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Optimization of reinforced concrete structures using population-based metaheuristic algorithms

Otimização de estruturas de concreto armado empregando algoritmos metaheurísticos baseados em populações

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ABSTRACT

For many industrial activities, ideal projects are achieved by comparing the solutions of alternative projects with those already executed. The feasibility of solutions plays an important role in these activities. For example, the underlying objective (cost, profit, etc.) estimated for each project solution is calculated and the best solution is adopted. This is the usual procedure followed by many constructors due to time and resource limitations. However, in many cases, this method is followed simply by a lack of knowledge of existing optimization procedures. In this context, a comparative study of population-based metaheuristic algorithms applied to a case study of a reinforced concrete beam design reinforced with a polymer matrix with carbon fibers will be presented. Evolutionary algorithms have the ability to determine the optimal values of the design variables without disregarding the restrictions on ACI-318 and ACI-440 standards while minimizing the reinforcement area for each beam (cost). The comparative study shows that not all algorithms presented violated design constraints. In addition, it can be said that the values found for the design variables present a low dispersion around the mean value of the objective function.

Keywords: Constrained Optimization; Evolutionary algorithms; Reinforced concrete

RESUMO

Para muitas atividades industriais, os projetos ideais são alcançados comparando a solução de projetos alternativos com os já executados. A viabilidade de soluções desempenha um papel importante nessas atividades. Por exemplo, o objetivo subjacente (custo, lucro, etc.) estimado para cada solução de projeto é calculado e a melhor solução é adotada. Este é o procedimento usual seguido por muitos construtores devido às limitações de tempo e recursos. No entanto, em muitos casos, esse método é seguido simplesmente pela falta de conhecimento dos procedimentos de otimização existentes.

Neste contexto, será apresentado um estudo comparativo de algoritmos metaheurísticos de base populacional aplicados a um estudo de caso de um projeto de viga de concreto armado reforçada com um material de matriz polimérica com fibras de carbono. Algoritmos evolutivos têm a capacidade de determinar os valores ótimos das variáveis de projeto sem desconsiderar as restrições das normas ACI-318 e ACI-440 enquanto minimiza a área da armadura de cada viga (custo). O estudo comparativo mostra que nem todos os algoritmos apresentados violaram as restrições de projeto. Além disso, pode-se dizer que os valores encontrados para as variáveis de projeto apresentam baixa dispersão em torno do valor médio da função objetivo.

Palavras-chave: Otimização com restrições; Algoritmos evolutivos; Concreto armado

1 INTRODUCTION

Optimizing means getting the best result for some circumstances and constraints. In the design, construction, and maintenance of any system, engineers need to make many technological and managerial decisions at various stages. These important issues are about economics, security, use, and the geometric shape adopted, for example, for a structural element or system. The ultimate goal of all these decisions is to minimize the effort required or maximize the desired benefit to meet the needs of the problem. Since the effort required can be expressed according to certain decision variables, optimization can be defined as the process of finding the conditions that provide the maximum or minimum value of a function (Rao, 2020; Kayabekir *et al.*, 2020).

For any optimization problem, the basic integrated components of the process are the optimization algorithm, an efficient numerical simulator, and a realistic representation of the physical processes that must be modeled and optimized. Most real-world problems are not linear in terms of their objective functions and constraints, it becomes necessary to use sophisticated optimization tools to deal with such problems. In addition, evaluations of objective functions can be time-consuming, which is true in some cases (Yang *et al.*, 2012; Yang, 2020).

One way to work around the problem of high computational processing time is through the use of metaheuristic algorithms to perform the design optimization step. One of the main advantages of nature-inspired algorithms is the quality of their

solutions, even potentially optimal solutions, and that they can be found quickly in practice. The philosophy of these methods is that they should work reasonably well most of the time, but not all the time because there are no guarantees that optimal solutions will be achieved (Yang, 2020; Nayak, 2020).

