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Special Edition

Pollutant dispersion model in landfills by GILTT technique

Modelo de dispersão de poluentes em aterros sanitários pela técnica GILTT

Josiane Konradt[I], Igor da Cunha Furtado[II], Régis Sperotto de Quadros[I], Daniela Buske[I], Guilherme Jahnecke Weymar[I]

[I] Universidade Federal de Pelotas, Pelotas, RS, Brazil
[II] Instituto Federal Sul-rio-grandense, Pelotas, RS, Brazil

ABSTRACT

The expansion of urban centers leads to an increase in the generation of municipal solid waste (MSW). In this way, due to the exorbitant amount of MSW produced annually, studies on the form of disposal of this waste are necessary, so that they are properly disposed of, due to environmental safety and reduction of risks to human health. Therefore, mathematical modeling is an essential tool in the study of the dispersion of pollutants in the porous medium, in this case, in relation to how the water table can be reached by these pollutants. Thus, this work intends to carry out a study of a two-dimensional model of the dispersion of pollutants in sanitary landfills, considering the governing equation in its dimensionless form. In the problem under study, a continuous and uniform leakage from a MSW storage cell is considered. The adopted model is solved by the Generalized Integral Laplace Transform Technique (GILTT) method, whose solution is analytical, except for the truncation error in the infinite series. The results obtained show that the proposed method is effective given the physical characteristics of the problem.

Keywords: Sanitary landfills; Dispersion of pollutants; GILTT; Porous media; Mathematical modeling

RESUMO

A expansão dos centros urbanos acarreta no aumento da geração de resíduos sólidos urbanos (RSU). Dessa forma, devido a exorbitante quantidade de RSU produzida anualmente, tornam-se necessários estudos acerca da forma de disposição desses resíduos, a fim que sejam destinados de forma adequada, em razão da segurança ambiental e redução de riscos à saúde humana. Logo, a modelagem matemática é uma ferramenta essencial no estudo acerca da dispersão de poluentes no meio poroso, neste caso, em relação a forma como o lençol freático pode ser atingido por esses poluentes. Assim, este trabalho pretende realizar um estudo de um modelo bidimensional da dispersão de poluentes em aterros sanitários, considerando a equação governante na forma adimensional. No problema em estudo, é considerado um vazamento contínuo e uniforme de uma célula de armazenamento de RSU. O modelo adotado é resolvido pelo método da Generalized Integral Laplace Transform Technique (GILTT), cuja solução obtida é analítica, exceto pelo erro de truncamento na série infinita. Os resultados obtidos mostram que o método proposto é eficaz com as características físicas do problema.

Palavras-chave: Aterros sanitários; Dispersão de poluentes; GILTT; Meio poroso; Modelagem matemática

1 INTRODUCTION

The generation of urban solid waste (MSW) has increased significantly over the years, one of the main causes is the accelerated growth of cities, associated with the consumption of industrialized and disposable products on a large scale. The Brazilian Association of Public Cleaning and Special Waste Companies (ABRELPE) prepares annual reports, where it publishes national and regional data on solid waste management in order to facilitate the structuring and implementation of actions, programs and public policies that allow overcoming the observed deficits and make the necessary advances to meet the current legislation and the new demands of society. Table 1 presents the generation and collection of MSW in Brazil in different years.

Table 1 – Generation and collection of urban solid waste in Brazil (T/day)

	2010	2017	2018/2019[1]	2020
Total generation	182,728	214,868	216,629	225,965
Total collection	161,084	196,050	199,311	208,437

Source: ABRELPE (2019), ABRELPE (2020), ABRELPE (2021)

It is observed that the amount of MSW generated per day is a significant number, however, a significant amount of MSW generated is still not collected, being disposed of in inappropriate places or incorrectly, that can cause negative consequences for the environment and human beings.

With the increase in the generation of household waste, the amount of materials disposed of for collection by urban cleaning services also grew, leading to a total of 76.1 million tons collected in 2020, which implies a collection coverage of 92.2% (ABRELPE, 2021). Although the municipality declares that it has collection selective, this does not mean that selective collection covers the entire boundary of the municipality, thus identifying the need for studies on a set of systems and measures, such as efficient management that guarantees the maximum reuse and recycling, promoting the social inclusion and economic emancipation of collectors of reusable and recyclable materials, positively impacting the recovery rates of such materials.

