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Engineering system design using the vibrating particles system algorithm

Projeto de sistemas de engenharia usando o algoritmo de sistemas de partículas vibrantes

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ABSTRACT

This contribution aims to apply the Vibrating Particles System Algorithm (VPSA) in engineering system design. In general, this optimization strategy is based on the free vibration simulation of a sub-damped system constituted by particles that gradually tend to equilibrium positions. In order to evaluate the capacity of this optimization strategy, three classical problems in the engineering context (welded beam design, pressure vessel design and tension/compression spring design) are studied. The obtained results demonstrate that the VPSA configures an interesting alternative to engineering system design compared with other heuristic approaches.

Keywords: Vibrating particles system algorithm; Heuristic optimization; Engineering system design

RESUMO

O presente trabalho tem por objetivo a aplicação do Algoritmo de Sistema de Partículas Vibrantes (ASPV) no projeto de sistemas de engenharia. De forma geral, esta estratégia de otimização é fundamentada na simulação da vibração livre de um sistema subamortecido constituído por partículas que aos poucos se aproximam de suas posições de equilíbrio. Para avaliar a capacidade desta estratégia de otimização, três problemas clássicos no contexto da engenharia (projeto de uma viga soldada, projeto de um vaso de pressão e o projeto de uma mola) são estudados. Os resultados obtidos demonstram que o ASPV configura como uma alternativa interessante em comparação com outras estratégias heurísticas.

Palavras-chave: Algoritmo de sistemas de partículas vibrantes; Otimização heurística; Projeto de sistemas de engenharia

1 INTRODUCTION

Recent advances in the field of optimization have been driven by the need to evaluate realistic case studies. For this purpose, the development and improvement of optimization strategies configure a research line of great interest to scientific community. This is due to the ability to escape from local optima, deal with mixed variables and are more appropriate strategies to solve multi-objective problems (Deb, 2001).

In recent years, optimization based on heuristic methods has been chosen as an approach to engineering system design, in which difficulties such as the presence of nonlinearities, multimodality, constraints and mixed variables (continuous, binary, discrete and integer) are often observed. Advances in the design area are due to popularization of these optimization techniques, which are increasingly diversified, allowing decision-making in applications with different levels of complexity (Lobato, 2008).

Optimization methods can be classified as heuristic (or random or non-deterministic) and classical (or deterministic). The former seeks the optimal solution considering the interaction between individuals in a population and are not based on information about the gradient of the objective function and constraints. On the other hand, classical methods aim to find the optimal solution considering information about the gradient of the objective function and constraints (Deb, 2001).

Regarding heuristic methods, the following advantages can be highlighted (Deb, 2001): conceptual simplicity, ease of implementation, ability to escape from local optima, ease of handling mixed variables and problems with discontinuity. On the other hand, the high number of objective function evaluations is considered as the main disadvantage.

Despite this significant disadvantage when compared to classical methods, the scientific community has expended efforts to develop and improve heuristic

methods. Among these, a new approach called the Vibrant Particles Systems Algorithm (VPSA) has excelled in applications in different fields of science and engineering. This optimization technique, proposed by Kaveh and Ghazaan (2017) for structure design, is based on the theory of vibrations, i.e., VPSA simulates the free vibration of an underdamped system consisting of particles that, gradually, converge to the equilibrium position. By combining random exploration with the refinement of the obtained results, the algorithm improves the quality of potential candidates (particles) with the oscillation around these positions during the optimization process (Kaveh and Bakhspoori, 2019). Applications of VPSA can also be found in photovoltaic system design (Gnetchejo *et al.*, 2019) and structure design (Talatahari, Jalili and Azizi, 2021; Kaveh, 2017).

This work aims to apply VPSA to engineering system design (welded beam design, pressure vessel design and tension/compression spring design), as well as compare it with other optimization strategies. This work is organized as follows. In Section 2, the VPSA is briefly presented. The proposed methodology is described in Section 3. The results of three applications are presented in Section 4. Finally, in the last section, the conclusions are drawn.