Optimized designs of reinforced concrete structures are subjects addressed in some articles found in the literature (Li *et al.*, 2021; Mergos, 2021; Kaveh *et al.*, 2020). A pioneering study on this subject can be found in Ceranic *et al.* (2001). This article reports the application of a simulated annealing algorithm in the minimization of the steel area of reinforced concrete containment structures. Simulated annealing is a stochastic relaxation technique with an analogy to the physical process of annealing in metals. The results indicated that the optimal solution is at the intersection of constraints so that design variables are directed to their lower limits until the limits of constraints are reached.

The optimization of the steel area and its location in a T-beam under bending is presented in Ferreira *et al.* (2003). In the study, the nonlinear behavior of concrete and the elasto-plastic behavior of steel are considered. The objective of this work is to obtain the optimal analytical detailing of the reinforcement of a section T. The results obtained for the steel area indicated a higher index of the economy than those obtained by traditional standards recommendations.

Akin and Saka (2015) present the application of the Harmony Search algorithm in the optimal sizing of reinforced concrete frames under seismic loads. The optimization algorithm is based on the musical improvisation process of musicians that aims to find perfect harmony. In the study, the objective function is selected as the total cost of the frame (cost of concrete, shapes, and steel). The optimal design produced satisfies the strength, ductility, ease of maintenance, and other restrictions related to the good design and indicated in the relevant specifications.

In this study, the main objective is to compare population metaheuristic algorithms through a case study of a reinforced concrete beam design reinforced

with a polymer matrix with carbon fibers material. For this, an optimization of the required area of the composite material used to reinforce the beam to shear loads will be presented. In other words, the minimization of the reinforcement area A_{CFRP} for each meter of beam. In addition, evolutionary algorithms will have the role of finding the optimal values of reinforcement design variables, such as: width w_f , the spacing s_f and the angle β . The design restrictions are related to: (i) spacing between polymeric reinforcements with carbon fiber, (ii) the expected effect of shear forces considered in the project and (iii) the efforts not foreseen as defined in the ACI-318 and ACI-440 standards.

2 THEORETICAL FOUNDATION

For the development of this section, the following references will be used: Karaboga (2005), Rao *et al.* (2011), Kaveh and Talatahari (2010), Kaveh and Dadras (2017), and Kaveh and Bakhshpoori (2019).

