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**Special edition** 

# **Feedback control applied to a low cost ball-in-tube air levitation plant**

Controle Realimentado aplicado em uma planta de levitação a ar de uma bola no tubo de baixo custo

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### **ABSTRACT**

Commercial educational tools for teaching feedback concepts in control systems engineering and mechatronics are generally expensive, large, complex and sensitive instruments; therefore, it cannot be used in universities with budget constraints, which greatly hinders the teaching of basic concepts of control theory and application. The mathematical tools that are taught in the disciplines of control theory in the undergraduate course are often abstract, especially in terms of practical application in an industrial setting, which makes it necessary to use didactic control plants to complement teaching and hands-on experimentation. This paper presents a low-cost ball and tube air levitation laboratory system as a teaching tool for engineering. Ball and tube laboratory setup is a dynamic benchmark system, designed to control the position of the ball on a vertical upward airflow that counteracts the gravitational force exerted on the ball without mechanical support. A blower feeds airflow, and the position of the ball is measurable by using an ultrasonic distance meter. As the purpose of the article is to serve as support material for undergraduate students who are taking their first steps in the study of control theory and feedback control, detailed modeling by the laws of physics are presented, followed by classic tools for the study of linear control and then a proportional-integral-derivative (PID) control system is developed. A modified structure of the classic PID is used and the results are compared with the classic structure

**Keywords:** Control engineering education, Ball and tube air levitation, mathematical modeling, Classic control tools, PID controller

#### **RESUMO**

Ferramentas educacionais comerciais para ensinar conceitos de controle realimentado em engenharia elétrica e mecânica são geralmente caras, grandes, complexas e possuem instrumentos muitos sensíveis;



portanto, não pode ser utilizado em universidades com restrições orçamentárias, o que dificulta muito o ensino de conceitos básicos de teoria e aplicação de controle. As ferramentas matemáticas que são ensinadas nas disciplinas de teoria de controle no curso de graduação são muitas vezes abstratas, principalmente em termos de aplicação prática em ambiente industrial, o que torna necessário o uso de plantas de controle didáticas para complementar o ensino e a experimentação prática. Este artigo apresenta um sistema de laboratório de levitação a ar com bola e tubo de baixo custo como uma ferramenta de ensino para engenharia. A planta para laboratório de bola e tubo é um sistema dinâmico padrão, projetado para controlar a posição da bola em um fluxo de ar ascendente vertical que neutraliza a força gravitacional exercida na bola sem suporte mecânico. Um soprador alimenta o fluxo de ar e a posição da bola é medida usando um sensor de distância ultrassônico. Como o objetivo do artigo é servir de material de apoio para alunos de graduação que estão dando seus primeiros passos no estudo da teoria e aplicação de controle e controle realimentado, são apresentadas modelagens detalhadas pelas leis da física, seguidas de ferramentas clássicas para o estudo de controle linear e então um sistema de controle proporcional-integral-derivativo (PID) é desenvolvido. Uma estrutura modificada do PID clássico é utilizada e os resultados são comparados com a estrutura clássica.

**Palavras-chave:** Educação em engenharia de controle, levitação da bola com ar no tubo, modelagem matemática, ferramentas de controle clássico, Controle PID

# **1 INTRODUCTION**

When teaching automatic control, there is always the question of which part is more important, the theoretical or the practical. Especially in courses such as electrical and mechanical engineering, where in current curricula, the number of hours devoted to this content is very low compared to control engineering. In line with Kheir, Astrom, Auslander, Cheok, Franklin, Masten and Rabins (1996), it can be based on two lines of thought mainly. The first one would be practical experience, while the second one would be theory and mathematics. In fact, nowadays, few people would discuss that control students need to have both a wide experience implementing solutions in real problems and plants and a deep understanding of the mathematics and theory behind these solutions. The first stream, the one based in practical experience, relies on the idea that something needs to be controlled and so, control systems engineering curricula should be based on hands-on and practice experiences. While this has been the traditional vision of engineering, it started changing around one hundred years ago, when the second stream, the one based in theory and mathematics, started to gain importance

(Froyd, Wankat & Smith, 2012; Saenz, Chacon, de la Torre, & Dormido, 2017). Therefore, achieving a balance between theoretical concepts and physical intuition, with a very limited workload, is a great challenge in teaching control.