In this way, according to Bassanezi (2002), quotes: "[...] mathematical modeling consists of the art of transforming reality problems into mathematical problems and solving them, interpreting their solutions in the language of the real world". So, the Mathematical modeling has been presented as an important tool to understand how phenomena of transport and dispersion of contaminants occur.

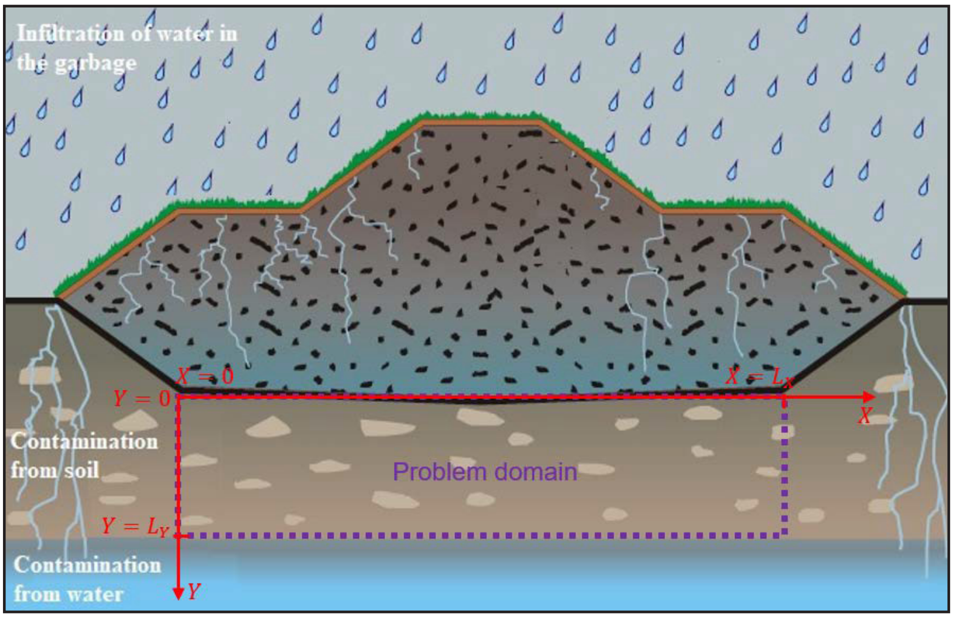
The present work intends to carry out a study of the two-dimensional mathematical model of the transport of contaminants in a sanitary landfill. For this, the dimensionless form of the effluent dispersion model in the porous medium will be considered, having as hypothesis the continuous and uniform leakage of pollutant from a MSW storage cell. For the resolution of the model, the Generalized Integral Laplace Transform Technique (GILTT) will be used in order to obtain the solution of the analytical form of the transient model of mass transport in a saturated porous medium.

2 METHODOLOGY

Through Figure 1 we can observe the two-dimensional model used in this work, where a simplified scheme of a MSW storage cell is represented, where the transport of the concentration of contaminants takes place through the porous medium until it reaches the water table, where $Y=0$ represents the boundary between landfill and soil and $Y=L_Y$ represents the boundary between soil and water table.

[1]The ABRELPE in 2018 and 2019 produced only one document on the panorama of solid waste in Brazil, where it considered that there was no increase over the course of a year.

Figure 1 – Section section of a landfill



Source: Adapted from CONDER (2017)

Equation (1), written in dimensionless form, describes the transport of pollutants in the saturated porous medium, and can be found in the work of Albuquerque (2018):

$$R\frac{\partial C^*}{\partial \tau} = L^*\frac{\partial^2 C^*}{\partial X^2} + \frac{\partial^2 C^*}{\partial Y^2} - Pe\frac{\partial C^*}{\partial Y}, \tag{1}$$

where Rrepresents the soil retardation factor, $C$is the concentration of the contaminant in the liquid phase, τ equals time, L^* is the relationship between the dimensions in X and Y of the problem $(L^* = (\frac{L_Y}{L_X})^2)$ and Pe is Péclet Number.

The initial condition of the problem is given by:

$$C^*(X, Y, 0) = C_0^*, \tag{2}$$

where C_0^* is the initial concentration of the contaminant in the MSW storage cell.

