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Special Edition

Performance of the ADF test in stationary series within structural breaks

Desempenho do teste ADF em séries estacionárias na presença de quebras estruturais

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ABSTRACT

The study of time series has been developed constantly, given the large volume of observed and measured data over the years. An important characteristic of time series is stationarity, which is mostly analyzed by unit root tests. It is a consensus in the literature that structural breaks, when present in the data series, can bias the result of the Augmented Dickey Fuller Test (ADF), the best known and most widely used method of stationarity investigation. So far, however, there is no consensus regarding the intensity that structural breaks can affect the power of the ADF Test, making the decision about using it difficult and possibly leading researchers to errors under those changes. Thus, this article analyzed the influence of level shift (LS) structural breaks in the stationarity analysis in annual time series using the ADF test through the rejection proportion of the null hypothesis. It was observed that this procedure tends to reject the null hypothesis in the presence of structural breaks in a possible confusion with the presence of a unit root. Furthermore, it was noted that, as the initial perturbation ω increased, the power of the test was rapidly reduced, mainly with level change breaks imputed in positions closer to the origin of the data series.

Keywords: Time series, Structural breaks, Level shift, ADF test

RESUMO

O estudo de séries temporais vem se tornando cada vez mais necessário, dado o volume de dados observados e medidos ao longo dos anos. Dentre as características mais importantes das séries temporais, tem-se a estacionariedade, que é majoritariamente analisada pelos testes de raiz unitária. É consenso na literatura que as quebras estruturais, quando presentes nas séries, podem viesar o resultado do Teste de Dickey-Fuller Aumentado (ADF). No entanto, até o momento, a intensidade com que as quebras afetam o poder do teste ADF não é clara, dificultando a tomada de decisão acerca da utilização deste teste e podendo induzir a erros quando a série contar com rupturas estruturais. Este trabalho comprova a influência de quebras estruturais do tipo level shift (LS) na análise de estacionariedade em séries temporais anuais estacionárias pelo do Teste ADF, quantificando seu poder. Conclui-se que o Teste ADF tende a não rejeitar a hipótese nula na presença de rupturas em uma possível confusão com a presença de raiz unitária. Ainda, ao aumentar a perturbação inicial ω, o poder do teste foi sendo rapidamente reduzido, principalmente com quebras de mudança de nível imputadas nas posições mais próximas à origem da série de dados.

Palavras-chave: Séries temporais, Quebras estruturais, Mudança de nível, Teste ADF

1 INTRODUCTION

The study of time series and structural breaks, which has been increasing in recent years, is based on considering characteristics of the data series. Among the most important is stationarity, which is normally evaluated by stationarity tests when considering the presence or absence of a unit root in the data series.

The concept of stationarity takes into account the analysis that the mean and variance are stable over a period considered, as mentioned by Hansen (2001). Structural changes, on the other hand, must be evaluated in the context of a model and deal with a change in the intercept, in the trend, or in both, over a considered period.

Tsay (1988) and Chen e Liu (1993) address four main types of perturbations that series can suffer, considering two types of outliers and two types of structural changes. The additive outlier (AO) and the innovational outlier are the most known types of outliers, while the level shift (LS) and the temporary change (TC) correspond to the types of structural breaks addressed.

The most used tests in the literature to assess the presence of a unit root are the Dickey-Fuller (DF) test, proposed by Dickey e Fuller (1979), and the augmented Dickey-Fuller test (ADF), proposed by Said e Dickey (1984). In addition, other tests also appear as important means of stationarity analysis, such as the Phillips Perron test and the KPSS, proposed by Phillips e Perron (1988) and Kwiatkowski et al. (1992), respectively.

Although it is widely used, in the presence of structural changes, the ADF test can present biased results according to Perron (1989), causing the null hypothesis of a unit root to be rejected and reducing the power of the test. However, so far, the intensity with this test power is affected is not clear in the literature, making the decision-making about whether or not to use the ADF test for stationarity analysis subjective or intuitive.

Thus, given that the presence of structural breaks in the series influence the the ADF test power, this work begins to fill a gap present in the literature, by quantifying the power of the ADF test in stationarity analysis in time series stationary with structural breaks for level changes to make it possible to make a decision on the use of that test.

