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Special Edition

Comparative analysis of deterministic and reliability-based structural optimization methods

Análise comparativa dos métodos de otimização estrutural determinístico e baseado em confiabilidade

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ABSTRACT

Optimization is the act of obtaining the best possible result under established conditions. Usually, the optimization of a structural design is done considering the structure's dimensions, the materials' properties, and the loads as deterministic values. This way, the optimization process can lead to a more economical design without guaranteeing that this structure is safe. In practice, there are always uncertainties about the final dimensions of the built structure, material properties, and loads. Then, the need arises to use design optimization techniques based on reliability to guarantee a project that is both economical and safe. This objective is achieved by including uncertainties in the optimization process. This article evaluates the parameters that determine the global minimum of the optimization methods DDO (Deterministic Design Optimization) and RBDO (Reliability-Based Design Optimization). This work aims to compare the structural optimization methods of DDO and RBDO through an example. The results are obtained through the methods implemented in the Python language and show that when comparing the two optimization methods, the presence of uncertainties alters the optimal solution.

Keywords: Optimization; Reliability; DDO; RBDO

RESUMO

A Otimização é o ato de se obter o melhor resultado possível sob condições estabelecidas. Usualmente, a otimização de um projeto estrutural é feita considerando as dimensões da estrutura, as propriedades dos materiais e os carregamentos como valores determinísticos. Dessa forma, o processo de otimização pode levar a uma estrutura mais econômica, mas, sem garantia que essa estrutura seja segura. Isso acontece porque na prática sempre existem incertezas sobre as dimensões finais da estrutura construída, propriedades dos materiais e carregamentos. Então, surge a necessidade de utilização de técnicas de otimização de projeto baseado em confiabilidade de forma a garantir um projeto ao mesmo tempo econômico e seguro. Isso é conseguido através da inclusão de incertezas no processo de otimização.

Neste artigo é feita a avaliação dos parâmetros que determinam o mínimo global dos métodos de otimização DDO (Deterministic Design Optimization) e RBDO (Reliability-Based Design Optimization). O objetivo deste trabalho é realizar uma análise comparativa dos métodos de otimização estrutural DDO e RBDO através de um exemplo. Os resultados são obtidos através de códigos dos métodos implementados na linguagem Python, e mostram que ao comparar os dois métodos de otimização a presença das incertezas altera a solução ótima.

Palavras-chave: Otimização; Confiabilidade; DDO; RBDO

1 INTRODUCTION

According to Kroetz (2019), structural optimization allows the conception of structures that meet desirable requirements and, combined with structural reliability, provides the necessary knowledge to obtain safe and viable systems. Formulating structural optimization problems containing uncertainty quantification is extremely important, especially for structural engineering.

For Shen et al. (2022), uncertainty is inevitable in the real physical world, and it is necessary to consider its effects on structural design and optimization processes. Thus, Ben-Tal *et al.* (2009) state that due to the uncertain nature of real-world engineering design problems, it has been well recognized that structural optimization that takes into consideration the uncertainties receives increasing attention and plays an essential role in practical engineering. One of the methods to incorporate uncertainty in an optimization problem is the RBDO (Reliability-Based Design Optimization), which focuses on finding the best solution that satisfies the target reliability constraints (Beck, 2019; Choi *et al.,* 2011).

For Guo *et al.* (2009), optimization problems are often formulated, assuming a deterministic scenario, where the parameters that affect the measurements are known and determined. Beck (2019) shows that deterministic optimization (Deterministic Design Optimization – DDO) is a formulation of a structural optimization problem using deterministic design variables. It involves the definition of an objective function to be minimized, taking into account its constraints, but does not take into account the uncertainties involved in the problem.

Melchers and Beck (2018) state that when deterministic optimization is used in formulating a problem that presents uncertainties, non-optimal solutions can be obtained. Thus, to get optimized solutions that are at the same time reliable and robust solutions, it is necessary to consider the uncertainties inherent in structural projects.

This work aims to conduct a comparative analysis of DDO and RBDO structural optimization methods through a tubular section steel column example, evaluating the importance and necessity of using reliability-based design optimization techniques to guarantee a project at the same time economical and safe.

2 THEORETICAL FOUNDATION

Several works have already been developed to analyze the performance of DDO and RBDO, highlighting the importance of comparative analysis of these methods.