One of the key elements for a practical approach to the assumptions and theories in control systems is the laboratory setups that can assist students in linking theoretical concepts to their practical application. Accordingly, the gap between industries and universities might reduce if correct simulation and implementation of real processes in laboratories become prevalent (Tootchi, Amirkhani, & Chaibakhsh, 2019).

Numerous vendors offer their products to academic institutions; unfortunately most of these devices come at a steep price. These products, such as linear and rotary inverted pendulums, DC motor experiments, coupled tank apparatuses, magnetic suspension control systems, ball-on-plate systems, magnetic levitation or simplified helicopters provide an excellent introduction to real control problems (Feisel & Rosa, 2005; Ma & Nickerson, 2006; Chen, Song, & Zhang, 2010; Wojtulewicz, Chaber, & Ławry´nczuk, 2016; Gunasekaran & Potluri, 2012). However, for most universities thousands or even tens of thousands of dollars invested in one workstation is financially prohibitive, in particular when in reality 5–10 of them are needed for a single class. This is not the only issue with currently available laboratory hardware, as these devices are usually paired with a closed-source software bundle that is yet again linked to commercial software such as MATLAB or LabView. Furthermore, students can only experiment in the laboratory and under close supervision; they may not take equipment home to continue working on their assignments. This, of course, is because of the high price, large size and delicate nature of these devices (Takács, Konkoly, & Gulan, 2019).

In recent decades, more precisely in 2005, we have been witnessing a revolution in teaching microcontroller and embedded software engineering thanks to the microcontroller unit (MCU) prototyping boards known collectively as the Arduino Ecosystem. The secret to the success of the Arduino Ecosystem boards is standardization.

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The Arduino Uno in particular became the gold-standard for education, since the hardware layout and functionality is unified, the software is free and well maintained and the support and community of helpful enthusiasts is especially active. The functionality of basic boards can be readily extended since the physical and electrical layout of the Uno board known as 'R3' is retained across numerous other products. As detailed by Garrigos, Marroqui, Blanes, Gutierrez, Blanquer and Canto (2017), extra hardware known as 'Shields' can be simply pushed onto the board through the header pins extending the base board functionality for classroom use. The Arduino has made a lasting appearance in the education of feedback control, system identification and signal processing concepts as well. The effect of Arduino-based control projects on student performance and satisfaction has been also quantified, demonstrating positive learning outcome and student satisfaction (Omar, 2018; Esposito, Mujica, Garcia, & Kovacs, 2015; Uyanik & Catalbas, 2018).

The design and analysis of control systems has been great interest for researchers and engineers. With the advancement of technology and complexity of industries, the crucial role of analysis and design of control systems has become more important over time. Control students need to have a solid background in implementing solutions in real plants and processes, and a deep understanding of the mathematics, statistics, and theories behind those solutions. Therefore, achieving a balance between theoretical knowledge and its practical applications is a significant challenge in control education.

One device that can be used to teach a wide range of algorithms used in control are air levitation ball and tube laboratory setups. Levitation systems have long been utilized in control system laboratories. Levitation is a process in which an object is floated against gravity by a physical force. An exciting example of levitation is based on airflow. Air levitation uses an air stream provided by a blower to obtain the levitating force on a levitator. This phenomenon stems from the Bernoulli principle (Qin & Duan, 2017). The aerial levitation control system is considered as an interesting device for educational purposes, due to its great versatility. Furthermore, the system is small and

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simple, which is very convenient to be transported from one laboratory to another. This type of lab support can be an environment to implement a variety of controllers, for example: PID controller, hybrid controller, predictive controller, fuzzy logic controller, slip mode controller and others that are used for nonlinear systems (Kuo, Li, & Guo, 2005).