The boundary conditions in dimensionless form and in the X direction are given per:

$$\frac{\partial C^*}{\partial X}(0, Y, \tau) = 0, \qquad \frac{\partial C^*}{\partial X}(1, Y, \tau) = 0, \tag{3}$$

where null flow conditions are used at the domain boundaries in X. The boundary conditions in the Y direction are given by:

$$C^*(X,0,\tau) = 1, \tag{4}$$

the Eq. (1) is subject to the interface condition Eq. (4) which corresponds to the continuous and uniform leakage of slurry in a MSW cell, in addition to:

$$\frac{\partial C^*}{\partial Y}(X,1,\tau) + BiC^*(X,1,\tau) = 0, \tag{5}$$

where Bi is the Biot number and this condition represents the convective flow located in the part of contact between the soil and the water table.

With dimensionless equations, represented by Eqs. (1) - (5) start solving the problem by applying the Superposition Method (Hahn and Özisik, 1993):

$$C^*(X,Y,\tau) = C^{**}(X,Y,\tau) + C_F(Y), \tag{6}$$

where C^{**} is an auxiliary function that carries with it the homogeneous boundary condition and C_F is the solution to the problem in the steady state. Thus, substituting Eq. (6) in the equation that governs the transport of pollutants in the porous medium we have:

$$R\frac{\partial C^{**}}{\partial \tau} = L^*\frac{\partial^2 C^{**}}{\partial X^2} + \frac{\partial^2 C^{**}}{\partial Y^2} + \frac{d^2 C_F}{dY^2} - Pe\left(\frac{\partial C^{**}}{\partial Y} + \frac{dC_F}{dY}\right). \tag{7}$$

From Eq. (7), we have two differential equations, an ordinary differential equation (ODE) and another partial differential equation (PDE).

The equation that represents the ODE is given by Eq. (8):

$$\frac{d^2 C_F}{dY^2} - Pe\frac{dC_F}{dY} = 0, \tag{8}$$

and the boundary conditions are given by the following equations:

$$C_F(0) = 1, \tag{9}$$

$$\frac{dC_F(1)}{dY} + BiC_F(1) = 0. \tag{10}$$

Using the boundary conditions given by Eq. (9) and Eq. (10), we obtain the analytical solution of Eq. (8):

$$C_F(Y) = \frac{e^{Pe}(Pe + Bi) - Bie^{PeY}}{e^{Pe}(Pe + Bi) - Bi}. \tag{11}$$

The EDP obtained from Eq. (7) is given by:

$$R\frac{\partial C^{**}}{\partial \tau} = L^* \frac{\partial^2 C^{**}}{\partial X^2} + \frac{\partial^2 C^{**}}{\partial Y^2} - Pe\frac{\partial C^{**}}{\partial Y}, \tag{12}$$

and their boundary conditions in the X direction are given by the following equations:

$$\frac{\partial C^{**}}{\partial X}(0, Y, \tau) = 0, \tag{13}$$

$$\frac{\partial C^{**}}{\partial X}(1, Y, \tau) = 0, \tag{14}$$

and its boundary conditions in the Y direction are:

$$C^{**}(X, 0, \tau) = 0, \tag{15}$$

$$\frac{\partial C^{**}}{\partial Y}(X, 1, \tau) + BiC^{**}(X, 1, \tau) = 0, \tag{16}$$

and the initial condition is given by:

$$C^{**}(X, Y, 0) = C_0^* - C_F(Y). \tag{17}$$

To obtain the solution $C^{**}(X, Y, \tau)$ of Eq. (12), the GILTT method was used. For this, first, the Sturm-Liouville auxiliary problem is taken considering the X direction:

$$\frac{d^2\varphi}{dX^2} + \frac{\lambda^2}{L^*}\,\varphi = 0, \tag{18}$$

$$\frac{d\varphi(0)}{dX} = 0, \tag{19}$$

$$\frac{d\varphi(1)}{dX} = 0, \tag{20}$$

where the differential equation has as solution the eigenfunctions (Hahn and Özisik, 1993):

$$\varphi_n(X) = \cos\left(\frac{\lambda_n}{\sqrt{L^*}} X\right), \tag{21}$$

applying the boundary conditions, we obtain the eigenvalues associated with each eigenfunction:

$$\lambda_n = n\pi\sqrt{L^*} \qquad\qquad n = 0, 1, 2, \ldots. \tag{22}$$

Concentration is expanded as a series in terms of the eigenfunctions:

$$C^{**}(X, Y, \tau) = \sum_{n=0}^{N} \varphi_n(X)\overline{C}_n(Y, \tau), \tag{23}$$

where $\overline{C}_n(\tau)$ are terms to be determined and N is the index at which the Eq. (23) converges, being the concentration expansion based on the eigenfunctions of the auxiliary problem .