2 LITERATURE REVIEW

2.1 Stationarity

The analysis of the stationarity of a time series is essential to forecasting and modeling with data, which has been used by many methods and evaluated by unit root tests. For Gujarati e Porter (2011), a process is stationary if some characteristics of the series do not change over time, such as mean, variance and the covariance for the same lag. Non-stationarity, on the other hand, is characterized by the variation of one of these characteristics in a considered interval, indicating the presence of a unit root.

For Bueno (2012), the stationarity allows predictions and inferences regarding the data, since "the non-stationarity causes the forecasts to become more inaccurate as we move away from the last sample point" (Silveira, 2017). In this way, the representation of a unit root process, or a random walk, can be equated as a first order regression model AR(1), according to Equation 1.

$$Y_t = \phi Y_{t-1} + u_t \tag{1}$$

In the case where $\phi = 1$, Y_t is characterized as a non-stationary process with the presence of a unit root. If $\phi < 1$, Y_t is a stationary process where there is no evidence of the presence of a unit root in the series. Also, u_t is a white noise term (with zero mean and constant variance).

2.2 DF and ADF Test

The presence of a unit root is usually assessed using unit root tests, where the Dickey-Fuller test and the Augmented Dickey-Fuller test are the most used in the literature. Starting from Equation 1, where σ = (ϕ -1) and Δ is the difference operator, we arrive at Equation 2, according to Gujarati e Porter (2011):

$$Y_t = \phi Y_{t-1} + u_t \tag{2}$$

The DF and ADF tests follow the Tau statistic, later recalculated and renamed in Dickey e Fuller (1979) and that, according to the regression equation format, can have a deterministic trend, a drift (a constant) or both, as mentioned in Bueno (2012). Thus, Equation 2 takes the form of Equation 3 and Equation 4 , as a constant or a trend and a constant are added.

$$\Delta Y_t = \beta_1 + \sigma Y_{t-1} + u_t \tag{3}$$

$$\Delta Y_t = \beta_1 + \beta_2 t + \sigma Y_{t-1} + u_t \tag{4}$$

Where β_1 corresponds to a constant and β_2 is the trend *t* coefficient. The DF test has its null hypothesis H_0 in the indication of non-stationarity (presence of a unit root), while the alternative hypothesis assumes the indication of stationarity in the series. Considering the observation of possible correlated noises in the DF test model, it was proposed an improvement through the proposition of the ADF test, which is based on the addition of values with a lag of ΔY , as proposed by Gujarati e Porter (2011) in Equation 5.

$$\Delta Y_t = \beta_1 + \beta_2 t + \sigma Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-i} + u_t \tag{5}$$

Thus, the sum of the number of differences in ΔY is added, followed by the noise term (u_t).

2.3 Outliers and Structural Breaks

The presence of outliers and structural breaks, also known as ruptures or breaks, is frequent in time series (Tsay (1988)). Thereby, two main types of outliers can be identified. The innovational outlier (IO) and the additive outlier (AO), in addition to two main types of structural breaks: level shift (LS) and temporary change (TC).

Shikida, Paiva and Junior (2016) consider three main methods to identify structural breaks: methods that identify the presence of breaks, the estimated date of the break, or even methods that analyze the relationship between unit root and structural breaks for stationarity analysis.

To identify the presence of breaks, the best known test is the Chow test, proposed by Chow (1960), which starts from the principle of knowing the break date and compares a regression to the date of the supposed break with the complete series. To estimate the break date, Bai (1994) addresses an estimation methodology based on the division of the data series into small samples that possibly contain breakouts, estimating their parameters by ordinary least squares (OLS) and analyzing their residuals.

Regarding the tests and methods that analyze the relationship between structural breaks and unit root, they have been proposed to circumvent the observation first made by Perron (1989), which shows that the ADF test tends to present biased results in favor of not rejecting the null hypothesis of unit root in the presence of structural breaks in the data series. Perron (1989) presents an improvement to the ADF test, but as a burden, the new test requires that the rupture date be known. Still, other tests have been proposed later and take into account the presence of structural breaks for stationarity analysis, such as the Zivot-Andrews test, shown in Zivot e Andrews (2002), that allows a break in the series.

