

Special Edition

Analysis of EAHE through a coupled mathematical model solved by Laplace transform and Gaver-Stehfest algorithm

Análise de TCSA's através de um modelo matemático acoplado resolvido por transformada de Laplace e algoritmo de Gaver-Stehfest

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ABSTRACT

Earth-air heat exchangers (EAHEs) are devices that are shown to be a great alternative to assist in the thermal comfort of buildings, mainly due to their low energy cost. The proposed model considers the heat exchange between the soil, the duct and the air, generating coupled equations. This work seeks to employ and verify EAHE applications in two scenarios, first in a literature experiment and then in an EAHE application in Pelotas. In both scenarios, it is considered that the soil and air temperature functions are time-dependent. The solution is obtained by applying the Laplace transform and for the inversion of the integral transform the numerical method known as the Gaver-Stehfest Algorithm was used. The obtained results show that the proposed model follows the behavior of the experimental data. In addition to the experimental data, the proposed model was compared with the analytical model by Nóbrega, where the results had a small variation (of 0.20°C between them) by considering the heat transfer process in the duct material, modeling it closer to the real situation.

Keywords: Earth-air heat exchanger, Laplace transform, Semi-analytical solution

RESUMO

Os trocadores de calor solo-ar (TCSA's) são dispositivos que se mostram como uma grande alternativa para auxiliar no conforto térmico de edificações, principalmente pelo seu baixo custo energético. O modelo proposto considera a troca de calor entre o solo, o duto e o ar, gerando equações acopladas. Esse trabalho busca empregar e verificar aplicações de TCSA em dois cenários, primeiro em um experimento de literatura e depois em uma aplicação de TCSA em Pelotas. Nos dois cenários, se considera que as funções temperaturas do solo e do ar são dependentes do tempo. A solução é obtida

aplicando a transformada de Laplace e para a inversão da transformada integral utilizou-se o método numérico conhecido como Algoritmo de Gaver-Stehfest. Os resultados obtidos mostram que o modelo proposto segue o comportamento dos dados experimentais. Além dos dados experimentais, o modelo proposto foi comparado como modelo analítico de Nóbrega, onde os resultados tiveram uma pequena variação (de 0,20 °C entre eles) por considerar o processo de transferência de calor no material do duto, modelando um caso mais próximo da situação real.

Palavras-chave: Trocador de calor solo-ar, Transformada de Laplace, Solução semi-analítica

1 INTRODUCTION

One of the biggest challenges today is the considerable increase in atmospheric pollution that human activities have generated. The large-scale generation of pollutants, especially those related to the burning of fossil fuels (such as, for example, CO₂, methane, etc.) have caused numerous changes in the environment (increase in global temperature, ocean warming, melting of polar ice caps, among others), as highlighted by the National Aeronautics and Space Administration.

Global warming and the energy crisis are issues of extreme importance for the entire world. And these themes gain greater importance, as they are directly related to the energy consumption of buildings. According to Qi et al. (2021), residential buildings are responsible for about 40% of total global energy consumption, and the main systems causing this consumption are heating, ventilation and air conditioning systems (responsible for 60% of total energy consumption of buildings). In the region of Pelotas-RS, the consumption of electricity in the summer months increases considerably due to the large number of air conditioners that are turned on to obtain thermal comfort in homes.

One of the roles of Science is to stimulate research that promotes sustainability. Thus, this work aims to discuss and investigate the earth-air heat exchanger (EAHE) as an alternative to help in thermal comfort with low energy cost. This type of device exploits the Earth's thermal inertia to dissipate the heat circulating through the heat exchanger duct. On hot days, the air circulating in the EAHE duct is cooled (giving heat to

the ground). This type of device can also be used in winter (cold days), for this purpose the air is heated (receiving heat from the ground).

EAHEs are already widely used in some European countries, such as Germany, Switzerland, among others. In Brazil many researches have been developed on this device. In this way, many studies appear with the objective of investigating different topics, such as the development and validation of complete or simplified models, as well as the analysis of their operational parameters (Brum et al., 2012, 2013; Minaei and Safikhani, 2021).