2 VIBRATING PARTICLES SYSTEM ALGORITHM

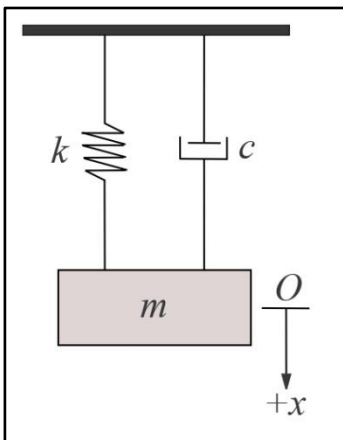
2.1 Conceptual Conception

The VPSA, proposed by Kaveh and Ghazaan (2017), is an optimization strategy originally developed to design trusses. The idea behind this approach consists of simulating the free vibration of systems with one degree of freedom and that presents viscous damping. In an optimization context, particles are solution candidates of a system that gradually tends toward its equilibrium position.

The theory of vibration provides the conceptual basis of the algorithm. In this case, vibrations can be classified into free and forced. In the first class, motion is maintained only by conservative forces. In the second, a periodic force is applied to the system (Kaveh and Ghazaan, 2017). Furthermore, generally in undamped vibrations, the effects of friction can be neglected. However, in fact, it is observed that all vibrations (free or forced) are damped at some level due to the presence of frictional forces (Kaveh and Ghazaan, 2017).

From these principles, the VPSA refers to the vibration of systems that can be represented according to Figure 1, which have a degree of freedom and viscous damping, which is characterized by direct proportionality and in the opposite direction of the friction force in relation to the velocity vector.

Figure 1 – Free vibration of a system with one degree of freedom with damping



Source: Adapted from Kaveh & Ghazaan (2017)

In this figure, a system constituted by a block of mass m , a spring of spring constant k and a damper of damping coefficient c can be observed.

By displacing the block at a distance x in relation to the equilibrium position (indicated by O), the block's equation of motion is given by Eq.(1):

$$m\ddot{x} + c\dot{x} + kx = 0 \quad (1)$$

The critical damping coefficient (c_c) is defined from the natural frequency (ω_n) of the system (Eq.(2) and Eq.(3)):

$$c_c = 2m\omega_n \quad (2)$$

$$\omega_n = \sqrt{k/m} \quad (3)$$

The value of the critical damping coefficient differentiates three types of systems: overdamped, critically damped, and underdamped (Kaveh and Ghazaan, 2017). In VPSA, only the underdamped system was considered as the solution; the other two types refer to a non-vibratory movement.

In this case, the solution of the differential equation (Eq.(1)) is given by Eq.(4):

$$x(t) = \rho e^{-\xi\omega_n t} \sin(\omega_D t + \phi) \quad (4)$$

$$\omega_D = \omega_n \sqrt{1 - \xi^2} \quad (5)$$

$$\xi = c/c_c \quad (6)$$

where the ω_D is the damped natural frequency and ξ is the damping rate.

The solution $x(t)$ requires the determination of constants ρ and ϕ , obtained by the definition of the initial conditions of problem.

2.2 Basic Steps

The basic steps considered in VPSA are presented as follows (Kaveh, 2017):

Step 1 - **Initialization:** The design problem (objective function, design space and constraints) and VPSA parameters are defined. In addition, the initial positions of particles are defined (randomly) inside the design space.

Step2 - **Evaluation of the candidates' solutions:** Each potential candidate (particle) is evaluated according to the objective function defined.

Step 3 - **Update of the particle positions:** For each particle, three equilibrium positions considering different weights are defined: (i) the best

solution found until the current generation (*HB*); (ii) a good particle (*GP*); and (iii) a bad particle (*BP*). To select *GP* and *BP*, a current population is ordered according to the values of the objective function. After this selection, these values are randomly chosen from the first and second half, respectively. To represent the effect of damping, a descending function, proportional to the number of generations, is proposed:

$$D = \left(\frac{q}{q_{\max}} \right)^{-\alpha} \quad (7)$$

where q is the actual generation, q_{\max} is the total number of generations and α is a constant defined by user.

Thus, the particle positions are updated as:

$$x_i^j(q+1) = w_1[DAr_1 + HB^j] + w_2[DAr_2 + GP^j(q)] + w_3[DAr_3 + BP^j(q)] \quad (8)$$

$$A = [w_1(HB^j - x_i^j(q))] + [w_2(GP^j(q) - x_i^j(q))] + [w_3(BP^j(q) - x_i^j(q))] \quad (9)$$

$$w_1 + w_2 + w_3 = 1 \quad (10)$$

where w_1 , w_2 and w_3 are parameters that represent the relative importance of *HB*, *GP* and *BP*, respectively, r_1 , r_2 and r_3 are random number in interval [0 1]; and x_i^j is the j -th variable of the i -th particle.