2.1 Optimization Algorithm Based on Teaching-Learning Process

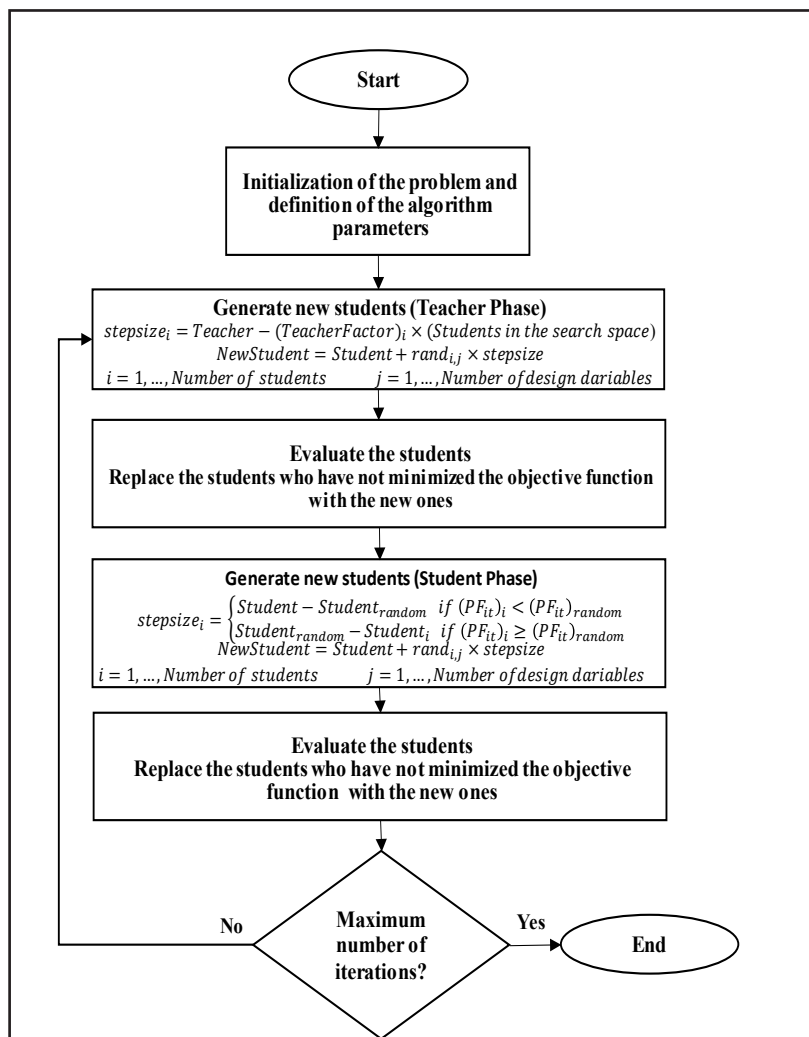
It is a population algorithm based on the school learning process. In this algorithm, two main components of a teaching and learning process are used: the influence of a teacher on the students and the interaction between the students themselves. The best student (the best objective function) is designated as the teacher in each iteration of the algorithm; and, the other students, are updated iteratively until the convergence of the objective function occurs for an optimal result of the problem under study.

Furthermore, the class's performance in learning the subject or the teacher's performance in teaching is treated as a normal distribution of the grades obtained by each student. The main difference between the two normal distributions is in their average value, that is, a teacher generally better transmits the content to students who have average or high grades. In this sense, the algorithm improves other students in the 'teacher phase' by using the difference between the teacher's knowledge and the

average knowledge of all students. Each student’s knowledge is obtained based on the position occupied by that student in the search space.

Regarding the formulation and structure of the algorithm, it begins with randomly generated initial students (students without instruction). In the cyclic body of the algorithm, two sequential search phases will be performed in each iteration of the algorithm: the teacher phase and the student phase. After each search phase, a replacement strategy is adopted to keep former students (with grades around or above average) or replace them with newly graduated students. In Figure 1, a flowchart for a better understanding of the algorithm optimization process based on the teaching-learning process is presented.

Figure 1 – Algorithm flowchart based on teaching and learning process



Source: Rao *et al.* (2011)

2.2 Optimization Algorithm Based on Imperialist Competition

It is a population algorithm with socio-political motivation. In this algorithm, the designer can determine the number of countries that will be classified as 'emperors', which leads the remaining countries, initially determined at random, to become their 'colonies' based on competence – thus forming empires or imperialist states. The structure of the algorithm can be summarized in four stages: formation of empires through a set of random initial solutions, movement of the colonies, emperor update and imperialist competition. With the exception of the first step, the others form the cyclic body of the algorithm.

The algorithm based on imperialist competition presents two main operating mechanisms: *(i)* improving the colonies of each empire through the intrinsic learning of the colonies with their respective emperors; and, *(ii)* through imperialist competitions between empires. The first case results in the 'strengthening' of the empires themselves. Thus, enabling the opportunity for each colony to assume the role of emperor (best objective function) of the respective empire of which it is part. On the other hand, imperialist competitions make it possible for one empire to lose its colonies to another, and there may also be the possibility that the weaker empire will collapse. The power of each empire depends not only on the quality or position of its emperor and the colonies, but also on the number of its colonies. With this, the process of searching for this algorithm is to find the optimal points of the design variables. More information on flowchart and pseudocode can be found in Atashpaz-Gargari and Luca (2007).