Therefore, this work aims to use and verify the model developed by Minaei and Safikhani (2021), which is a very complete model in the sense that the modeling of the problem considers the heat exchange between the soil, the duct and air, considering the physical properties of each. Furthermore, this research proposes to work with air and soil temperature functions that vary with time (Nóbrega, 2021), bringing more complexity to the model.

2 METHODOLOGY

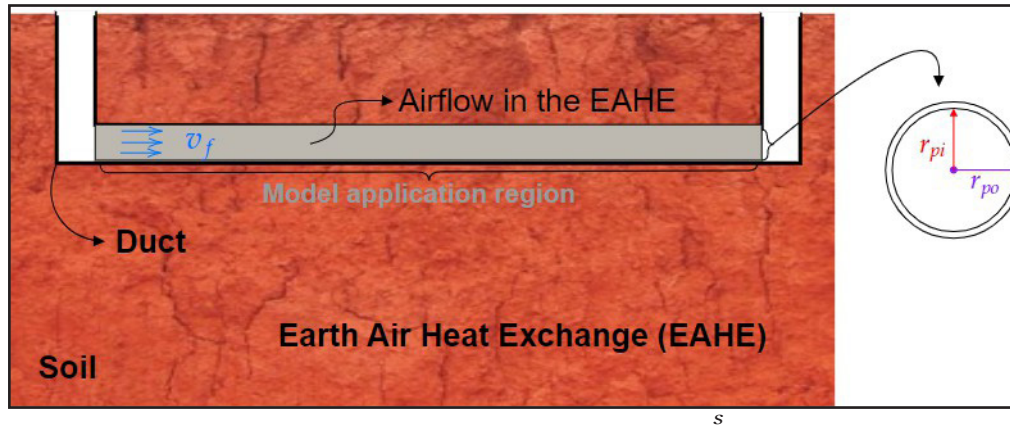
For the development of this work, the following are presented: the mathematical model; the technique used to obtain the model solution; data from the literature of an experiment carried out with EAHE in the city of Viamão by Vaz et al. (2011). Thus, to reproduce part of the experiment by Vaz et al. (2011), the simulations considered a pipeline with 0.11 m in diameter and 25.77 m in length and, finally, a case study in the region of Pelotas. The experiment with EAHE and the case study were used to verify the model and proposed solution.

2.1 Model

Figure 1 is a schematic representation of a EAHE, where you can visualize the inlet and outlet of the air flow, as well as the cross-sectional area of the duct and the application region of the model under study. In the illustration, the variables r_{po} and

r_{pi} are the outer and inner radius of the duct, respectively, and v_f is the velocity of the fluid (air).

Figure1 – Schematic representation of a EAHE



Source: Author's

The construction of the model was based on the following references: Minaei and Safikhani (2021); Nóbrega (2021) and Bejan and Kraus (2003). Using the law of conservation of energy, for the air inside the heat exchanger, we have

$$\rho_f c_{p_f} A \frac{\partial T_f}{\partial t} = -\dot{m} c_{p_f} \frac{dT_f}{dx} - \frac{T_f(x) - T_{po}(x)}{R_p}, \quad (1)$$

where the subscript f refers to the fluid (air), ρ is the specific gravity, c_p is the specific heat, A is the area, m mass and T_f is the temperature of the fluid and T_{po} is the temperature in the exchanger (duct).

For the conditions of the problems under study, the following equations were used:

$$T_f(x, 0) = T_g(t), \quad (2)$$

$$T_f(0, t) = T_i(t), \quad (3)$$

where T_g is the soil temperature and T_i is the air in let temperature in the exchanger, both functions dependent on time (Nóbrega, 2021).

To model the soil temperature around the exchanger, the heat equation in cylindrical coordinates is used, given below:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{1}{\alpha_s} \frac{\partial T}{\partial t}, \quad (4)$$

where T is the soil temperature and α_s is the thermal diffusivity of the soil.