Parameters A and D present a similar effect to ρ and $e^{-\xi\omega_n t}$ (see Eq.(4)), while the value of $\sin(\omega_D t + \phi)$ is considered equal to unity. It is important to mention that the effect of the bad particle (*BP*) is considered in the update of positions. Thus, p between 0 and 1 is defined and, for each particle, its value is compared with a random number (*rand*) in the range [0 1]. If $p < rand$, then the weight of *BP* is equal to zero ($w_3 = 0$). As a consequence, $w_2 = 1 - w_1$.

Step 4 - **Treatment of site constraints:** considering the possibility of design space violation, a technique based on harmonic search is used (Kaveh and Talatahari, 2009), in which the *HMCR* parameter specifies whether the

replacement of the violated component will be carried out with the *HB* of a random particle or with a new random value inside the search space.

Furthermore, if *HB* is chosen, the *PAR* parameter determines whether this value should be modified by a new neighborhood value or not.

Step 5 - **Stopping Criterion:** Steps 2 to 4 are repeated until the established stopping criterion is satisfied and the optimal solution is presented.

3 METHODOLOGY

As mentioned earlier, the VPSA is applied to engineering system design (welded beam design, pressure vessel design and tension/compression spring design). For this purpose, in each application, 20 particles and 200 generations are considered. In addition, Table 1 presents other parameters considered. It is important to mention that these parameters were chosen from preliminary simulations.

Table 1 – Parameters considered by VPSA in all applications

Parameter	Value
α	0.01
w_1	0.3
w_2	0.3
w_3	0.4
p	0.1
<i>HMCR</i>	0.95
<i>PAR</i>	0.1

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In order to deal with inequality constraints, the Exterior Penalty Method (Vanderplaats, 1999) is considered. In this case, the original constrained problem is converted into unrestricted problem by defining the pseudo-objective function

(*POF*) and penalty factor (r_p), considered equal to 10^9 . Thus, when one or more constraints are violated, the original objective function (*OF*) is penalized as:

$$POF(x) = OF(x) + r_p \sum_{j=1}^m (\max(0, g_j(x)))^2 \quad (11)$$

where m is the number of inequality constraints.

It is important to emphasize that the stopping criterion is the number of generations (200). In addition, the VPSA was executed 30 times to obtain the average, worst and standard deviation values.

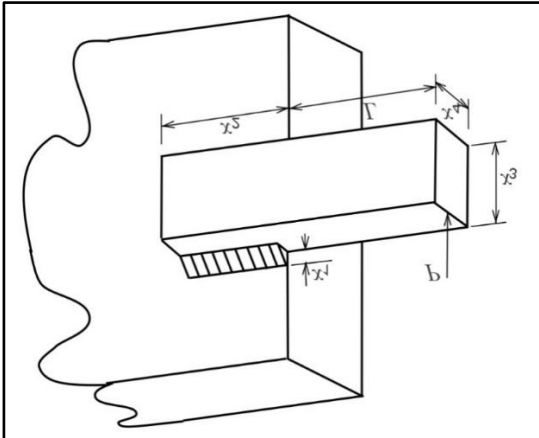
4 RESULTS AND DISCUSSION

In this section, the proposed methodology is applied to engineering system design. For this purpose, three benchmark design problems are evaluated. The results obtained by using the VPSA are compared with those obtained considering other heuristic strategies.

4.1 Welded Beam Design Problem

The first application considers the welded beam design problem to minimize the fabricating cost (Eq.(12)) subject to constraints on shear stress (τ), bendingstress (σ), buckling load (P_c), end deflection (δ), and side constraint. The design variables (geometric characteristics of the weld and beam) are indicated by x_1 , x_2 , x_3 and x_4 , according to Figure2.