Substituting the Eq. (23) expansion into Eq. (12):

$$R\frac{\partial}{\partial \tau}\left[\sum_{n=0}^{N}\overline{C}_n(Y,\tau)\varphi_n(X)\right] = L^*\left[\sum_{n=0}^{N}\overline{C}_n(Y,\tau)\varphi_n''(X)\right] + \frac{\partial^2}{\partial Y^2}\left[\sum_{n=0}^{N}\overline{C}_n(Y,\tau)\varphi_n(X)\right] - Pe\frac{\partial}{\partial Y}\left[\sum_{n=0}^{N}\overline{C}_n(Y,\tau)\varphi_n(X)\right], \tag{24}$$

and applying the integral operator $\int_0^1 (.)\varphi_m(X)dX$ on both sides of the equation, noting that, by Eq. (18), $\varphi_n''(X) = -\lambda_n^2\varphi_n(X)$, get:

$$R\frac{\partial\overline{C}_n(Y,\tau)}{\partial \tau}\left[\sum_{n=0}^{N}\int_0^1 \varphi_m(X)\varphi_n(X)dX\right] = -\lambda_n^2\overline{C}_n(Y,\tau)\left[\sum_{n=0}^{N}\int_0^1 \varphi_m(X)\varphi_n(X)dX\right] + \frac{\partial^2\overline{C}_n(Y,\tau)}{\partial Y^2}\left[\sum_{n=0}^{N}\int_0^1 \varphi_m(X)\varphi_n(X)dX\right] - Pe\frac{\partial\overline{C}_n(Y,\tau)}{\partial Y}\left[\sum_{n=0}^{N}\int_0^1 \varphi_m(X)\varphi_n(X)dX\right]. \tag{25}$$

Therefore, in all summations of Eq. (25) the terms are all null, except when m = n. Therefore, Eq. (25) boils down to:

$$R\frac{\partial\overline{C}_n(Y,\tau)}{\partial \tau} = -\lambda_n^2\overline{C}_n(Y,\tau) + \frac{\partial^2\overline{C}_n(Y,\tau)}{\partial Y^2} - Pe\frac{\partial\overline{C}_n(Y,\tau)}{\partial Y}. \tag{26}$$

For the resolution of PDE, Eq. (26), we solve the auxiliary Sturm-Liouville problem in Y:

$$\frac{d^2\psi}{dY^2} + \beta^2\psi = 0, \tag{27}$$

$$\psi(0) = 0, \tag{28}$$

$$\frac{d\psi(1)}{dY} + Bi\psi(1) = 0, \tag{29}$$

where the differential equation has as solution the eigenfunctions (Hahn and Özisik, 1993):

$$\psi_k(Y) = \mathrm{sen}(\beta_k Y), \tag{30}$$

applying the boundary conditions, we obtain the eigenvalues associated with each eigenfunction, they must satisfy the following transcendental equation:

$$\beta_k \cot(\beta_k) + Bi = 0, \tag{31}$$

the eigenvalues β_k are the roots of the transcendental Eq. (30) and were calculated by the Newton-Raphson method.

Next, the solution of PDE Eq. (26) is expanded as a series in terms of the eigenfunctions:

$$\overline{C}_n(Y,\tau) = \sum_{k=0}^{K} \psi_k(Y)\widetilde{C}_k(\tau), \tag{32}$$

where $\int_0^1 (.)\psi_l(Y)dY$ are terms to be determined and K is the index at which Eq. (32) converges.