 In this article, the goal is to design a low-cost setup with educational functionality, which can increase the accessibility of the control experiences for students and improve the quality of teaching. The setup is in the final stages of fabrication and testing and a complete mathematical model is presented, using the laws of physics. A case study is presented, along with classic control

application tools, discretization, obtaining reduced order models. Results are validated for four controller tuning methods.

The paper is organized as follows. Section II presents a complete mathematical modeling using the laws of physics of the ball and pipe air levitation problem. Section III provides all content for plant discretization, application of Smith's Method to obtain a reduced order plant model and discretization and obtaining the difference equations of the controllers. At the end of the section, the results for four different tuning methods are presented and compared with respect to performance and control signal. A conclusion puts an end to the paper in Section IV.

### **2 MATHEMATICAL MODELING**

The development of the physical equations of an air levitated ball-tube system was already done in previous works such as Timmerman and Weele (1999), Escano, Ortega and Rubio (2005), Jernigan, Fahmy and Buckner (2009), Tootchi, Amirkhani and Chaibakhsh (2019), and Chacon, Saenz, De la Torre, Diaz and Esquembre (2017).

The mathematical modeling step of a physical system is an important part in the design and analysis of any form of control strategies. The mathematical modeling step of a physical system is an important part. From a model that faithfully describes all

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the characteristics of the dynamics of the real system, the designer will have enough information to choose the best control strategy to apply to the system. A very common task among control designers is to use the control designed in the system model for validation and testing of the same, before application in the real system ( Amirkhani & Nahvi, 2016).

It is not very common in the literature, a detailed modeling of the levitation system of balls and tubes using the laws of physics, due to several complex phenomena involved in the system. But, as in this article, one of the objectives is to review all the physical modeling part, a detailed modeling is presented based on the work of Tootchi, Amirkhani and Chaibakhsh (2019).

The high-speed air hits the bottom of the ball, creating a high-pressure zone under it. This high-pressure air moves over the curved surface of the ball with high velocity, creating a low-pressure zone. If the ball tends to move away from the middle of the stream, the atmospheric air around this low-pressure zone pushes it back to the middle because of its relatively high pressure. Hence, the lateral motion of the ball becomes stable in the middle of the flow. The vertical motion of the ball around this equilibrium point is stable as well, given by the balancing of gravity and the air drag.

#### **2.1 System Model**

The mathematical models of the air levitation system are developed in several previous studies, concisely (Chacon, Saenz, De la Torre, Diaz & Esquembre; 2017; Chołodowicz & Orlowski, 2017). The system dynamic equations are nonlinear. This is due to the nonlinear description of the air stream, which subjects to the Bernoulli's equation (Qin & Duan, 2017). This equation makes a crucial prediction about the relationship between the pressure and the velocity of a moving ideal fluid:

$$
p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \tag{1}
$$

where,  $^1$  ρ $\mathsf{v}^2$  is  $\cancel{\mathsf{k}}$ inetic energy and  $\rho$ gy is gravitational potential energy,  $\bm{\mathsf{p}}_1$  and  $\bm{\mathsf{p}}_2$  are the static pressures of at cross-section,  $\rho$  is the density of the following air,  $y_1$  and  $y_2$  are the different distances between the ball and the bottom of the pipe,  $v_1$  and  $v_2$  are the mean velocities of fluid flow at the cross-section. Once a fluid moves at high speed, it has less pressure than the same fluid at low speed. In experiments, one side of the ping-pong ball is in contact with low pressure that creates a gradient pressure, the other side of the ping-pong ball is in contact with a higher pressure that results in the pingpong ball being pushed towards the low pressure region. Placing the ball in the tube allows to achieve higher heights but the Bernoulli principle still works. This is because the tube accumulates the air increasing the speed around the ball and allow the ball to go higher. As air is blown out from the blower, it flows at high speed and this creates a low pressure region at the top of the tube. The still air around the ball is at a higher pressure and pushes on the ball and causes it to stay floating.