Equation (4) is subject to the conditions (initial and boundary):

$$T(r, t = 0) = T_G(t), \quad (5)$$

$$T(r \rightarrow \infty, t) = T_G(t), \quad (6)$$

$$-k \frac{\partial T}{\partial r} (2\pi r) \Big|_{r=r_{po}} = \frac{T_f - T_{po}}{R_p}, \quad (7)$$

in which R_p is the conductive resistance to heat, represented by:

$$R_p = R_{cond} + R_{conv} = \frac{1}{2k_p\pi} \ln\left(\frac{r_{po}}{r_{pi}}\right) + \frac{1}{2\pi r_{pi} h_{conv}}, \quad (8)$$

for the convective transfer coefficient (h_{conv}) (Bejan and Kraus, 2003) will be used:

$$h_{conv} = \frac{k_f}{2r_{pi}} \frac{\left(\frac{f}{8}\right) (Re - 1000) Pr}{1 + 12.7 \sqrt{\left(\frac{f}{8}\right) (Pr^{\frac{2}{3}} - 1)}} \quad (9)$$

where Re and Pr are the Reynolds and Prandtl numbers, respectively, and $f = [0.79 \ln(Re) - 1.64]^{-2}$.

2.2 Solution by Laplace transform

The proposed solution was based on the work of Minaei and Safikhani (2021), modifying soil and inlet air temperature conditions. Initially, to facilitate the resolution of the problem, the following variable substitutions are applied: $\theta = T - T_G$ and $x^* = \frac{x}{L}$.

Thus, Equations (1) to (7) are:

$$m \frac{\partial \theta_f}{\partial t} + \frac{d\theta_f}{dx^*} + c [\theta_f(x^*) - \theta_{po}(x^*)] = 0. \quad (10)$$

$$\theta_f(x^*, 0) = 0, \quad (11)$$

$$\theta_f(x^* = 0, t) = T_i - T_G = \theta_i(t). \quad (12)$$

For heat transfer in the soil around the heat exchanger, the following heat equation was used:

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} = \frac{1}{\alpha_s} \frac{\partial \theta}{\partial t}, \quad (13)$$

and the conditions become:

$$\theta(r, t = 0) = 0, \quad (14)$$

$$\theta(r \rightarrow \infty, t) = 0, \quad (15)$$

$$-k \frac{\partial \theta}{\partial r} (2\pi r) \Big|_{r=r_{po}} = q_{po}. \quad (16)$$

To solve the proposed problem, the Laplace transform is applied to Equations (10) to (16), resulting in:

$$m s \bar{\theta}_f + \frac{d\bar{\theta}_f}{dx^*} + c [\bar{\theta}_f(x^*, s) - \bar{\theta}_{po}(x^*, s)] = 0. \quad (17)$$

$$\bar{\theta}_f(x^*, 0) = 0, \quad (18)$$

$$\bar{\theta}_f(x^* = 0, s) = \bar{\theta}_i(s). \quad (19)$$

$$\frac{\partial^2 \bar{\theta}(r, s)}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{\theta}(r, s)}{\partial r} - \frac{s}{\alpha_s} \bar{\theta}(r, s) = 0. \quad (20)$$

$$\bar{\theta}(r \rightarrow \infty, t) = 0, \quad (21)$$

$$-k \frac{\partial \bar{\theta}}{\partial r} (2\pi r) \Big|_{r=r_{po}} = \frac{\bar{\theta}_f(x^*, s) - \bar{\theta}_{po}(x^*, s)}{R_p}. \quad (22)$$

To solve Equation (20), a variable change is used: $y = r\sqrt{\frac{s}{\alpha_s}}$, and it is considered that $\bar{\theta}(r,s) = g(y)$, obtaining a differential Bessel equation (with well-known general solution). Finally, the boundary conditions (Equations (21) and (22)) are applied, obtaining:

$$\bar{\theta}(r,s) = \frac{\bar{\theta}_f(x^*,s) - \bar{\theta}_{p_o}(x^*,s)}{R_p} \frac{1}{(2\pi r_{p_o})k\sqrt{\frac{s}{\alpha_s}}} \frac{K_0\left(r\sqrt{\frac{s}{\alpha_s}}\right)}{K_1\left(r_{p_o}\sqrt{\frac{s}{\alpha_s}}\right)}. \tag{23}$$