2.3 Optimization Algorithm Based on the Behavior of an Artificial Bee Colony

It is an evolutionary algorithm based on swarm intelligence and the behavior of a group of bees in looking for food. In the algorithm, each candidate solution is represented by a food source and the quality of the nectar is associated with the optimal objective function of this solution. These food sources are modified by bees in a repetitive way, with the aim of achieving food sources with better nectar.

Bees are categorized into three types: employed bees, observant bees and exploitative bees. All present different tasks in the colony. Bees make modifications with different strategies according to their tasks. The bees employed, for example, try to modify the food sources and share their information with the observant bees. Observant bees select a food source based on information from the bees employed. Exploitative bees conduct random searches in the vicinity of the hive. Therefore, the algorithm searches in three completely different sequential phases for each iteration.

A random number of bees are initially generated as part of the algorithm's procedure. The size of the colony (or population) is the first parameter provided by the designer. With this, each employed bee generated will try to find a new source of food by looking around their corresponding food source. For this, a size of steps, based on random permutation, is used to direct the search for food – here, at this stage, it is disregarded the source of food already found by bees. Equation (1) mathematically represents this phase.

$$\begin{aligned} \text{stepsize} &= \text{rand}_{(i,j)} \cdot (HB - HB[\text{permute}(i)(j)]) \\ \text{newHB} &= HB + \text{stepsize} \end{aligned} \quad (1)$$

where $\text{rand}_{(i,j)}$ is a random number chosen from the continuous uniform distribution in the range $[-1, 1]$, permute are different line permutation functions, i is the number of bees and j is the number of dimensions of the problem.

After, new observant bees are generated. At this stage, the bees employed share their information (quality and nectar position) of the food sources with the observant bees. The number of bees employed and observant is the same. Each observant bee is attracted to an employed bee with the probability P_i , and it selects a food source associated with that bee employed to generate a new food source for possible modification. A selection probability scheme, such as the roulette selection scheme, is used in the algorithm and can be represented by the following expression:

$$P_i = PFit_i / \sum_{i=1}^{nHb} PFit_i \quad (2)$$

where $PFit_i$ is the penalized objective function of the i -th food source. Sequentially, a neighborhood food source (HB_{rws}) is determined by adding a random step (Equation (3)).

$$\begin{aligned} stepsize &= rand_{(i,j)} \cdot (HB_{rws} - HB[permute(i)(j)]) \\ newHB &= \begin{cases} HB_{rws} + stepsize, & \text{if } rand < mr \\ HB_{rws}, & \text{else} \end{cases} \end{aligned} \quad (3)$$

where mr is a variable that describes the rate of change and determines whether the watcher bee's chosen food supply will be altered or not. $rand$ is a real number randomly chosen in the range $[0,1]$. Generally, this parameter is considered 0.8 based on the literature on the application of the algorithm.

Finally, employed bees that cannot modify their food sources after a certain number of attempts become random. The respective food source of the bees will be abandoned and a new source of food of random location will be generated near the hive. This phase only makes it possible to evaluate new candidate solutions for the algorithm, although in most cases they will be considered unfeasible solutions.

2.4 Penalty Methods for Constrained Functions

The basic idea of the penalty function approach is to define the $PFit$ on Equation (4) so that if there are violations of any restriction, the cost function F_{it} (Equation 5) is penalized by adding a positive value. The advantage of this method lies in transforming the optimization problem with constraint to an unconstrained case. That is, all constraints are incorporated into the new objective function. However, this introduces free parameters whose values need to be set to solve the problem appropriately. There are several types of penalty functions, the most popular, and adopted in this study, is called quadratic loss function (Arora, 2017; Yang, 2014).