Since the ball moves through a fluid, a drag force appears. It could be said that the drag force grows as the flow velocity increases. Nevertheless, the relationship between those two parameters is not as simple as that, and it needs to be properly modeled. According to Stokes' law, the drag force of a small spherical object moving through a Newtonian fluid is given as in 2; where  $F_d$  is the drag force,  $\mu$  is the coefficient of viscosity, *v* is the velocity of the object relative to the fluid, and *r* is the radius of the sphere (Bridges & Robinson, 2020)

$$
F_d^{(\text{viscous})} = 6\pi r \mu v. \tag{2}
$$

However, when the spherical object moves at higher speeds, the flow stops being smooth and becomes turbulent. This requires more energy and makes the drag force to switch to the quadratic regime as it is shown in equation 3; *A* is the cross section of the spherical object and ρ is the density of the fluid.

$$
F_d^{\text{(inertial)}} = \frac{\rho A v^2}{2}.
$$
\n<sup>(3)</sup>

Nonetheless, a final correction needs to be made to the expression of the drag force of equation 3. An spherical object that travels through a finite-size medium (such a channel or a tube), does not have the same behavior as that same object traveling through an infinite-size medium. So, a proper way to integrate the nature of the medium into the drag force equation is through the so-called drag coefficient (C<sub>a</sub>). This coefficient can be empirically measured, and is a very accurate way of describing the size and composition of the fluid medium.

This mean that, for the particular case of the ball-tube system, the drag force that affects the ball would be the one shown in equation 3; where ρ is the density of the air,  $C_d$  is the drag coefficient, A is the cross section of the ball,  $v_f$  is the velocity of the air inside the tube and *y* is the position of the ball in the tube.

$$
F_d = \frac{1}{2} \cdot \rho \cdot C_d \cdot A \cdot (v - \dot{y})^2. \tag{4}
$$

Besides that, there are two more forces that act upon the ball. All the forces acting over the ball are shown in equations 5, which according to Newton's second law give us the dynamic equation 6 that describes the system (Timmerman & Weele, 1999; Escano, Ortega & Rubio, 2005; Jernigan, Fahmy, & Buckner, 2009). The force diagram can be seen in figure 3.1.1.

$$
F_g = m \cdot g
$$
  
\n
$$
F_b = \rho \cdot g \cdot V_b
$$
  
\n
$$
F_d = \frac{1}{2} \cdot \rho \cdot C_d \cdot A \cdot (v - \dot{y})^2.
$$
\n(5)

$$
m \cdot \ddot{y} = F_d + F_b - F_g. \tag{6}
$$

*F <sup>g</sup>* is the weight force of the ball, where *m* is the mass of the ball and *g* the gravitational acceleration. Finally,  $F_b$  is the buoyancy force, where  $V_b$  is the ball's volume.

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Figure 1 – Balance of forces acting on the ball. Beige arrow: Drag force and buoyancy force. Red arrow: weight force of the ball



Source: Authors

So, if equation 6 is rewritten using the expressions of the forces, the dynamic equation of the system would be equation 7.

$$
\ddot{y} = \frac{1}{2m} \cdot C_d \cdot \rho \cdot A(v_f - \dot{y})^2 + \frac{1}{m} \cdot \rho \cdot g \cdot V_b - g. \tag{7}
$$

Moreover, the ball will reach its steady state when it does not move; in other words, when  $y = y = 0$ . Besides that, the velocity of the air at the equilibrium point will be defined as v<sub>eq</sub> = v<sub>ƒ</sub> − y˙, so the gravitational force can be rewritten as in equation 8.

$$
g = \frac{\rho \cdot C_d \cdot A}{2(m - \rho V_b)} \cdot v_{eq}^2. \tag{8}
$$

Combining equation 8 and equation 7, it can be obtained a new dynamic equation 3.1.1.8. This equation describes the dynamics of the system, and it takes the friction force due to the airflow into account. That is why equation 9 is a nonlinear equation that can be linearized using Taylor's expansion. In the case of this project, the approaches for the controller design that are going to be used are linear ones; so, equation 8 needs to be linearized.