Figure 2 – Welded beam design problem



Source: Adapted from Coello & Montes (2002)

Mathematically, this problem can be represented by the following expressions (Coello and Montes, 2002).

$$\min f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2) \quad (12)$$

$$g_1(x) = \tau(x) - \tau_{max} \leq 0 \quad (13a)$$

$$g_2(x) = \sigma(x) - \sigma_{max} \leq 0 \quad (13b)$$

$$g_3(x) = x_1 - x_4 \leq 0 \quad (13c)$$

$$g_4(x) = x_4 - 240 \leq 0 \quad (13d)$$

$$g_5(x) = 0.125 - x_1 \leq 0 \quad (13e)$$

$$g_6(x) = \delta(x) - \delta_{max} \leq 0 \quad (13f)$$

$$g_7(x) = P - P_c(x) \leq 0 \quad (13g)$$

$$0.00254 \leq x_1 \leq 0.0508 \quad (13h)$$

$$0.00254 \leq x_2 \leq 0.254 \quad (13i)$$

$$0.00254 \leq x_3 \leq 0.254 \quad (13j)$$

$$0.00254 \leq x_4 \leq 0.0508 \quad (13k)$$

where:

$$\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2};$$

$$\tau' = \frac{P}{\sqrt{2}x_1x_2};$$

$$\tau'' = \frac{MR}{J};$$

$$M = P\left(L + \frac{x_2}{2}\right);$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1+x_3}{2}\right)^2};$$

$$J = 2\left[\sqrt{2}x_1x_2\left(\frac{x_2^2}{12} + \left(\frac{x_1+x_3}{2}\right)^2\right)\right];$$

$$\sigma(x) = \frac{6PL}{x_4x_3^2};$$

$$\delta(x) = \frac{4PL^3}{Ex_4x_3^3};$$

$$P_c(x) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right);$$

$$L = 0.3556 \text{ m};$$

$$P = 26689.33 \text{ N};$$

$$E = 206.8427 \text{ GPa};$$

$$\sigma_{max} = 206.8427 \text{ MPa};$$

$$\tau_{max} = 93768.7 \text{ kPa};$$

$$\delta_{max} = 0.00635 \text{ m}.$$

This problem has been studied by various authors considering different strategies. In this case, approaches based on Genetic Algorithms were considered by Deb (1991), Coello (2000) and Coello and Montes (2002). He and Wang (2007) considered the Particle Swarm Algorithm as the optimization strategy, Montes and Coello (2008) used evolutionary strategies and Kaveh and Talatahari (2010) considered the Improved Ant Colony Optimization.

Table 2 presents the obtained results considering VPSA and other strategies to solve the welded beam design problem.

Table 2 – Best solution considering different optimization approaches for the welded beam design problem

Reference	x_1 [m]	x_2 [m]	x_3 [m]	x_4 [m]	$f(x)$ [\$]
Deb (1991)	0.006322	0.156794	0.207744	0.006434	2.433116
Coello (2000)	0.005304	0.086881	0.228537	0.005334	1.748309
Coello & Montes (2002)	0.005232	0.088172	0.229114	0.005245	1.728226
He & Wang (2007)	0.005140	0.090023	0.229825	0.005225	1.728024
Montes & Coello (2008)	0.005073	0.091746	0.229553	0.005234	1.737300
Kaveh & Talatahari (2010)	0.005225	0.088167	0.229532	0.005226	1.724918
Present Work	0.005225	0.088153	0.229536	0.005226	1.724947

Organized by the authors (2023)

This table shows that the VPSA strategy was able to obtain a good estimate for the optimal solution in relation to the best solution reported in the literature. In addition, the design space vector obtained is consistent with the corresponding optimal solution. As presented in Table 3, the average and worst values obtained demonstrate that the VPSA strategy always converges to a solution close to that reported in the literature. It is important to mention that the value of the standard deviation obtained by VPSA is the smallest among all, which demonstrates the ability of this optimization algorithm.

Concerning the number of objective function evaluations (n_{eval}), Table 3 shows that the value required by VPSA (4020) is much lower than the values required by Coello and Montes (2002), Montes and Coello (2008) and Kaveh and Talatahari (2010). Thus, a reduction of approximately 94.99%, 83.92% and 77.16% is observed, respectively. On the other hand, in relation to studies conducted by Coello (2000) and He and Wang (2007), an increase can be observed in the value of this parameter of 45.53% and 63.19%, respectively.