Considering that in $r = r_{p_o}$ we have: $\bar{\theta}(r_{p_o},s) = \bar{\theta}_{p_o}(x^*,s)$, using Equations (22) and (23), then follows:

$$\bar{\theta}_{p_o}(x^*,s) = \frac{K_0\left(r_{p_o}\sqrt{\frac{s}{\alpha_s}}\right)}{\underbrace{R_p 2k_s \pi r_{p_o} \sqrt{\frac{s}{\alpha_s}} K_1\left(r_{p_o}\sqrt{\frac{s}{\alpha_s}}\right) + K_0\left(r_{p_o}\sqrt{\frac{s}{\alpha_s}}\right)}_{=\chi(s)}} \bar{\theta}_f(x^*,s). \tag{24}$$

Equation (17) can be rewritten using (24):

$$\frac{d\bar{\theta}_f}{dx^*} + [m s + c(1 - \chi(s))]\bar{\theta}_f = 0. \tag{25}$$

Thus, Equation (25) is solved by the method of separation of variables and using the condition in $x^* = 0$ Equation (19) we have the following particular solution for $\bar{\theta}_f$:

$$\bar{\theta}_f(x^*,s) = \bar{\theta}_i(s)\exp([-m s - c(1 - \chi(s))]x^*). \tag{26}$$

Therefore, using the relation (24), we have the solution $\bar{\theta}_{p_o}(x^*,s)$:

$$\bar{\theta}_{p_o}(x^*,s) = \bar{\theta}_i(s)\chi(s)\exp([-m s - c(1 - \chi(s))]x^*). \tag{27}$$

And substituting the Equations (26) and (27) in the solution transformed into $\bar{\theta}$ (23), we get:

$$\bar{\theta}(r,x^*,s) = \bar{\theta}_i(s) \frac{K_0\left(r\sqrt{\frac{s}{\alpha_s}}\right)}{K_0\left(r_{p_o}\sqrt{\frac{s}{\alpha_s}}\right)} \chi(s) \exp([-m s - c(1 - \chi(s))]x^*). \tag{28}$$

Due to the complexity of calculating the inverse Laplace transform in Equations (26), (27) and (28), we will use the Gaver- Stehfest algorithm (Stehfest, 1970), which is a very efficient numerical inversion method known in the literature. This method produces an approximate value for the solution using the following equation:

$$f(t) = \frac{\ln 2}{t} \sum_{i=1}^N V_i F\left(\frac{\ln 2}{t} \cdot i\right), \quad (29)$$

where N is an even number, F is the transformed function and V_i is defined by:

$$V_i = (-1)^{N/2+i} \sum_{k=\lceil \frac{i+1}{2} \rceil}^{\min\{i, N/2\}} \frac{k^{N/2} (2k)!}{(N/2 - k)! (k)! (k - 1)! (i - k)! (2k - i)!}. \quad (30)$$

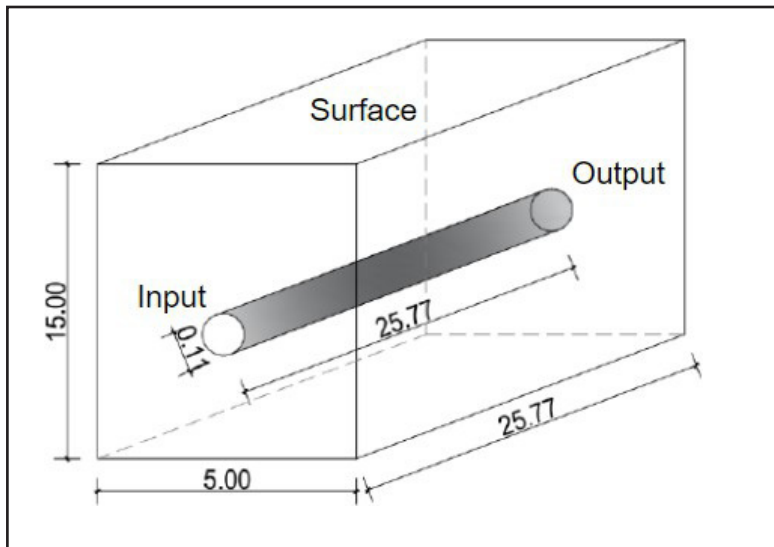
The parameter N is the number of terms used in the summation of Equation (29) and can be optimized to avoid rounding errors.