Beck and Gomes (2012) present the effects of uncertainty and expected failure costs in an optimal structural design, comparing three different formulations of structural optimization problems: DDO, RBDO, and RO (Risk Optimization). Results show that even when optimal safety factors are used as constraints in DDO, the formulation reduces manufacturing costs but increases total expected costs. When system failure probability is used as a constraint in RBDO, this solution reduces manufacturing costs but increases total expected costs. RO produces the topology and ideal balance point between economy and safety.

Zhao *et al.* (2016) presented a comparison between deterministic optimization and considering reliability with three methodologies, including the double-loop approach (The Performance Measurement Approach, PMA) and the decoupled approaches (the so-called Hybrid method and the sequential optimization and reliability assessment, SORA). The stochastic response surface method (SRSM) was applied for reliability analysis. The results show that more efficient optimal solutions are found using reliability-based optimization.

Zhang *et al.* (2018) investigate the differences between deterministic and reliability-based design optimization by applying shock-resistance design to thin-walled foam-filled structures. The results demonstrate that the optimized systems can considerably improve the energy absorption capacity with greater reliability when using the RBDO method, and the deterministic optimization returns non-viable solutions for this problem.

Faes and Valdebenito (2020) propose an approach to solve a particular class of problems in RBDO: minimizing the failure probability of a linear system subjected to an uncertain load. That is, the solution of the RBDO problem is reduced to the resolution of a single deterministic optimization problem followed by single reliability analysis. The application and capabilities of the proposed approach are illustrated through three examples.

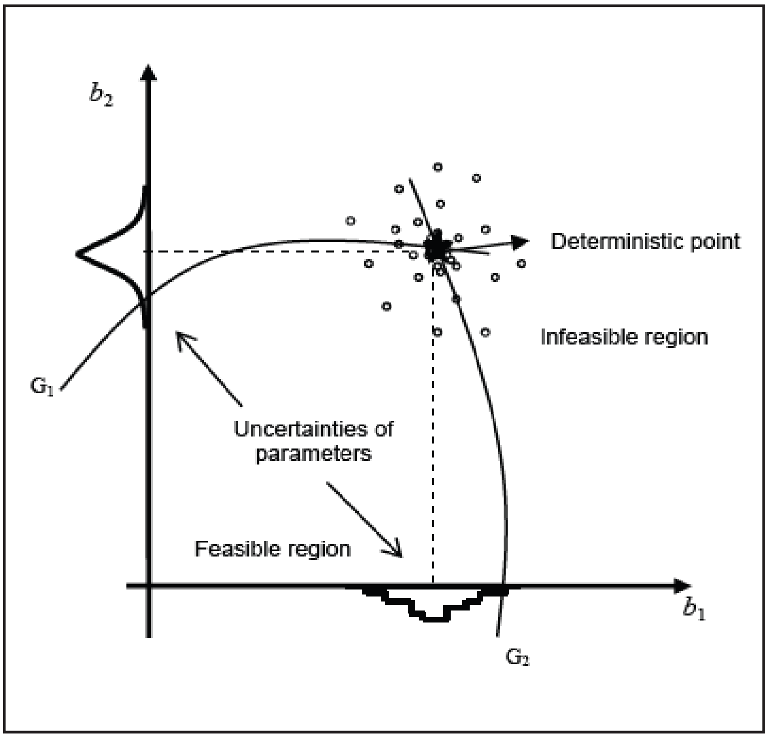
3 METHODOLOGY

The methodology employed in elaborating this work consists of developing codes for comparative analysis through the DDO and RBDO methods. The optimization problem codes are implemented in Python language.

For Arora (2016), it is essential to correctly formulate a design optimization problem because the optimal solution will be as good as the formulation. If a critical constraint is forgotten in the formulation, the optimal solution is likely to be violated.

For Choi *et al.* (2011), in traditional optimization problems, all input information, design variables, and constraints are deterministic. In real-world applications, design variables are uncertain, with probability distributions, and the resulting optimal point can be spread across a single deterministic solution. Some scattered solutions are viable, and some are not, resulting in premature failures and unusable projects or products. To consider the uncertainties in variables, physical models, and solution algorithms, a reliability-based design that considers the randomness of the variables must be carried out to minimize the risk of failure. Figure 1 shows an optimal point for a constrained minimization problem and the need to include uncertainties in the optimization process.