Substituting the Eq. (32) expansion into Eq. (26):

$$R\frac{\partial}{\partial\tau}\left[\sum_{k=0}^{K}\psi_k(Y)\widetilde{C}_k(\tau)\right] = -\lambda_n^2\left[\sum_{k=0}^{K}\psi_k(Y)\widetilde{C}_k(\tau)\right] + \frac{\partial^2}{\partial Y^2}\left[\sum_{k=0}^{K}\psi_k(Y)\widetilde{C}_k(\tau)\right] - Pe\frac{\partial}{\partial Y}\left[\sum_{k=0}^{K}\psi_k(Y)\widetilde{C}_k(\tau)\right], \tag{33}$$

and applying the integral operator $\int_0^1 (.)\psi_l(Y)dY$ on both sides of the equation, noting that, by Eq. (27), $\psi_k''(Y) = -\beta_k^2\psi_k(Y)$:

$$\begin{aligned}\widetilde{C}_k'(\tau)\left[\sum_{k=0}^{K} R\int_0^1 \psi_k(Y)\psi_l(Y)dY\right] = \widetilde{C}_k(\tau)\Bigg\{ -\lambda_n^2\left[\sum_{k=0}^{K}\int_0^1 \psi_k(Y)\psi_l(Y)dY\right] - \\ \beta_k^2\left[\sum_{k=0}^{K}\int_0^1 \psi_k(Y)\psi_l(Y)dY\right] - Pe\left[\sum_{k=0}^{K}\int_0^1 \psi_k'(Y)\psi_l(Y)dY\right]\Bigg\},\end{aligned} \tag{34}$$

can be rewritten in the form:

$$\widetilde{C}_k'(\tau)\left[\sum_{k=0}^{K} R\int_0^1 \psi_k(Y)\psi_l(Y)dY\right] + \widetilde{C}_k(\tau)(\lambda_n^2+\beta_k^2)\left[\sum_{k=0}^{K}\int_0^1 \psi_k(Y)\psi_l(Y)dY\right] + \left[\sum_{k=0}^{K} Pe\int_0^1 \psi_k'(Y)\psi_l(Y)dY\right] = 0. \tag{35}$$

Then, we rewrite Eq. (35) in matrix form:

$$A \cdot Z'(\tau) + B \cdot Z(\tau) = 0, \tag{36}$$

where $Z(\tau) = \{\widetilde{C}_k\}$, with k = 0,1,2,...; $A = \{a_{k,l}\}$, where $a_{k,l} = R\int_0^1 \psi_k(Y)\psi_l(Y)dY$ and $B = \{b_{k,l}\}$, where $b_{k,l} = (\lambda_n^2+\beta_k^2)\int_0^1 \psi_k(Y)\psi_l(Y)dY + Pe\int_0^1 \psi_k'(Y)\psi_l(Y)dY$.

Still, considering F = A^{-1} · B, we rewrite Eq. (36) of the form:

$$Z'(\tau) + F \cdot Z(\tau) = 0. \tag{37}$$

Note that the matrix ODE, Eq. (37), is subject to an initial condition. We will use the initial condition of the problem, applying the same procedures that were performed in the EDP:

$$\overline{C}_n(Y,0) = C_0^* - C_F(Y). \tag{38}$$

The initial condition of the matrix differential equation, Eq. (37) is obtained by applying the same procedures, carried out in EDP, in Eq. (17). Obtaining as a result:

$$Z(0) = A^{-1} \cdot H, \tag{39}$$

where $H = R\int_0^1 [C_0^* - C_F(Y)]\psi_l(Y)dY$ and thus the initial condition is well defined.

Finally, we solve the matrix ODE Eq. (37). Applying the Laplace transform, denoted by $\mathcal{L}\{(.),\tau \to s\}$, on both sides:

$$s\mathcal{L}\{Z(\tau),\tau \to s\} - Z(0) + F \cdot \mathcal{L}\{Z(\tau),\tau \to s\} = 0. \tag{40}$$

Considering that the F matrix of Eq. (40) is diagonalizable, we have:

$$F = X \cdot D \cdot X^{-1}, \tag{41}$$

where D is the diagonal matrix whose elements are the eigenvalues of F , X is the matrix whose columns are linearly independent eigenvectors of F and X^{-1} is its inverse. Substituting Eq. (41) into Eq. (40), applying some properties and isolating $\mathcal{L}\{Z(\tau), \tau \to s\}$ we obtain if:

$$\mathcal{L}\{Z(\tau), \tau \to s\} = X \cdot (s \cdot I + D)^{-1} \cdot X^{-1} \cdot Z(0). \tag{42}$$