Even in the face of the existence of other unit root tests that take breakpoints into account, the ADF test is still mostly used in the literature to assess the stationarity of time series, since the vast majority of studies do not report whether the presence of a structural break has been tested ot not.

Kaiser e Maravall Herrero (1999) and Chen e Liu (1993) state that the AO, LS and TC are well defined by models, unlike the IO, which is observed mainly in seasonal time series. Thus, considering the scope of annual time series and adapted by Kaiser e Maravall Herrero (1999), the typologies of AO, LS and TC are shown (Figure 1).

Considering the imputation of an outlier or change in the structure of the series from time t = 25, the additive outlier represents an isolated peak from the mentioned instant. Note that this outlier concerns a non-standard observation. The change of level (level shift) is graphically represented by a step function, acting on the series from t= 25 and maintaining its effect until the end of the observed series. The temporary change is illustrated as a peak that takes a certain period of time to disappear and causes the series to return to its initial level according to the damping value δ, which is defined in the interval $0 < \delta < 1$.

Therefore, Tsay (1988) defines Equation 6 as a starting point for additive outliers and structural changes such as level change and temporary change.

$$Z_t = Y_t + f(t) \tag{6}$$

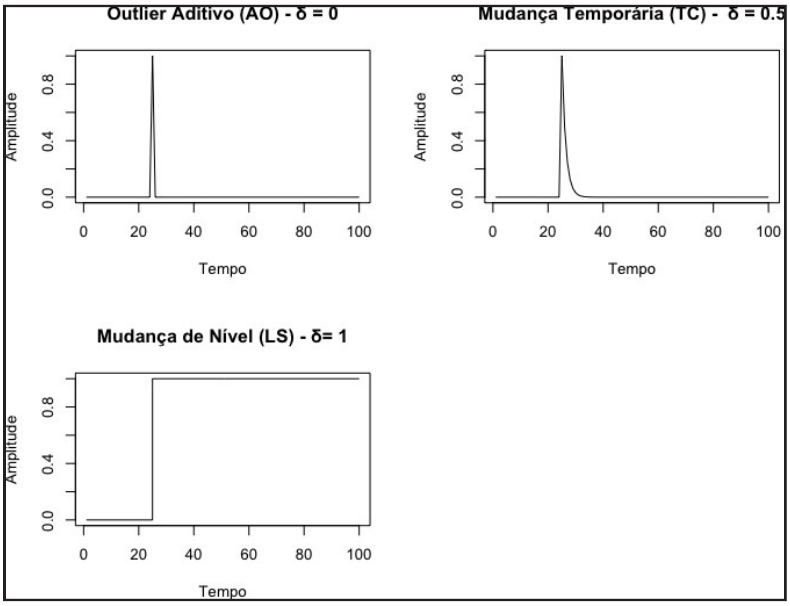
The function $f(t)$ represents exogenous disturbances in the Yt series that the Z_t series undergoes (outliers or breaks). Conside- ring that Chen and Liu (1993) determine some variables that make up $f(t)$), we have Equation 7. Hyndman e Athanasopoulos (2018) define B as the notation backshift according to Equation 8.

$$Z_t = Y_t + \omega L(B) I(t) t_1 \tag{7}$$

$$BY_t = Y_{t-1} \tag{8}$$

The break amplitude ω determines the amplitude of the imputed initial disturbance, depending on the existence or not of the disturbance and determining the addition of the term $\omega L(B)I(t)t_1$ to the Y_t series. The term $L(B)$ depends on the type of outlier or change in the structure of the series. For situations of level change,the Equation 9 is set, according to Tsay (1988), Chen e Liu (1993) and Trívez (1995).

Figure 1 – AO, LS e TC breaks and outliers



Source:

$$L(B) = \frac{1}{(1-B)} \tag{9}$$

So, a level change is a change in the level of the series that lasts until the end of the observation period, starting at $t = d$ Where d is the moment when the break was imputed, extending to all $t > d$. Thus, for $t < d$, $Y_t = Z_t$ and for $t > d$, $Y_t = Z_t + f(t)$.