$$PF_{it}(\mathbf{h}(x), \mathbf{g}(x), r) = r \left\{ \sum_{i=1}^p [h_i(x)]^2 + \sum_{i=1}^m [g_i(x)]^2 \right\}; \quad (4)$$

$$g_i(x) = \max(0, g_i(x))$$

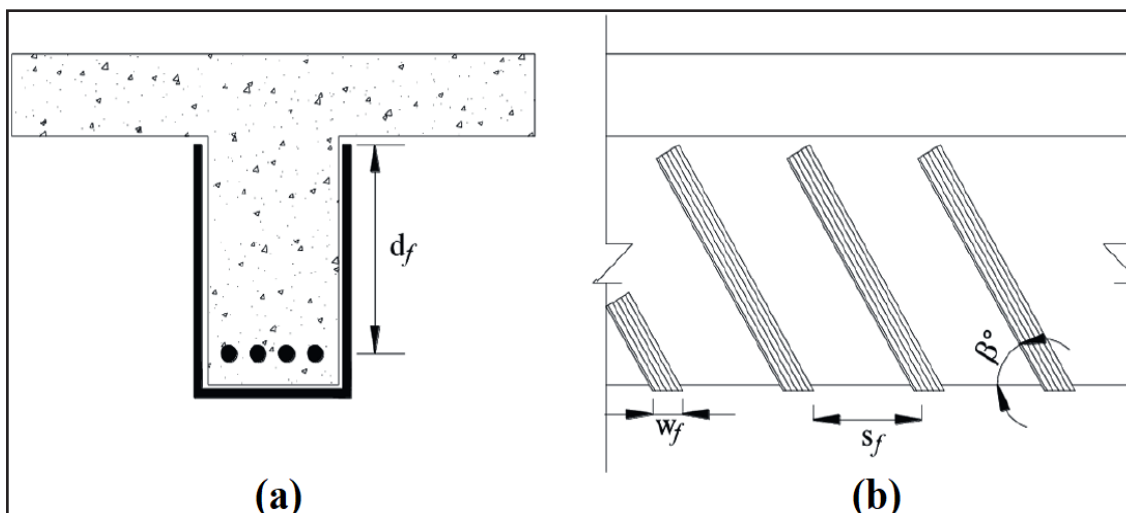
where $r > 0$ is a scalar penalty parameter, $\mathbf{h}(x)$ is the vector of equality constraints, $\mathbf{g}(x)$ is the vector of inequality constraints, and p and m represent the total number of restrictions considered by the designer for equality cases $h_i(x)$ and inequality $g_i(x)$, respectively. In addition, constraints with values greater than or equal to zero, i.e., $h_i(x) \geq 0$ or $g_i(x) \geq 0$, represent the violation of the evolutionary algorithm during the process of searching for the optimal value of the design variables.

$$F_{it} = F_{it} + PF_{it}(\mathbf{h}(x), \mathbf{g}(x), r) \quad (5)$$

3 RESULTS AND DISCUSSIONS

The cross-section of the reinforced concrete T-beam and the design variables are shown in Figure 2.

Figure 2 – (a) Reinforced concrete beam reinforced with composite material and (b) design variables



Source: Kayabekir *et al.* (2018)

The optimization problem related to minimizing the reinforcement area that will be used to the design of a T-beam is represented in Eq. (6) and Table 1 presents the geometric data of the proposed problem.

$$\begin{aligned}
 & \text{Minimize } A_{CFRP} = \frac{w_f (2d_f + b)}{s_f} \times 1000 \text{ mm} \\
 & \text{subject to } \begin{cases} g_1(x): s_f \leq d/4 \\ g_2(x): V_{added} < 0.7R \frac{(2t_f w_f f_{fe})(\sin\beta + \cos\beta)d_f}{s_f + w_f} \\ g_3(x): \frac{(2t_f w_f f_{fe})(\sin\beta + \cos\beta)d_f}{s_f + w_f} \leq \frac{2\sqrt{f'_c b_w d}}{3} - V_s \end{cases} \quad (6)
 \end{aligned}$$

where b_w is the width of the beam, h is the height of the beam, d is the effective depth of the beam, t_f is the thickness of the reinforcement, R is a reduction factor, h_f is the thickness of the slab, f'_c is the strength of concrete to compression, f_{fe} is the effective tensile strength of the reinforcement, V_{added} is the additional shear force, V_s is the shear force capacity of steel rebar, $d_f = d - h_f$ is the depth covered by the reinforcement, $g_1(x)$ is a design restriction regarding the spacing between polymeric reinforcements with carbon fiber, $g_2(x)$ is a restriction on the unforeseen effect of shear efforts and $g_3(x)$ is a restriction on the shear effects foreseen in the project.