$$
\ddot{y} = g \cdot \left(\frac{m - \rho \cdot V_b}{m}\right) \cdot \left(\left(\frac{v_f - \dot{y}}{v_{eq}}\right)^2 - 1\right). \tag{9}
$$

#### **2.2 Linearization**

The linearization step for this system is very straightforward. Let  $x = \frac{v_f - \dot{y}}{v}$ . Then the equation 9 is of the type  $f = g(x^2 - 1)$ , and it can be easily linearized around the equilibrium point ( $v_{eq} = (v_w - y')$ , or  $x = 1$ ) using Taylor's approximation ( $f(x) \approx f(x_0)$ )  $+f'(1x_0(x-x_0))$ 

$$
\ddot{y} = \frac{2g}{v_{eq}} \left( \frac{m - \rho \cdot V_b}{m} \right) \cdot (v_f - \dot{y} - v_{eq}). \tag{10}
$$

Determining system behavior at a point of operation is a critical step, mainly due to the nonlinear behavior of the physical system. The identification of the process transfer function is carried out in the frequency domain with the open loop tests performed over the system. The system is with one input and one output (SISO). The input signal is wind speed generated by blower and output is an accretion of the position of the ball. Assuming the system is well described by the linearized model, the transfer function between ball position and wind speed is:

$$
G_0(s) = \frac{Y(s)}{V(s)} = \frac{1}{s} \cdot \frac{b}{(s+b)}.
$$
\n(11)

Where *V* (*s*) and *Y* (*s*) are wind speed and increment of ball's position about the equilibrium point, respectively and  $b=\frac{2g\cdot(m-\rho V_b)}{mv_{eq}}$ . Considering the fan can be modeled as a first-order process, the transfer function between the input voltage and the wind speed is represented as:

$$
G_1(s) = \frac{V(s)}{U(s)} = \frac{k_v}{(\tau \cdot s + 1)}.
$$
\n(12)

Where  $V(s)$ ,  $U(s)$  are wind speed and input voltage, respectively. In addition,  $k_{v}$  is the sensitivity gain that relates the input voltage to the wind speed at steady state, and τ is the time constant of the fan. It should be noted that there be existed a delay in the operator, as well as measurement delay, in the ball and pipe system, which needs to be included in the model. The fan used in this setup has electronic components that can bring out a delay in the time of performing the command. Moreover. A low pass filter has also been used to reduce the signal noise of the measurement, which results in a delay in the system. It is necessary to mention, the values of these delays are not clear and are estimated at the identification phase. In the present paper, the effect of the two delays is assumed to be  $T_{d}$ , cumulatively.

$$
e^{-T_d \cdot s} = \frac{1 - T_d \cdot s}{T_d \cdot s + 1}.\tag{13}
$$

Finally, the transfer function of the entire system defines as follows:

$$
G_f(s) = G_0(s) \cdot G_1(s) = \frac{Y(s)}{U(s)} = \frac{b \cdot k_v \cdot (1 - T_d \cdot s)}{s \cdot (s + b) \cdot (\tau \cdot s + 1) \cdot (T_d \cdot s + 1)}.
$$
\n(14)

As this article is intended for undergraduate students, who are taking their first steps in the area of classical control theory. We will adopt a normalized system, so that the system of the equation 14 can be approximated by a fourth-order linear system, thus simplifying the controller design stage.

$$
G(s) = \frac{Y(s)}{U(s)} = \frac{1}{(s+1)^4}.
$$
\n(15)

On a final note, it should be addressed the fact that the equations which were presented, do not take the dynamics of the actuator into account nor they consider the non-linearities of the system.

### **3 CONTROLLER DESIGN**

In this paper, the control objective is to impel the ball to track a reference trajectory

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by regulating the voltage of the blower. The difference between a setpoint and the current position of the ball (position Error) is the input of the controller. The output of the controller will be the control command, sent to the blower driver to control the blower's rotation speed. Eventually, the position of the ball will be variate according to the generated airflow. Proportional-integral-derivative(PID) controller was adopted, due to the fact that it is a control strategy widely used in industrial plants. The PID controller is favorite for its simple functionality which allows for straightforward operation.