2.3 Model verification

To verify the model, two simulations are presented comparing the results with: 1) experimental data from Vaz et al. (2011), referring to the installation of a EAHE in the city of Viamão, in the state of Rio Grande do Sul; 2) the application of the EAHE in Pelotas, according to the methodology presented by Nóbrega (2021). Below are the data from 1) and 2).

1) Vaz experiment (2011) Experimental data from Vaz et al. (2011) refer to the installation of a EAHE in the city of Viamão, in the state of Rio Grande do Sul. The dimensions of the duct, used in the experiment, can be found in Figure 2.

Figure 2 – Simulation domain of the simplified model (dimensions in meters)



Source: Vaz et al. (2011)

The air mass flow was 0.0364 kg/s, the duct material used was PVC and the thermo-physical properties of the air and soil are shown in Table 1.

Table1 – Thermo-physical properties of air and soil

	Specificmass $\rho[\text{kgm}^{-3}]$	Thermalconductivity $k[\text{Wm}^{-1}\text{K}^{-1}]$	Specificheat $c_p[\text{Jkg}^{-1}\text{K}^{-1}]$	Dynamicviscosity $\mu[\text{kgm}^{-1}\text{s}^{-1}]$
Air	1.16	0.0242	1010	1.798×10^{-5}
Soil	1800	2.1	1780	-

Source: Vaz et al. (2011)

Air and soil temperatures ($^{\circ}\text{C}$) (at a depth of $z = 1.6\text{m}$) were adjusted by least squares, according to the work by Brum et al. (2012), and are provided, respectively, by the following functions:

$$T_{air}^e(t) = 20.49 + 5.66 \sin\left(\frac{2\pi}{365}t - 5.30\right), \tag{31}$$

$$T_{soil}(t) = 20.49 + 3.03 \sin\left(\frac{2\pi}{365}t - 5.92\right). \tag{32}$$

2) Application of the EAHE in Pelotas - case study

The case study was carried out in the locality called Trevo do Contorno (geographical location 31° 45' 24.8"S and 52° 6' 2.6"W).

For the simulations, the limiting cases of a homogeneous soil composed exclusively of clay or saturated clay are considered, where their thermo-physical properties are presented in Table 2.

Table2 – Thermo-physical properties of air and soil

	Specificmass $\rho[\text{kgm}^{-3}]$	Thermalconductivity $k[\text{Wm}^{-1}\text{K}^{-1}]$	Specificheat $c_p[\text{Jkg}^{-1}\text{K}^{-1}]$
Clay	1600	0.25	890
Saturatedclay	2000	1.58	1550

Source:Nóbrega(2021)

The dimensions and material of the EAHE were the same as the heat exchanger in Vaz's experiment (2011). The air and soil temperature curves are obtained in the same way as the previous experiment, and can be found in the work of Nóbrega (2021). The equations are presented below:

$$T_{air}^e(t) = 18.37 + 6.31 \sin\left(\frac{2\pi}{366}t + 1.20\right), \quad (33)$$

$$T_{soil}(t,z) = 18.37 + 6.31 \sin\left(\frac{2\pi}{366}t + 1.20 - \gamma z\right) \exp(-\gamma z). \quad (34)$$

3 RESULTS

In this section, the results obtained with the model are presented: for the experiment by Vaz and the application of the EAHE in Pelotas (Nóbrega, 2021). In both cases, the proposed model was compared with the analytical model by Nóbrega (2021), whose solution is given by the equation:

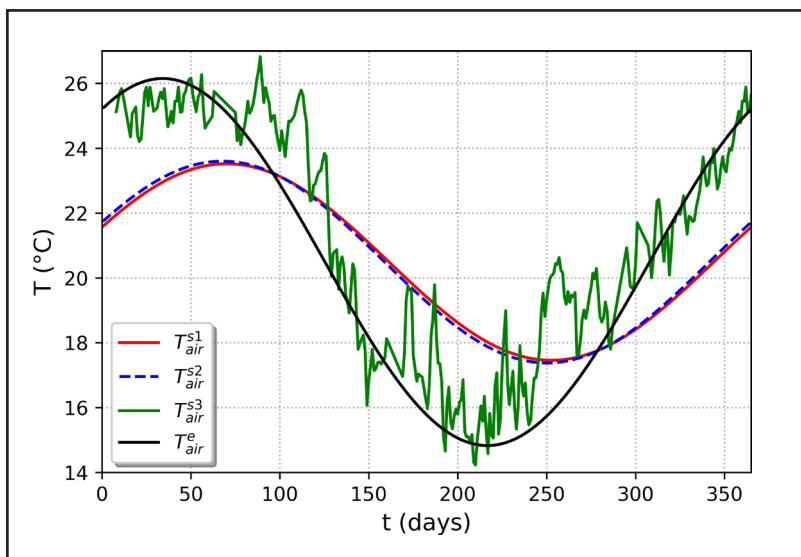
$$T_{air}^s = T_{air}^e + \varepsilon(T_{soil} - T_{air}^e), \tag{35}$$

where T_{air}^e and T_{air}^s the inlet and outlet air temperatures, respectively, and T_{soil} is the soil temperature. For this solution consider $\varepsilon = 1 - \exp(-NUT)$, where $NUT = \frac{h_{conv}A}{\dot{m}c_{p_f}}$.

3.1 Vaz Experiment (2011)

In Figure 3 are shown the temperature curves at the inlet (T_{air}^e) and outlet (T_{air}^{s1} - model presented in this work; T_{ar}^{s2} analytical model presented by Nóbrega (2021) and T_{air}^{s3} data from experiment Vaz et al. (2011)) of EAHE as a function of time