Figure 1 – Need for Reliability-based Design Optimization



Source: Choi *et al.* (2011)

According to Beck (2019), some elementary examples of reliability-based optimization approach are built from the fundamental reliability problem. The fundamental problem involves two statistically independent random variables with Normal probability distribution:

$$R \sim N(\mu_R;\ \sigma_R), \qquad S \sim N(\mu_S;\ \sigma_S)$$

Where is it resistance or capacity, is solicitation or demand; is the mean and is the standard deviation. The limit state function is linear and given by g(x)=R-S=0. For this problem, the reliability index (β) is given by:

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R{}^2 + \sigma_S{}^2}}. \tag{1}$$

3.1 Method DDO

According to Kharmanda *et al.* (2009), system safety can be considered by assigning safety coefficients to specific structural parameters in DDO. By using these safety factors, the optimization problem consists of minimizing an objective function $f(x)$ subject to geometric or physical constraints $g(x) < 0$ in the following:

Minimize: $f(\mathrm{x})$

Subject to: $g(x) < 0 \quad K = 1,\ldots,K$ (2)

where x is determined by the vector of deterministic design variables.

Beck (2019) shows that to compare the DDO method with the RBDO, it is necessary to rewrite the fundamental problem as:

Determine: μ_R^* that minimizes: μ_R,

Subject to: $\mathrm{g}\ (\mu_R) = y\mu_S - \mu_R \leq 0$ (3)

Where the mean resistance μ_R is the only design variable and $y \geq 1$ is a safety coefficient. It is also required that $\mu_S > 0$, to avoid the trivial solution: $\mu_R = \mu_S = 0$, for which the structure disappears. Eq. (3) represents a constrained optimization problem. For a formal solution, the Langrangian function is written, which adds the constraint to the objective function, the development of the method is described by Beck (2019).

3.2 Method RBDO

For Beck (2019), the RBDO is obtained from Eq. (3), and can be written as:

Determine: μ_R^* that minimizes: μ_R

Subject to: $\beta(\mu_R) \geq \beta_T$ (4)

where μ_R is again the design variable, and β_T the target reliability index. Using formal optimization nomenclature and Eqs. (1) and (4), the problem is rewritten as:

Determine: μ_R^* that minimizes: μ_R,

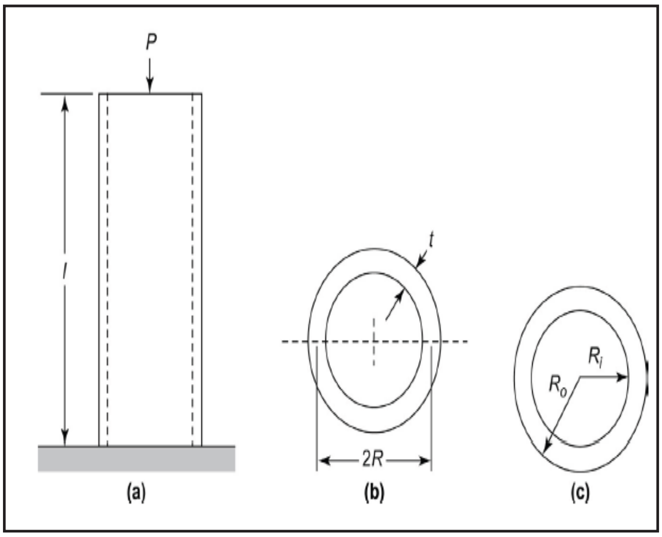
Subject to: $g(\mu_R) = \beta_T - \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \leq 0.$ (5)

As in the DDO method, the RBDO is presented in detail in Beck (2019).

4 RESULTS AND DISCUSSIONS

This section presents a selected example from the literature. The study consists of a comparative analysis of the DDO and RBDO methods.

Figure 2 – Tubular column (a); Design variables (b)



Source: Arora (2016)

The example is adapted from Arora (2016). It consists of determining the minimum mass of a steel column with a tubular section of length l = *5 m,* modulus of elasticity E = 207GPα, and density ρ = 7833 *kg/m*^{3}, supporting a load *P* without buckling. The column is fixed at the base and free at the top, as shown in Figure 2.

