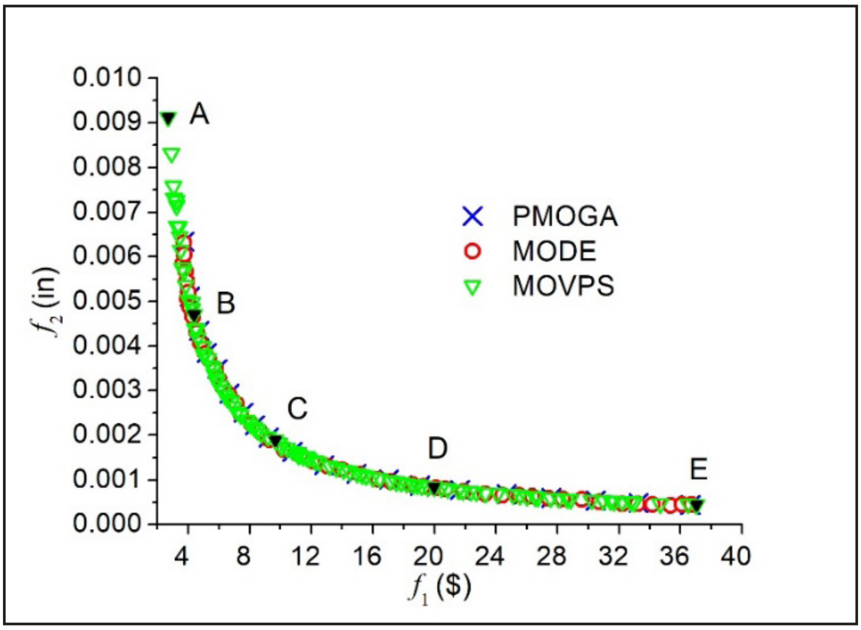
where: $\tau = \sqrt{\tau_1^2 + \tau_2^2 + \frac{x_2 \tau_1 \tau_2}{\sqrt{0.25\left(x_2^2 + (x_1 + x_3)^2\right)}}}$; $\tau_1 = \frac{6000}{\sqrt{2} x_1 x_2}$; $\tau_2 = \frac{6000(14 + 0.5 x_2)\sqrt{0.25\left(x_2^2 + (x_1 + x_3)^2\right)}}{2\left(0.707 x_1 x_2 \left(x_2^2/12 + 0.25(x_1 + x_3)^2\right)\right)}$;

$\sigma = \frac{504000}{x_3^2 x_4}$; $P_c = 64746.022(1 - 0.0282346 x_3) x_3 x_4^3$.

To solve this problem, the following lateral constraints are considered: 0.125 in ≤ x_1, x_4 ≤ 5 in, 0.1 in ≤ x_2, x_3 ≤10 in. In Eq. (18), the coefficients 1.10471 and 0.04811 are related to the material cost per unit of volume. In addition, the following constants are considered (Castro, 2001): F = 6000 lb, τ_{max} = 13600 psi, E = 30×10^6 psi, σ_{max} = 30000 psi, u_{max} = 0.25 in and L = 14 in.

The optimal solutions obtained by each optimization strategy are presented in Figure 6. This figure shows the conflicting nature between the objectives, as well as the agreement between the MOVPS solution and those obtained by PMOGA and MODE. The most evident difference refers to the fact that, in relation to solutions that preferentially minimize the cost (f_1), MOVPS was able to obtain some configurations not observed by the other algorithms (close to point A).

Figure 6 – Pareto curve for the welded beam design



Source: Authors (2023)

The parameters considered by MOVPS are: $p_m = 0.05$; $NP = 100$; $q_{max} = 240$ (24100 objective function evaluations). About PMOGA and MODE algorithms, 100200 and 25050 evaluations of the objective function vector were necessary to solve this problem, respectively.

In Table 2 some points (A, B, C, D and E) of the solution found by the proposed methodology are presented. In this case, the beam width (x_3) approaches the maximum limit (10 in). In relation to other design variables, it appears that the increase in thickness ($x_1$and x_4) accompanied by the reduction in beam length (x_2) leads to the minimization of beam deflection at the expense of increasing the price, and vice versa. The extreme points A and E prioritize, in particular, the lowest cost ($2.66) and the lowest deflection (0.0004 in), respectively. Points B and D represent those that tend to favor one objective or the other, while point C seeks to balance the fulfillment of both objectives, resulting in a cost of $8.0255 at a deflection of 0.0022 in.

Table 2 – Some points on the Pareto curve obtained by MOVPS for the welded beam design problem

	A	B	C	D	E
x_1 (in)	0.2394	0.3080	0.6920	0.8604	1.1167
x_2 (in)	5.2426	3.7144	1.3983	1.0727	0.8962
x_3 (in)	9.9671	10.0000	9.9797	10.0000	10.0000
x_4 (in)	0.2523	0.4349	0.9855	2.3275	5.0000
f_1 ($)	2.6600	4.0960	8.0255	17.7553	37.0673
f_2 (in)	0.0088	0.0050	0.0022	0.0009	0.0004

Organized by the authors (2023)

5 CONCLUSIONS

This work aimed to extend the VPS to the multi-objective context. The proposed methodology was applied to two classic design problems in the engineering area. The results obtained demonstrate that MOVPS was able to obtain, in each application, a good approximation to the Pareto curve in terms of convergence and diversity. It is worth noting that the results obtained by MOVPS were through a smaller number of evaluations of the objective function in relation to the algorithms considered for comparison. Furthermore, it is noteworthy that the MOVPS was easy to calibrate, i.e.; for the applications studied, no difficulties were observed when choosing the parameters of the proposed methodology. As a suggestion for future work, the MOVPS will be applied in engineering system design considering the effect of uncertainty and robustness.

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