#### **3.1 Classical PID Equation - Standard Isa**

$$
u(t) = k_p e(t) + \frac{k_p}{t_i} \int e(t)dt + k_p t_d \frac{d}{dt} e(t)
$$
\n(16)

where the error *e*(*t*) = setpoint *−* entrada.

To implement in a microcontroller (For example, microcontrollers from the Arduino ecosystem), it is necessary to discretize the continuous equation above.

A trapezoidal sum was used to approximate the integral and for the derivative term we will use finite differences (regressive).

$$
\int e(t)dt = \sum \left(\frac{e(k) - e(k-1)}{2}\right)T_s,\tag{17}
$$

$$
\frac{d}{dt}e(t) = \frac{e(k) - e(k-1)}{T_s}.\tag{18}
$$

Where  $T<sub>s</sub>$  is the sampling time. Choosing the correct sampling time is very important for digital systems. A good choice can be made using the method proposed by Zigle-Nichols.

$$
T_s < \frac{\theta}{4}, \qquad (19)
$$
\n
$$
\tau < T < \tau
$$

$$
\frac{1}{10} \le T_s \le \frac{1}{20}.\tag{20}
$$

Thus, the PID digital transfer function can be written as:

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$$
C(z^{-1}) = \frac{u(k)}{e(k)} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}},
$$
\n(21)

where,

$$
q_0 = k_p \left( 1 + \frac{T_s}{2t_i} + \frac{t_d}{T_s} \right),
$$
  

$$
q_1 = -k_p \left( 1 - \frac{T_s}{2t_i} + \frac{2t_d}{T_s} \right),
$$
  

$$
q_2 = \frac{k_p t_d}{T_s}.
$$

With this, we can manipulate the 21 equation to obtain the control law that will be implemented.

$$
u(k)(1 - z^{-1}) = q_0e(k) + q_1z^{-1}e(k) + q_2z^{-2}e(k),
$$
  

$$
u(k) - u(k)z^{-1} = q_0e(k) + q_1z^{-1}e(k) + q_2z^{-2}e(k),
$$
  

$$
u(k) = u(k)z^{-1} + q_0e(k) + q_1z^{-1}e(k) + q_2z^{-2}e(k).
$$

Now we apply the inverse z-transform to obtain the difference equation (which will be implemented in the microcontroller).

$$
u(k) = u(k-1) + q_0 e(k) + q_1 e(k-1) + q_2 e(k-2).
$$
\n(22)

#### **3.2 Modified structure I+PD**

Consider the case where the reference input is a step function. The PID control involve a step function in the manipulated signal. Such a step change in the manipulated signal may not be desirable in many occasions. Therefore, it may be advantageous to move the proportional action and derivative action to the feedback path so that these actions affect the feedback signal only. It is called the I+PD control. The manipulated signal is given by:

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$$
u(t) = k_c \frac{1}{t_i s} R(s) - k_c \left( 1 + \frac{1}{t_i s} + t_d s \right) B(s).
$$
\n(23)

Notice that the reference input R(s) appears only in the integral control partThus, in I+PD control, it is imperative to have the integral control action for proper operation of the control system.

Therefore, the difference exclusion of this structure is represented as follows:

$$
u(k) = u(k-1) - p_0(y(k) - y(k-1)) + p_1(r(k) - y(k)) - p_2(y(k) - 2y(k-1) + y(k-2)).
$$
\n(24)

where,

 $p_0 = k_p \left( 1 - \frac{T_s}{2t_i} \right),$  $p_1 = \left(\frac{k_p T_s}{t_i}\right),$  $p_2 = \left(\frac{k_p t_d}{T_s}\right).$ 

#### **3.3 Approximation by a reduced order model (Smith's Method)**

The Doctor. Cecil Smith (1972) proposed that the values of  $\theta$  and  $\tau$  be selected in such a way that the model and the real responses coincide at two points that present a high rate of variation. The intermediate values determined from the graph are the magnitude of the value applied at the input and the magnitude of the state of change at the output, two times being adopted, in which the first is when the output reaches 28%, and the second, 63% of the final output value.