Assuming the column wall is thin ($R \gg t$), the cross-sectional area of the material (*A*) and the moment of inertia (*I*) are given by:

$$A = 2\pi Rt \tag{6}$$

$$I = \pi R^3 t \tag{7}$$

Where *R* is the mean radius of the column and *t* is the wall thickness.

The total mass of the column to be minimized is given as:

$$Mass = \rho(Al) = 2\rho l\pi Rt. \tag{8}$$

The first constraint is that the stress (*P/A*) must not exceed the allowable stress (*S*) of the material to avoid failure and is given by:

$$\frac{P}{2\pi Rt} - S \leq 0. \tag{9}$$

The column must not buckle under the applied load *P*, which implies that the applied load must not exceed the buckling load ($P \leq P_{cr}$), given by:

$$P_{cr} = \frac{\pi^2 EI}{4l^2} \tag{10}$$

$$P - \frac{\pi^3 ER^3 t}{4l^2} \leq 0. \tag{11}$$

The example presented in Arora (2016) is deterministic, so there are no random variables. To make a comparison with the DDO and RBDO methods, a target reliability index was adopted in this example ($\beta_T = 3$), the safety coefficients $y_s = 1{,}70$ and $y_P = 1{,}20$ and two statistically independent random variables with Normal probability distribution, to characterize a design problem, but resulting in the same stress and loading, as shown in *S* and *P*:

$S \sim N\ (\mu_S;\ \sigma_S) = (297600;\ 29760)$ kN /m²

$P \sim N\ (\mu_P;\ \sigma_P) = (5882.3529;\ 588.23529)$ kN /m.

4.1 Method DDO

The DDO method can be written as:

determine $d^* = \{R^*, t^*\}$

that minimizes: $f\,(d) = 2\rho l\pi Rt$ (12)

Subject to: $g_1\,(d) = y_S\, y_P\, \mu_P - \mu_S\, 2\pi Rt \leq 0$ (13)

$g_2\,(d) = y_S\, y_P\, \mu l^2 - \pi^3\, ER^3 t \leq 0.$ (14)

The solution starts with the Lagrangian function that includes the objective function and its constraints, as shown in:

$$\mathcal{L}(d,u,s) = 2\rho l\pi Rt + u_1(g_1(d) + s_1^2) + u_2(g_2(d) + s_2^2) \quad (15)$$

where are the Lagrangian multipliers, and are the slack variables. The conditions necessary for a point $d^* = \{ R^*, t^*\}$ be solution are shown in:

$$\frac{\partial \mathcal{L}}{\partial R} = 2\rho l\pi t - 2u_1\mu_S\pi t - 3u_2\pi^3 ER^2 t \quad (16)$$

$$\frac{\partial \mathcal{L}}{\partial t} = 2\rho l\pi R - 2u_1\mu_S\pi R - u_2\pi^3 ER^3 \quad (17)$$

$$\frac{\partial \mathcal{L}}{\partial u_1} = \gamma_S \gamma_P \mu_P - \mu_S 2\pi R t + s_1^2 \quad (18)$$

$$\frac{\partial \mathcal{L}}{\partial u_2} = \gamma_P \mu_P 4 l^2 - \pi^3 E R^3 t + s_2^2 \quad (19)$$

$$\frac{\partial \mathcal{L}}{\partial s_1} = 2 u_1 s_1 \quad (20)$$

$$\frac{\partial \mathcal{L}}{\partial s_2} = 2 u_2 s_2. \quad (21)$$

To determine the global minimum, it is necessary to check all possible ways to satisfy the switching conditions, as shown in Table 1.

Table 1 – Numerical solution of the DDO method

	Case 1 **(u_1, u_2, s_1, s_2)** **(0,0,1,1)**	**Case 2** **(u_1, u_2, s_1, s_2)** **(1,1,0,0)**	**Case 3** **(u_1, u_2, s_1, s_2)** **(1,0,1,0)**	**Case 4** **(u_1, u_1, s_1, s_2)** **(0,1,0,1)**
R (m)	0.0	0.1575	0.4870	0.3184
t (m)	0.0	0.0405	0.0132	0.0048
f (*R*,*t*)	0.0	1579.2339	1579.2339	378.1110
Point	-	A	B	C

Source: Authors (2022)

It can be seen from Table 1 that the constraint on allowable stress (g_1) and buckling load (g_2) are active at point A, in relation to B, it is active only at constraint g_1. Point C does not satisfy the global minimum condition, it is just a local minimum. Therefore, points that accompany the allowable stress constraint (g_1 from point A) give the DDO solution.