3 MODELING

Starting from Equation 2, used to represent an autoregressive model of order 1 - AR(1), a model without constant and without trend is set, according to Equation 10.

$$Y_t = \phi Y_{t-1} + u_t \tag{10}$$

Also considering Equation 7, which shows that the perturbation $f(t)$, we have Equation 11), followed by Equation 12 of the particular case of level change structural breaks.

$$Z_t = \phi_1 Y_{t-1} + u_t + \omega L(B) I(t) t_1 \tag{11}$$

$$Z_t = \phi_1 Y_{t-1} + u_t + \frac{\omega}{(1-\delta B)} I(t) t_1 \tag{12}$$

4 METODOLOGY

Considering that this work analyzes the influence of level shift structural breaks, the methodology for planning and carrying out experiments and simulations is followed under different scenarios for stationarity analysis using the ADF test with the aid of the free software R (R Core Team et al., 2020).

Thus, starting from an AR(1) model with imputation of LS structural breaks for annual series without constant and without trend, the influence of the variable ω with other variables is analyzed (Table 1)), for the scenario of stationary series considering a ϕ = 0.6 and a 5% significance level.

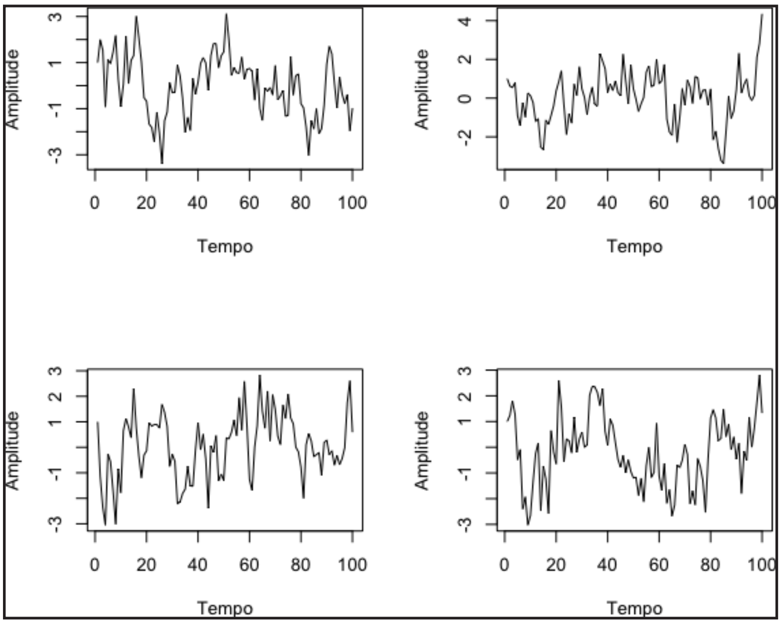
Then, 1000 replications of a series with 100 elements will be considered and a break imputation in five different positions (quantiles 0.10; 0.20; 0.40; 0.60 and 0.80) and the proportion of rejection of the null hypothesis after the ADF test is applied.

Table 1 – Scenarios for AR(1) with no constant and no trend, with ϕ = 0.6

Variable	Attribution
Series lenght (n)	100
Type of break	LS
Break amplitude/perturbation (w)	0; 1, 1,5; 2; 2,5; 3; 3,5; 4 e 4,5
Break positions	Quantiles 0,10; 0,20; 0,40; 0,60 e 0,80
Replications (N)	1000
Interpretation	Rejection frequency of H0