Table 3 – Statistical results considering different optimization approaches for the welded beam design problem

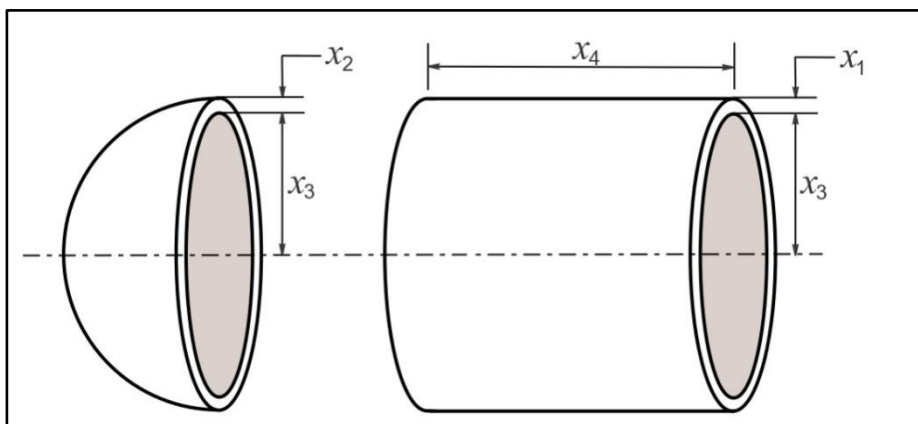
Reference	Best Value [\$]	Worst Value [\$]	Average [\$]	Standard Deviation [\$]	n_{eval}
Deb (1991)	2.433116	-	-	-	1450
Coello (2000)	1.748309	1.785835	1.771973	0.011220	2190
Coello & Montes (2002)	1.728226	1.993408	1.792654	0.074713	80200
He & Wang (2007)	1.728024	1.782143	1.748831	0.012926	1480
Montes & Coello (2008)	1.737300	1.994651	1.813290	0.070500	25000
Kaveh & Talatahari (2010)	1.724918	1.775961	1.729752	0.009200	17600
Present Work	1.724947	1.747262	1.729120	0.004945	4020

Organized by the authors (2023)

4.2 Pressure Vessel Design Problem

Second application considers a cylindrical vessel capped at both ends by hemispherical heads as shown in Figure 3.

Figure 3 – Pressure vessel design problem



Source: Adapted from Kaveh & Talatahari (2010)

The objective is to minimize the total cost, including the cost of material, forming, and welding. For this purpose, four design variables are considered: x_1 is the thickness of the shell, x_2 is the thickness of the head, x_3 is the inner radius, and x_4 is the length of cylindrical section of the vessel, not including the head. In

this case, the variables x_1 and x_2 are integer multiples of 0.0625 inch, the available thickness of rolled steel plates, and x_3 and x_4 are continuous (Kannan and Kramer, 1994). Mathematically, this problem can be formulated as:

$$\min f(x) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3 \quad (14)$$

$$g_1(x) = -x_1 + 0.0193x_3 \leq 0 \quad (15a)$$

$$g_2(x) = -x_2 + 0.00954x_3 \leq 0 \quad (15b)$$

$$g_3(x) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296 \leq 0 \quad (15c)$$

$$g_4(x) = x_4 - 240 \leq 0 \quad (15d)$$

$$x_1 = \{0.0015875y \mid y \in \mathbb{Z}, 1 \leq y \leq 21\} \quad (15e)$$

$$x_2 = \{0.0015875y \mid y \in \mathbb{Z}, 1 \leq y \leq 21\} \quad (15f)$$

$$0.254 \leq x_3 \leq 5.08 \quad (15g)$$

$$0.254 \leq x_4 \leq 5.08 \quad (15h)$$

It is important to highlight that, originally, VPSA is not able to deal with mixed variables (continuous, integer, discrete and binary). Thus, to solve this case study, the approach proposed by Lobato (2008) to deal with mixed variables is considered. In summary, this approach consists of redefining the domain of integer, discrete and binary variables to transform them into a continuous domain. As a consequence, the VPSA strategy can be applied without any modification in the original algorithm. A complete description of this strategy can be found in Lobato (2008).

Table 4 presents the obtained results considering the proposed methodology and other strategies (Branch-and-Bound Method (Sandgren, 1988); Lagrange Multiplier Method (Kannan and Kramer, 1994); Genetic Algorithms (Deb, 1997; Coello, 2000; Coello and Montes, 2002), Particle Swarm Algorithm (He and Wang, 2007)).

This table shows that the VPSA strategy was able to obtain a good estimate for the optimal solution when compared with the best values reported in the literature. This demonstrates that the strategy proposed by Lobato (2008) to deal with mixed variables was efficient in controlling discrete variables.

The average and worst values presented in Table 5 demonstrate that the VPSA strategy always converges to a solution close to that reported in the literature. Concerning the reported best solution, the value of the standard deviation obtained by VPSA is the smallest among all.