Applying the inverse Laplace transform, denoted by $\mathcal{L}^{-1}\{(.), s \to \tau\}$, on both sides of Eq. (42), it is concluded that:

$$Z(\tau) = X \cdot \mathcal{L}^{-1}\{(s \cdot I + D)^{-1}, s \to \tau\} \cdot X^{-1} \cdot Z(0). \tag{43}$$

Performing the inversion of the matrix ($s \cdot I + D$) and applying the inverse Laplace transform, we have:

$$\mathcal{L}^{-1}\{(s \cdot I + D)^{-1}, s \to \tau\} = \begin{pmatrix} e^{-d_0\tau} & 0 & \cdots & 0 \\ 0 & e^{-d_1\tau} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-d_N\tau} \end{pmatrix} = G(\tau). \tag{44}$$

With the matrix $G(\tau)$ defined in Eq. (44), it is concluded that the solution of the matrix ODE Eq. (37) is:

$$Z(\tau) = X \cdot G(\tau) \cdot X^{-1} \cdot Z(0). \tag{45}$$

Therefore, the solution of the two-dimensional model of the dispersion of pollutants in porous media, represented by Eqs. (1) - (5) is given by:

$$C^*(X, Y, \tau) = \sum_{n=0}^{N} \varphi_n(X) \left[\sum_{k=0}^{K} \psi_k(Y) \widetilde{C}_k(\tau) \right] + C_F(Y), \tag{46}$$

where $\varphi_n(X)$ is defined by Eq. (21), $\psi_k(Y)$ is defined in Eq. (30) and $\widetilde{C}_k(\tau)$ is defined by Eq. (45).

3 RESULTS AND DISCUSSIONS

From the solution of the two-dimensional model of transport of pollutants in the porous medium, the results obtained through the online software Google Collaboratory (Python language) will be presented and analyzed.

In Table 2 the results are presented considering the following numbers (of eigenvalues and eigenfunctions) for the truncations of the series given in Eq. (46): $N = K = 5; 10$ and 20. The analysis of the concentrations were performed at positions $(X, Y) : (0; 0.2); (0; 0.4); (0; 0.6)(0; 0.8)$ and at the instant $\tau = 0.4$.

Table 2 – Convergence of the dimensionless concentration field for the case Pe = 2, R = 1 and Biot Infinite

C(X; Y ; τ)	5	10	20
C(0; 0.2; 0.4)	0.914118840	0.914227273	0.914222281
C(0; 0.4; 0.4)	0.796514870	0.796524815	0.796524468
C(0; 0.6; 0.4)	0.627241522	0.627315094	0.627316729
C(0; 0.8; 0.4)	0.376452771	0.376447769	0.376450952

Source:authors

Table 3 – presents the approximate relative error, based on truncation of sums.

Table 3 – Relative approximate error based on the approximations made in Eq. (46) ($N = K = 5; 10$ and 20)

Relative approximate error (%) - rae	5	10	20
rae - [C(0; 0.2; 0.4)]	-	0.01186067637	0.00054605026
rae - [C(0; 0.4; 0.4)]	-	0.00124855752	0.00004359728
rae - [C(0; 0.6; 0.4)]	-	0.01172809706	0.00026060265
rae - [C(0; 0.8; 0.4)]	-	0.00132876591	0.00084538384

Source:authors

Observing the Table 2 and Table 3, for $N = K = 20$, it can be seen that the results have a precision of 3 decimal places. Thus, in the following simulations $N = K = 20$ were used.

Table 4 presents the distribution of the concentration of contaminants in position $X = 0.5$ for different times and considering $R = 1$ and $Pe = 2$. It can be observed that if $Pe = 2$, that is, a low value, the pollutant flow is slow. For results with τ $= 0.2$, that is, a short time, the concentration has a significant variation, however, with τ > 0.5 the concentration practically does not change. Furthermore, it is observed that considering a value of Y closer to the surface and consequently to the sanitary landfill, it is noticed that the dimensionless concentration is a value closer to 1, which would be the maximum concentration, mainly in comparison to the value of $Y = 0.8$, as it is a value more adjacent to the water table and the concentration of the contaminant is lower.