Source: authors

Figure 2 – Examples of time series generated from an AR(1) with ω = 0

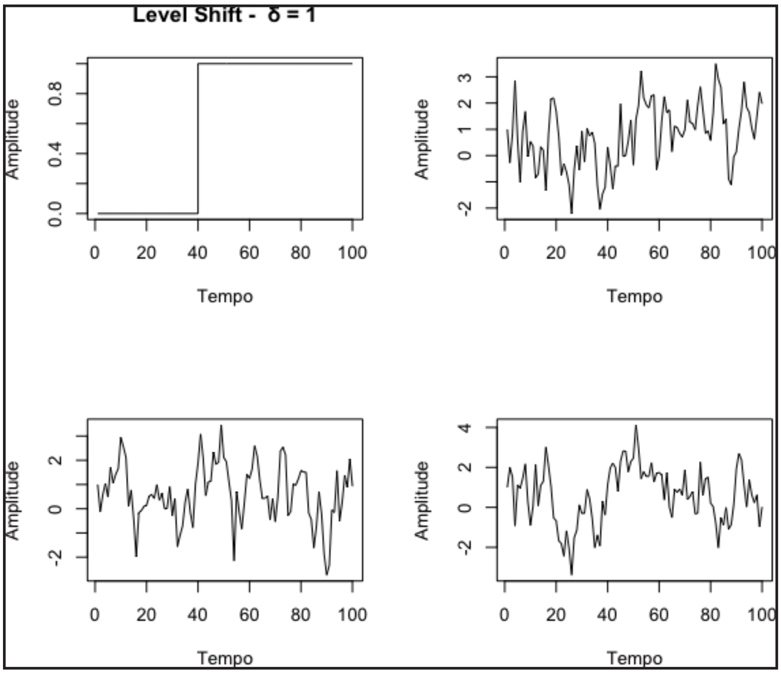


Source: authors

4 RESULTS

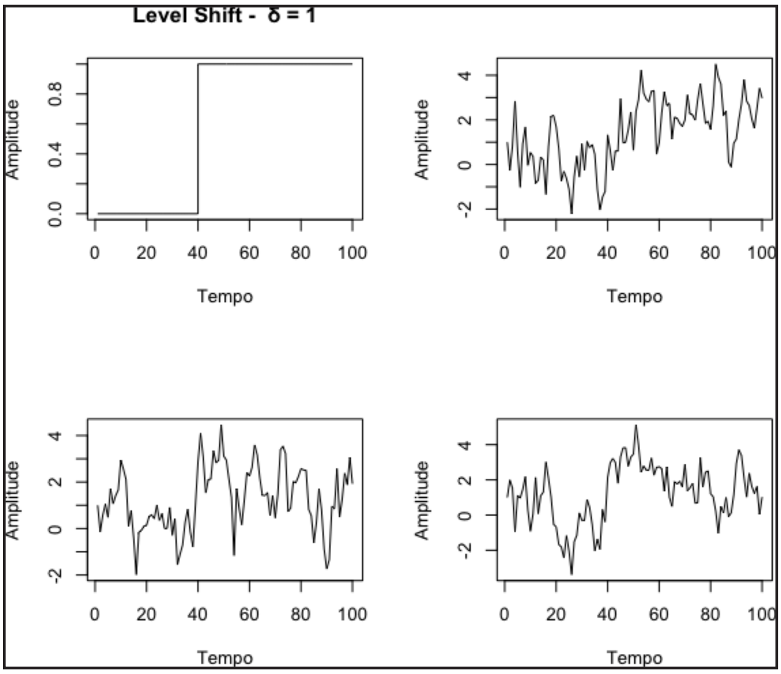
According to selected variables (Table 1), some experiments were carried out and, graphically, examples of the generated series are shown. Where ω = 0 (Figure 2),), the time series without the effect of level changes are shown (Equation 10). Where ω = 1, ω = 2 and ω= 3 (Figures 3, 4 and 4) were imputed LS breaks in the quantile (0.40), or t = 40, when modeled by Equation (12).