Figure 3 - Temporal evolution of temperature at inlet (T_{air}^e) and outlet (T_{air}^{s1} - proposed model; T_{ar}^{s2} analytical model by Nóbrega (2021) and T_{air}^{s3} experiment data by Vaz et al. (2011)) from EAHE



Source: Vaz et al. (2011) from EAHE

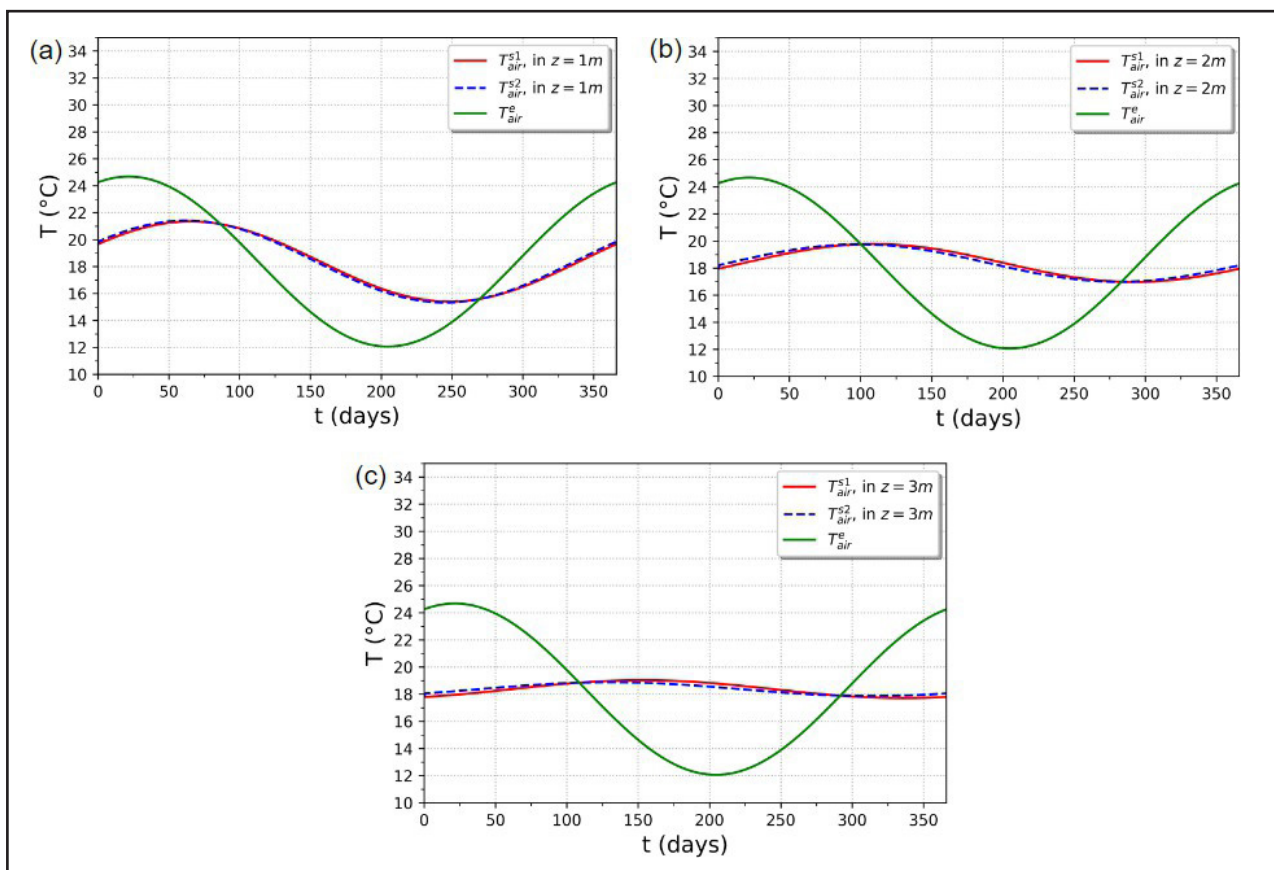
Through the results it is possible to observe that when comparing the solutions of the proposed model (T_{air}^{s1}) with the analytical model of Nóbrega (T_{ar}^{s2}), the results are very similar with a maximum variation of 0.20 °C, where the small discrepancy