Table 4 – The best solution considering different optimization approaches for the pressure vessel design problem

Reference	x_1 [m]	x_2 [m]	x_3 [m]	x_4 [m]	$f(x)$ [\$]
Sandgren (1988)	0.0285750	0.0158750	1.2115800	2.9896054	8129.1036
Kannan& Kramer (1994)	0.0285750	0.0158750	1.4805914	1.1097260	7198.0428
Deb (1997)	0.0238125	0.0127000	1.2275566	2.8620466	6410.3811
Coello (2000)	0.0206375	0.0111125	1.0242271	5.0800000	6288.7445
Coello & Montes (2002)	0.0206375	0.0111125	1.0692739	4.4870129	6059.9463
He & Wang (2007)	0.0206375	0.0111125	1.0691182	4.4893611	6061.0777
Montes & Coello (2008)	0.0206375	0.0111125	1.0692914	4.4966692	6059.7456
Kaveh & Talatahari (2010)	0.0206375	0.0111125	1.0692982	4.4865989	6059.7258
Present Work	0.0206375	0.0111125	1.0693423	4.4860519	6059.6149

Organized by the authors (2023)

Table 5 –Statistical results considering different optimization approaches for the pressure vessel design problem

Reference	Best Value [\$]	Worst Value [\$]	Average [\$]	Standard Deviation [\$]	n_{eval}
Coello (2000)	6288.7445	6308.1497	6293.8432	7.4133	2190
Coello & Montes (2002)	6059.9463	6469.3220	6177.2533	130.9297	80200
He & Wang (2007)	6061.0777	6363.8041	6147.1332	86.4545	1480
Montes & Coello (2008)	6059.7456	7332.8799	6850.0049	426.0000	25000
Kaveh & Talatahari (2010)	6059.7258	6150.1289	6081.7812	67.2418	-
Present Work	6059.6149	6113.6617	6078.0933	16.9090	4020

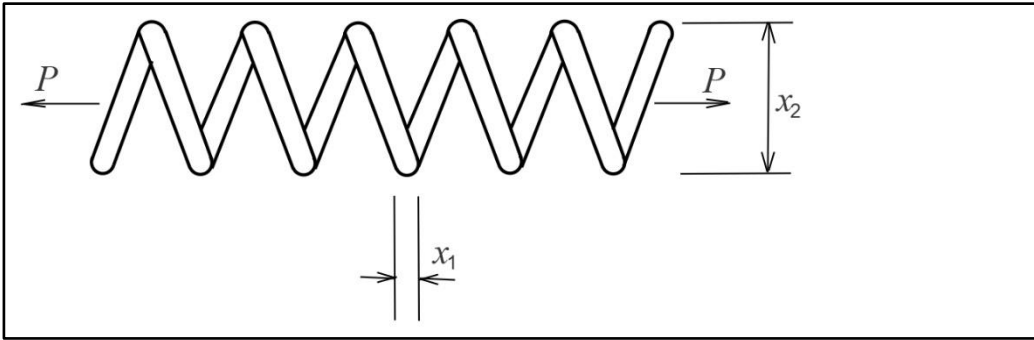
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Concerning n_{eval} presented in Table 5, it shows that the value required by VPSA (4020) is much lower than the values required by Coello and Montes (2002) and Montes and Coello (2008), i.e., a reduction of approximately 94.99% and 83.92% is observed. On the other hand, concerning the works by Coello (2000) and He and Wang (2007), there is an increase in the value of this parameter of 45.53% and 63.19%, respectively.

4.3 Tension/Compression String Design Problem

The last application consists of minimizing the weight of a tension/compression string subject to constraints on shear stress, surge frequency, and minimum deflection as presented in Figure 4 (Arora, 1989). In this case, the design variables are the mean coil diameter (x_1), the wire diameter (x_2), and the number of active coils (x_3).