Table 4 – Results of the simulations considering $N = K = 20$; $X = 0.5$; $R = 1$; $Pe = 2$

			Y			
Time (τ)	0	0.2	0.4	0.6	0.8	1
0.1	1	0.7003558244	0.5147904328	0.3983390333	0.2612129508	0
0.2	1	0.8471679881	0.7076633965	0.5545755738	0.3396328895	0
0.3	1	0.8974409395	0.7742738091	0.6090937972	0.3672226542	0
0.4	1	0.9143937450	0.7967454860	0.6274958060	0.3765392628	0
0.5	1	0.9201111771	0.8043243577	0.6337022984	0.3796815689	0
0.6	1	0.9220394599	0.8068804397	0.6357955290	0.3807413589	0
0.7	1	0.9226898009	0.8077425150	0.6365015011	0.3810987884	0
0.8	1	0.9229091378	0.8080332623	0.6367396003	0.3812193366	0
0.9	1	0.9229831123	0.8081313210	0.6368199027	0.3812599932	0
1	1	0.9230080612	0.8081643927	0.6368469858	0.3812737052	0

Source:authors

Table 5 presents the distribution of the concentration of contaminants in position $X = 0.5$ for different times and considering $R = 1$ and $Pe = 10$.

Table 5 – Simulation results considering $N = K = 20$; $X = 0.5$; $R = 1$; $Pe = 10$

	Y					
Time (τ)	0	0.2	0.4	0.6	0.8	1
0.02	1	0.4800113068	0.4705237516	0.5979105352	0.7135120931	0
0.05	1	0.5013257032	0.6182477117	0.7996391136	0.8148676265	0
0.1	1	0.8645453934	0.9130902644	0.9491513581	0.8572503000	0
0.2	1	0.9952639609	0.9949329986	0.9807792388	0.8645002171	0

Source:authors

It can be noticed that considering the value of Pe a higher value, the concentration also increases, therefore, the soil will be contaminated more quickly by the pollutant, this occurs because the contaminant drains quickly and contaminates the soil less, in this way, the pollutant reaches the water table considerably, that is, the diffusion process is smaller than the advection process. Furthermore, for $\tau = 0.2$ it is observed that the concentration stability was not achieved, requiring a longer time for it to occur.

4 CONCLUSIONS

The results obtained are consistent with the dynamics of the problem. Regarding the Péclet number, as a higher value is considered for this parameter, the concentration grows, so the soil will be contaminated more quickly by the pollutant and the water table will be significantly affected. The results of the soil depth parameter, showed that considering a value closer to the surface and consequently to the landfill, that is, values close to 0, it is clear that the dimensionless concentration is closer to the maximum achievable. As next steps, we intend to continue investigating the model and the results presented, creating graphs so that the results are more accessible.

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Authorship contribution

1 – Josiane Konradt

Student of the Graduate Program in Mathematical Modeling
https://orcid.org/0000-0002-4689-8616 • josianekonradt@gmail.com
Contribution: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Project administration, Software, Supervision, Validation, Visualization, Writing - original draft, Wriing - review & editing

2 – Igor da Cunha Furtado

Teacher of basic education
https://orcid.org/0000-0002-0776-3225 • igorjara@gmail.com
Contribution: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Project administration, Software, Supervision, Validation, Visualization, Writing - original draft, Wriing - review & editing

3 – Régis Sperotto de Quadros

Professor of Higher Education
https://orcid.org/0000-0002-9720-8013 • quadros99@gmail.com
Contribution: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Project administration, Software, Supervision, Validation, Visualization, Writing - original draft, Wriing - review & editing

4 – Daniela Buske

Professor of Higher Education.
https://orcid.org/0000-0002-4573-9787 • danielabuske@gmail.com
Contribution: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Project administration, Software, Supervision, Validation, Visualization, Writing - original draft, Wriing - review & editing

5 – Guilherme Jahnecke Weymar

Professor of Higher Education.
https://orcid.org/0000-0001-8216-9122 • guilhermejahnecke@gmail.com
Contribution: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Project administration, Software, Supervision, Validation, Visualization, Writing - original draft, Wriing - review & editing

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