4.2 Method RBDO

The RBDO method can be written as:

determine $d^* = \{R^*, t^*\}$ t

hat minimizes: $f(d) = 2\rho l\pi Rt$ (22)

Subject to: $g_1(d) = \beta_T - \beta_S(d) \leq 0 = \beta_T - \frac{2\mu_S\pi Rt - \mu_P}{\sqrt{{\sigma_P}^2 + 4\pi^2R^2t^2{\sigma_S}^2}} \leq 0$ (23)

$$g_2(d) = \beta_T - \beta_P(d) \leq 0 = \beta_T - \frac{\pi^3ER^3t - 4l^2\mu_P}{\sqrt{16l^4{\sigma_P}^2}} \leq 0. \quad (24)$$

The solution starts with the Lagrangian function that includes the objective function and its constraints, as shown in:

$$\mathcal{L}(d,u,s) = 2\rho l\pi Rt + u_1(\beta_T - \beta_S(d) + s_1^2) + u_2(\beta_T - \beta_P(d) + s_2^2) \quad (25)$$

where are the Lagrangian multipliers, and are the slack variables. The conditions necessary for a point $d^* = \{R^*, t^*\}$ be solution are shown in:

$$\frac{\partial\mathcal{L}}{\partial R} = 2\rho l\pi t - u_1\frac{\partial\beta_S}{\partial R} - u_2\frac{\partial\beta_P}{\partial R} \quad (26)$$

$$\frac{\partial\mathcal{L}}{\partial t} = 2\rho l\pi R - u_1\frac{\partial\beta_S}{\partial t} - u_2\frac{\partial\beta_P}{\partial t} \quad (27)$$

$$\frac{\partial\mathcal{L}}{\partial u_1} = \beta_T - \beta_S(d) + s_1^2 \quad (28)$$

$$\frac{\partial\mathcal{L}}{\partial u_2} = \beta_T - \beta_P(d) + s_2^2 \quad (29)$$

$$\frac{\partial\mathcal{L}}{\partial s_1} = 2u_1s_1 \quad (30)$$

$$\frac{\partial\mathcal{L}}{\partial s_2} = 2u_2s_2. \quad (31)$$

It is necessary to check all possible ways of satisfying the switching conditions to determine the global minimum, as shown in Table 2.

It can be seen from Table 2 that the DDO and RBDO methods are very similar, and the conclusions regarding the points are the same. What differentiates the two methods is that the loading uncertainties are used in RBDO making point A more restricted in relation to the DDO.

Table 2 – Numerical solution of the RBDO method

	Case 1 (u_1, u_2, s_1, s_2) (0,0,1,1)	**Case 2 (u_1, u_2, s_1, s_2) (1,1,0,0)**	**Case 3 (u_1, u_2, s_1, s_2) (1,0,1,0)**	**Case 4 (u_1, u_2, s_1, s_2) (0,1,0,1)**
R (m)	0.0	0.1561	0.4945	0.1647
t (m)	0.0	0.0313	0.0099	0.0219
f (*R*,*t*)	0.0	1203.4030	1203.4030	886.3344
Point	-	A	B	C

Source: Authors (2022)