between them arises from the fact that the proposed model considers the duct material during the heat transfer process. Thus, the proposed model has an extra thermal resistance, causing the curve to have a small delay when compared to the analytical model by Nóbrega (2021). It is also possible to verify that the model result follows the experimental data, and that it can be used to make first estimates on the use of EAHE.

3.2 Application of the EAHE in Pelotas

In Figures 4 and 5 show the results obtained for different EAHE installation depths ($z = 1\text{ m}$, $z = 2\text{ m}$ and $z = 3\text{ m}$), where the proposed model (T_{air}^{s1}) and Nóbrega's analytical model (T_{ar}^{s2}) - Equation (35), considering a homogeneous soil of clay and saturated clay, respectively.

Figure 4 - Temporal evolution of the temperature at inlet (T_{air}^e) and outlet (T_{air}^{s1} - proposed model and T_{ar}^{s2} analytical model of Nóbrega (2021)) of the EAHE, considering the clay soil

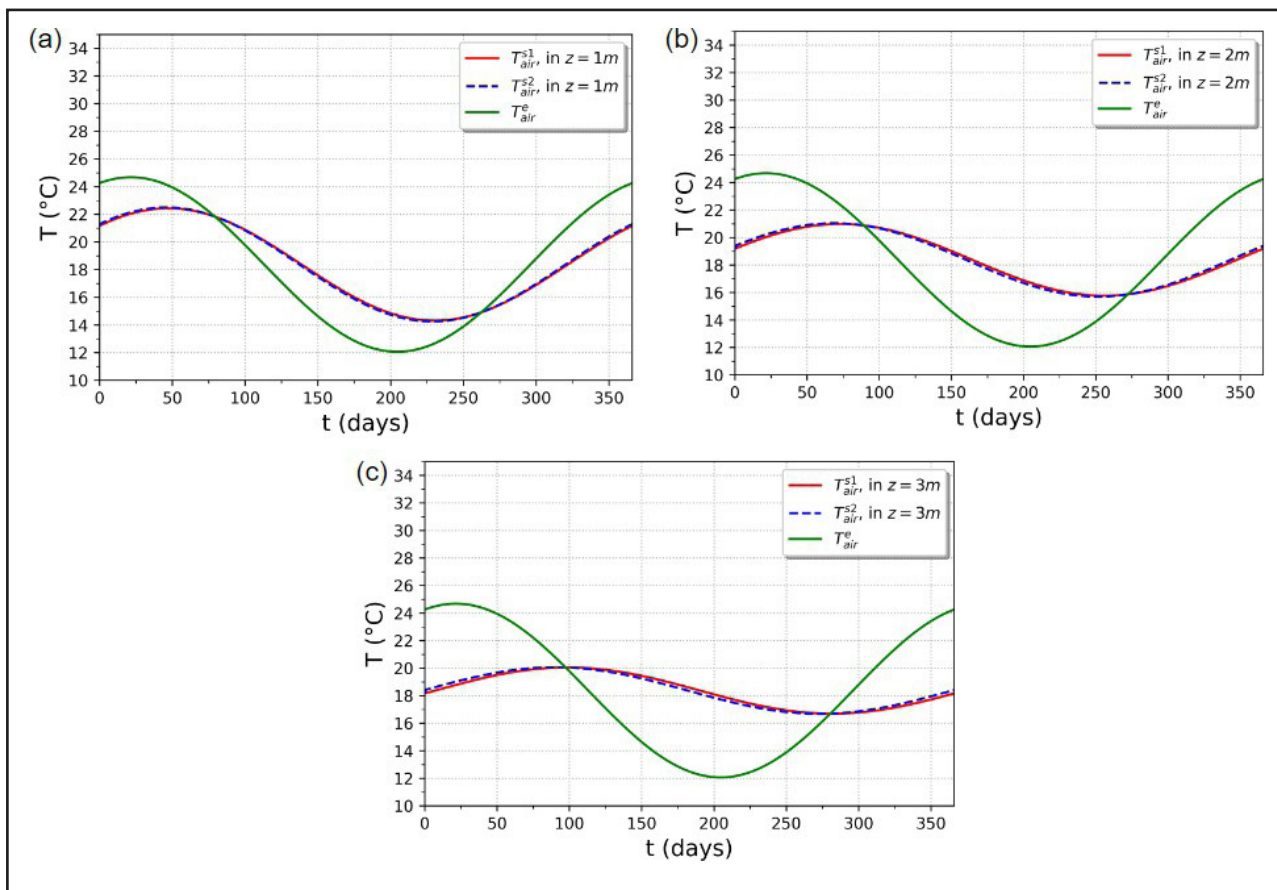


Source: Nóbrega (2021)

In the same way as the previous experiment, it can be seen that the curves (T_{air}^{s1} and T_{ar}^{s2}) are not superimposed, this is due to the fact that the model under study considers heat transfer in the duct material.

Through the graphs it is possible to verify that in both cases (clay soil and saturated clay) the two solutions used generate curves close to each other. By the behavior of the curves, it is possible to verify that a EAHE in Pelotas has the potential to reduce (at the peak of summer) or increase (at the peak of winter) the air temperature. For example, a EAHE installed at a depth of 2 m, in the case of clay soil, Figure 4(b), the reduction/increase is about 6°C; for a saturated clay soil, Figure 5(b), the reduction/increase is about 5°C

Figure 5 – Temporal evolution of the temperature at in let (T_{air}^e) and outlet (T_{air}^{s1} -proposed model and T_{ar}^{s2} analytical model of Nóbrega (2021)) of the EAHE, considering the saturated clay soil



Source: Nóbrega (2021)

4 CONCLUSIONS

The solution proposal presented is valid, as the results obtained by the model are similar to the analytical model by Nóbrega, already existing in the literature and widely used in several applications. The small difference between the results is due to the fact that the proposed model considers the heat transfer in the duct material, simulating a situation closer to reality.

The methodology is simple to implement, as it uses an integral transform technique, and for the inverse Laplace transform it uses the Gaver Stehfest Algorithm.

The coupling between the equations that govern this problem represents the iteration of heat exchange between soil, duct and air, this transient iteration between soil, duct and air that the model considers is an important factor to investigate in the implementation of a EAHE, what many times some analytical models simplify (working in steady state). New studies are needed to investigate the potential of the model and the techniques employed.

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