Table 1 – Geometric data and search intervals for the optimization process

Design Variables	Unit	Magnitude
b_w	mm	200
h	mm	500
d	mm	450
t_f	mm	0.165
R	-	0.5
h_f	mm	100
f'_c	MPa	20
f_{fe}	MPa	3790
w_f	mm	10-100
s_f	mm	0-d/4
β	°	0-90
V_{added}	kN	50
V_s	kN	50

Source: Authors, 2023

The optimization results, compared with the values provided by Kayabekir *et al.* (2018), are presented in Table 2. The optimization process is repeated 10 times for each metaheuristic algorithm used. Table 2 also shows the results obtained by the simulated metaheuristic algorithms related to: (i) the best minimization of the objective function (area of reinforcement) A_{otim} , (ii) the average of the value of the objective function for each algorithm A_{mean} , (iii) to the standard deviation of the value of the objective function for each algorithm σ , and (iv) to the optimal values obtained for the reinforcement design variables w_f , s_f and β . The algorithms used for the results presented are: optimization algorithm based on the teaching-learning process (TLP), optimization algorithm based on imperialist competition (IC) and the optimization algorithm based on the behavior of an artificial bee colony (ABC). As can be seen in Table 2, all simulated algorithms showed lower results regarding the value obtained for the objective function (A_{CFRP}) when compared to the value provided by Kayabekir *et al.* (2018).

Table 2 – Optimal and average results obtained from the simulations of metaheuristic algorithms

Algorithm	$(A_{\text{CFRP}})_{\text{otim}}$ (mm ²)	$(A_{\text{CFRP}})_{\text{mean}}$ (mm ²)	σ (mm ²)	$(w_f)_{\text{otim}}$ (mm)	$(s_f)_{\text{otim}}$ (mm)	$(\beta)_{\text{otim}}$ (°)
TLP	238.830	238.832	6.00	28.63*	86.65*	65.01*
IC	239.270	241.408	2286.58	33.87	102.99	64.19
ABC	238.920	239.450	326.50	31.59	96.10	64.02
Kayabekir <i>et al.</i> (2018)	243.091	243.091	5.97×10^{-11}	30	90	65

Source: Authors, 2023

Also, the TLP metaheuristic algorithm was the one that best minimized the objective function, although this reduction is very small, 1.75% lower than the Reference

value. For all simulated cases, there was no restriction violation, that is, the value of the penalized function is equal to the value of the objective function. Thus, indicating that the reinforced concrete structure will not suffer from cracks resulting from shear deformations after insertion of polymer matrix reinforcements with carbon fibers. All algorithms demonstrate good performance and desired accuracy in the results of design variables.

4 CONCLUSIONS

The present study brought a comparison of population-based metaheuristic algorithms applied to an optimized design of reinforced concrete beams reinforced with composite material according to ACI-318 and ACI-440 standards. All simulated cases presented results close to the reference value found in the literature, both for the design variables and for the value of the objective function. In addition, by the value of the standard deviation found for each algorithm, the values are considered homogeneous, that is, they present a low dispersion around the value of the mean of the objective function.

Studies, to better understand the behavior of each of the algorithms presented, are necessary to propose improvements and adaptations in the codes used. For this, test functions for optimization could be used to better evaluate the characteristics of optimization algorithms, such as convergence rate, precision, robustness, etc. and thus expand to new optimized designs of reinforced concrete structures.

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