Figure 4 – Tension/Compression String Design Problem



Source: Adapted from Coello & Montes (2002)

Mathematically, this case study is formulated as (Coello and Montes, 2002).

$$\min f(x) = (x_3 + 2)x_2x_1^2 \quad (16)$$

$$g_1(x) = 1 - \frac{x_2^3x_3}{71.785x_1^4} \leq 0 \quad (17a)$$

$$g_2(x) = \frac{4x_2^2 - x_1x_2}{12.566(x_2x_1^3 - x_1^4)} + \frac{1}{5.108x_1^2} - 1 \leq 0 \quad (17b)$$

$$g_3(x) = 1 - \frac{140.45x_1}{x_2^2x_3} \leq 0 \quad (17c)$$

$$g_4(x) = \frac{(x_1 + x_2)}{1.5} - 1 \leq 0 \quad (17d)$$

$$0.05 \leq x_1 \leq 2.00 \quad (17e)$$

$$0.25 \leq x_2 \leq 1.30 \quad (17f)$$

$$2 \leq x_3 \leq 15 \quad (17g)$$

This classical application has been studied by various authors. Belegundu (1982) used the Lagrange Multiplier Method associated to Fletcher's algorithm (Fletcher, 1975), Arora (1989) evaluated a set of strategies (Constraint Correction,

Cost Reduction, Constraint Correction at Constant Cost and Constraint Correction at a Specified Increase in Cost) to solve this design problem. Coello (2000) and Coello and Montes (2002) used Genetic Algorithms. He and Wang (2007) used Particle Swarm Algorithm as an optimization strategy. Montes and Coello (2008) used evolutionary strategies and Kaveh and Talatahari (2010) considered the Improved Ant Colony Optimization.

Table 6 presents the obtained results considering different strategies.

Table 6 – Best solution considering different optimization approaches for the tension/compression string design problem

Reference	x_1 [m]	x_2 [m]	x_3	$f(x)$ [$10^{-6}m^3$]
Belegundu (1982)	0.0012700	0.0080239	14.250000	0.2102952
Arora (1989)	0.0013563	0.0101391	9.185390	0.2086073
Coello (2000)	0.0013076	0.0089322	11.632201	0.2081976
Coello & Montes (2002)	0.0013205	0.0092447	10.890522	0.2078044
He & Wang (2007)	0.0013139	0.0090842	11.244543	0.2077060
Montes & Coello (2008)	0.0013117	0.0090261	11.397926	0.2080829
Kaveh & Talatahari (2010)	0.0013174	0.0091821	11.000000	0.2071817
Present Work	0.0013070	0.0089143	11.643316	0.2077465

Organized by the authors (2023)

As observed in earlier applications, the obtained results presented in this table demonstrate that the VPSA strategy was able to obtain a good approximation to the best solution reported in the literature. In addition, the average, worst and standard deviation values presented in Table 7 proved that the proposed methodology always converged to a close value of the best solution reported.

Finally, concerning the n_{eval} presented in Table 7, it can be observed that the value required by VPSA (4020) is much lower than the values required by Coello and Montes (2002) and Montes and Coello (2008), i.e., a reduction of, approximately 94.99% and 83.92% is observed. On the other hand, concerning

the works by Coello (2000) and He and Wang (2007), there is an increase in the value of this parameter of 45.53% and 63.19%, respectively.

Table 7 – Statistical results considering different optimization approaches for the tension/compression string design problem

Reference	Best Value [10 ⁻⁶ m ³]	WorstValue [10 ⁻⁶ m ³]	Average [10 ⁻⁶ m ³]	Standard Deviation [10 ⁻⁶ m ³]	<i>n_{eval}</i>
Coello (2000)	0.2082	0.2101	0.2092	6.45×10 ⁻⁴	2190
Coello & Montes (2002)	0.2078	0.2126	0.2088	9.67×10 ⁻⁴	80200
He & Wang (2007)	0.2077	0.2118	0.2086	8.52×10 ⁻⁴	1480
Montes & Coello (2008)	0.2081	0.2701	0.2206	1.58×10 ⁻²	25000
Kaveh & Talatahari (2010)	0.2072	0.2111	0.2084	5.72×10 ⁻⁴	-
Present Work	0.2077	0.2111	0.2087	9.08×10 ⁻⁴	4020

Organized by the authors (2023)

5 CONCLUSIONS

This work aimed to apply the VPSA optimization strategy in three engineering design problems with different levels of complexity. The obtained results are in agreement with those reported by other authors, which demonstrates the quality of this emerging approach, especially in relation to binomial convergence versus computational cost. In addition, this optimization algorithm was associated with a strategy used to treat discrete variables. From a statistical point of view, for each application, the VPSA strategy always converged to a good estimate of optimal solution, demonstrating the capacity of this heuristic optimization strategy. As a proposal for future works, the VPSA will be applied in the multi-objective context.

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