Considering the plant model defined in the equation 15. To define the parameters of the PID controller of this plant, first you must model a 1<sup>st</sup> order plant with transport delay (FOPDT). Through Smith's method, the temporal response of the plant to the step

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was traced and then the points were marked at the points 0*.*283*y*(*∞*) and 0*.*632*y*(*∞*), as shown in the figure below.





Source: Authors

The plant parameters are calculated using the relation:

$$
\tau = 1.5(t_2 - t_1) \rightarrow \tau = 1.5(4.35 - 2.69) = 2.49
$$
\n
$$
\theta = (t_2 - \tau) \rightarrow \theta = (4.35 - 2.49) = 1.86
$$
\n(26)\n
$$
K_p = 1
$$
\n(27)

The FOPDT model is defined as:

$$
\hat{G}_p(s) = \frac{e^{-1.86s}}{(2.49s + 1)}.\tag{28}
$$

With these parameters obtained and using the following design criteria for controller tuning ( $\xi = 0.8$ ,  $\omega_n = 0.5$  rad/s), the following parameters are obtained for four different types of PID controllers.

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The next step is the discretization of the plant. There are several ways to choose the sampling period, the following way is used:

$$
Ts = \frac{1}{(4)\alpha} \to \mathbf{0.7},\tag{29}
$$

where α refers to the dominant closed-loop pole. The transfer function using zeroorder hold (ZOH) is:

$$
\frac{0.0057(z+5.7240)(z+0.5706)(z+0.0570)}{(z-0.4966)^4}.
$$
\n(30)

Representing by equation the difference:

$$
y(k) = 1.9863y(k-1) - 1.4796y(k-2) + 0.4898y(k-3) - 0.0608y(k-4)
$$
  
+ 0.0058u(k-1) + 0.0365u(k-2) + 0.0209u(k-3) + 0.0011u(k-4) (31)

#### **3.4 Simulation of the results**

Four simulations were performed, one for each type of PID controller tuning. Analyzing the over-signal of the controllers (see Figure 3), it can be observed that the tuning by Aström-Hägglund, because its control signal is the smallest among all simulated tunings. In second place is the Chien-Hrones-Reswick tuning, followed by AMIGO and, finally, the Ziegler-Nichols tuning obtained the highest control signal.



Figure 3 – Control signal from the PID controller tunings

Regarding the response time, all controllers have practically the same value, as can be seen in the Figure 4. Thus, it can be concluded that for the specifications of ξ  $= 0.8$  and  $\omega_n = 0.5$  rad/s, the PID controller adjusted by Aström-Hägglund obtained the best results.



Figure 4 – 4*th* order plant response with PID controller

For the I+PD controller structure, four simulations were also performed for the step response, one for each type of tuning. Analyzing the response time for the design specifications of  $\xi$  = 0.8,  $\omega_n$  = 0.5 rad/s the best tuning is Ziegler-Nichols (see Figure 5). Evaluating the oversignal, the best result obtained was the Aström-Hägglund tuning.



Figure 5 – 4*th* order plant response with I+PD controller

Regarding the control signal, it is observed that the modified structure I+PD presented a considerable improvement in relation to the classic PID structure (see Figure 6). The tuning that presented the best result, considering the control signal, was the Aström-Hägglund, followed by Ziegler-Nichols.



Figure 6 – Control signal from the I+PD controller tunings

# **4. CONCLUSION**

In this work a complete modeling was presented using the laws of physics, of the classic ball and pipe air levitation setup. A case study, considering a model of a fourth-order plant, was used and classic control techniques widely used in industrial applications were addressed. A reduced order model of the plant was obtained using Smith's Method for the design of the PID controller and a modified I+PD structure. Four controller tuning methods are used, and the performance and control signal are evaluated and compared. This work was a first contact of undergraduate students with the application of feedback control. As future work, control identification techniques will be addressed, and a comparison between the identified models with the real system will be evaluated.

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