Figure 3 – Examples of time series generated from an AR(1) with ω = 1



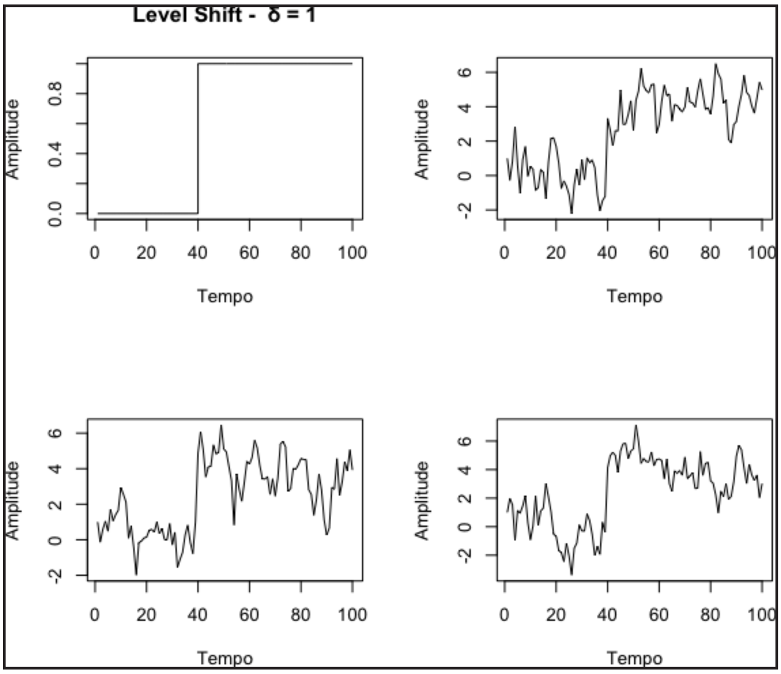
Source: authors

Figure 4 – Examples of time series generated from an AR(1) with ω = 2



Source: authors

Figure 5 – Examples of time series generated from an AR(1) with ω = 4



Source: authors

Note that the greater the amplitude of the initial perturbation ω, the more visible the structural break with a typology of level change becomes. Table 2 shows the proportion of rejection of H_0 for different ω considered. It is possible to observe that the proportion of cases in which the null hypothesis is rejected decreases when considering initial positions in the series (quantiles 0.10 and 0.20 and 0.40) in general and as we approach to the end of the series (quantiles 0.60 and 0.80), the effect of the break affects the test less. When considering $\varphi = 0.6$, the ADF test is expected to reject the null hypothesis.

Assuming a significance level of 5%, it is expected that among the 1000 series, at least 950 reject H_0 and a maximum of 50 do not reject the null hypothesis, to consider the test satisfactory in this first analysis.

Thereby, based on the test results obtained in Table 2, it is possible to observe that without the imputation of breaks, the ADF test proved to be extremely satisfactory, rejecting the null hypothesis in 998 of the 1000 series. However, it was found that the power of the ADF test to reject H_0 decreases as the level shift breaks are added and the intensity of the breaks increases, as well as the imputation position changes.

By varying ω, it is noticed that even if it is not possible to visually detect a level change, the ADF test starts to have its power reduced with values of ω= 1, indicating that even if the scientist does not notice the presence of the break visually, it must be tested and considered.

Table 2 – Rejection Frequency of H_0 for ϕ_1 = 0,6

ω	Q(0,10)	Q(0,20)	Q(0,40)	Q(0,60)	Q(0,80)
0	998	998	998	998	998
0,5	995	994	994	995	996
1	940	945	963	981	990
1,5	709	755	850	936	982
2	298	354	553	776	932
2,5	61	69	209	498	856
3	8	3	42	221	697
3,5	1	0	4	70	497
4	0	0	0	7	288
4,5	0	0	0	2	142

Source: authors

For values of ω = 0.5, the aforementioned test rejected H_0 at least 994 times, and still can be considered satisfactory.

When ω = 1, the test is unsatisfactory from position t = 40 or quantile (0.40) until the end of the series rejecting H_0 less than 950 times.

For ω = 1.5, the ADF test proves to be satisfactory only when the imputed break was in the last position of the tested series, t = 80 or the quantile (0.80). For values where ω > 1.5, a high incidence of non-rejection of the null hypothesis was observed.

6 CONCLUSIONS

As mentioned, there is a consensus in the literature that the ADF test may have its power affected by time series with structural breaks. However, the intensity with which this power is affected according to the characteristics of the series and the structural break is not clear, making it difficult to make a decision regarding the use of that test.

Through the simulations, it was confirmed that the power of the test is reduced by imputing level change structural breaks. Also, the test power reduction was analyzed quantitatively, noting that variables such as the initial break amplitude and the imputation position influence the power of the ADF test, making it possible to quantify the test power according to imputed breaks and analyze its performance through these scenarios.

Thus, therefore, the present work began to fill the gap present in the literature on this issue, allowing the reduction in the power of the ADF test to be confirmed and quantified in the presence of stationary time series with breaks in their structures.

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