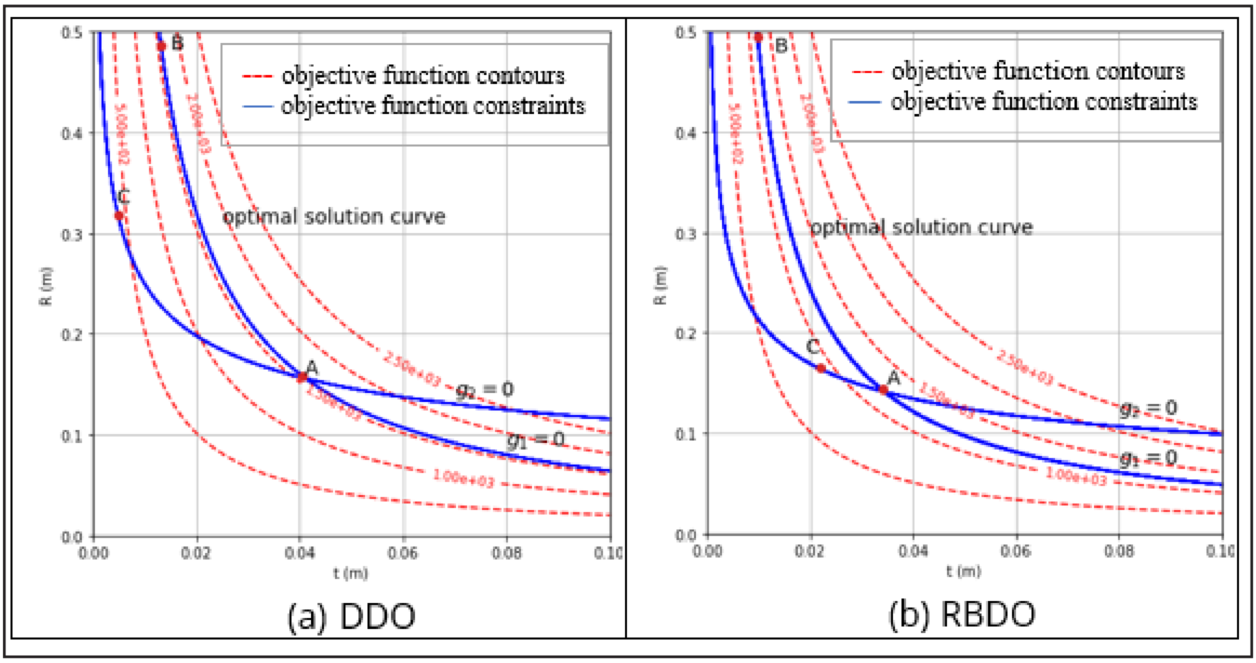
The graphical solution of the DDO and RBDO methods can be interpreted through Fig. 3(a) and (b). In the figure, the dashed red lines represent the contour curves of the objective function ($f(d) = 2\rho l\pi Rt$), as a function of the design variables R and t, it also shows the full lines in blue, which represent the constraints in allowable stress (g_1) and buckling load (g_2). Therefore, as shown in Tables 1 and 2, the graphical solution of the methods is given by all points that accompany the allowable stress constraint (g_1) from point A.

Thus, in this example, the contours of the objective function are parallel to the constraint allowable stress constraint (g_1). Since (g_1) is active at the optimal point, the example has infinitely many optimal points, that is, the entire AB curve in Fig. 3(a) and (b). One can read the coordinates of any point on the AB curve as an optimal solution.

Through the graphical solution, it is noticeable that when using the DDO version, the optimal solution curve is more comprehensive than the RBDO. This is because, in the deterministic version, loading uncertainties are not considered. Thus, when using the optimized version based on reliability, the solution curve is much more restricted, thus showing the importance of uncertainty analysis in structural optimization.

The graphical solution of the DDO and RBDO methods is shown in Figure 3.

Figure 3 – Graphical solution of the DDO and RBDO methods



Source: Authors (2022)

According to Beck (2019), the problems solved by the DDO and RBDO methods can be very similar, when the deterministic and probability constraints have the same degree of nonlinearity. This is not always the case, since the linearity of the reliability constraint depends not only on the shape of the performance function, but also on the probability distribution of the random variables. If the variables in this example had not normally distributed, the reliability indices would not be valid, and an iterative solution would be required. In this case, the reliability constraints would become non-linear, making the RBDO solution distinct from the DDO solution.

5 CONCLUSIONS

This article presented a comparative study of structural optimization methods (DDO and RBDO), considering the results of the minimum mass of a tubular steel column. The performance of these methods was analyzed through an example to identify the differences between deterministic and reliability-based optimization.

The DDO method presented a more comprehensive result than the RBDO, this occurred because this method does not consider the loading uncertainties of the structure studied, and generally, the optimal solution converges on the constraint limit.

Despite being close to the DDO, the optimal solution of the RBDO method appeared more limited, thus showing the importance of considering the uncertainties in formulating a structural optimization problem.

In deterministic optimization, there is no space to accommodate the uncertainties of the design variables. Thus, real-world problems solved by DDO can become less significant or even unfeasible.

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Contribution: Software, Supervision and Writing – review & editing

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