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Special Edition

Fractional calculus applied to the Cournot-type adjustment model

Cálculo fracionário aplicado a um modelo de ajustes do tipo Cournot

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ABSTRACT

In this contribution, we propose the incorporation of memory in a Cournot-type model for the process of adjusting the production of firms in a duopoly market, through the calculation of fractional order. Under the simplifying assumption that the inverse demand and costs of firms are affine functions of the quantities produced, we show numerically some conditions on memory (fractional derivative order) and on the speed of the adjustment process, so that the model proposed converges to Cournot's generalized stationary points. Under these assumptions, we show that the firm that has more memory obtains greater profit.

Keywords: Generalized Cournot model; Memory; Duopoly

RESUMO

Nesta contribuição, propomos a incorporação de memória em um modelo do tipo Cournot para processo de ajustes de produção de firmas em um mercado em duopólio, através do cálculo de ordem fracionária. Sob uma premissa simplificadora, de que a demanda inversa e os custos das firmas são funções afim das quantidades produzidas, mostramos, numericamente, algumas condições sobre a memória (ordem da derivada fracionária) e sobre a velocidade do processo de ajuste, para que o modelo proposto convirja para os pontos estacionários generalizados de Cournot. Sob tais premissas, mostramos que a firma que possui maior memória obtém maior lucro.

Palavras-chave: Modelo de Cournot generalizado; Memória; Duopólio

1 INTRODUCTION

Oligopolies can be defined as a market structure with imperfect competition, in which a small group of companies dominates the production of a specific good or service, according to Gremaud et. al. (2003). In particular, in this type of market, assuming that there is no cooperation between companies, the demand curve for a company's products depends on the reaction of other companies (Zibiani, 2015).

The mathematical modeling of this type of market in a duopoly model (oligopoly consisting of only two firms) is due to the economist Antoine Augustin Cournot initiated the pioneering work in the area in Cournot (1897), which considers that: I) the products are identical; II) firms do not cooperate; III) firms have market power, that is, each firm's decision alters the market price; IV) firms compete for quantities and not for price; V) Each firm acts strategically in order to maximize profit, given the decision of the competing firm, with the hypothesis that its own decision does not change the decision of the other firms, e.g. Martin (1993). Let $P_i(q_1(t), q_2(t))$ represent the inverse demand function and $C_i(q_i(t))$ represent the cost function, with $q_i(t)$ representing the quantity produced by firm i at time $t > 0$. Cournot (1897) constructed the profit function $\mathrm{II}_i(q_1(t), q_2(t))$ for each firm i under such assumptions. Cournot then demonstrated that the function that represents the best response of the firm i given a (exogenous) response of the firm j to the market's $j \neq i$ can be constructed from the firm's marginal profit i (a partial derivative of the function Pi_i with respect to the quantity q_i) (Cournot ,1897). In particular, equilibrium states occur when marginal profits intersect, that is, when, simultaneously, firms find their best response, which no longer changes Martin (1993).

The versatility of the Cournot model allows it to be used, with the necessary adaptations, in a large number of market situations. For example, in Agliari, Gardini and Puu (2000) this model is used to model an oligopoly with three competitors. The number of companies is generalized to n firm in Ahmed and Agiza (1998). There is also the possibility of considering that the firms that make up the oligopoly produce non-homogeneous products, that is, non-identical products. Agliari, Naimzada and Pecora (2016) investigated a market with such characteristics. For recent references on variations of the Cournot model proposed in the literature, see Culda, L. C, Kaslik, E., Neam,tu (2022).

However, as far as the authors are aware, all variations of the Cournot model are myopic, that is, a mathematical modeling of competition between firms in oligopoly where the entire history of the market is not taken into account.

The main contribution of this article focuses on the proposal and analysis of a Cournot-type model whose competition memory of each firm in an oligopoly is considered. To this end, we propose a Cournot-type model, whose adjustment dynamics is modeled by fractional order derivatives, whose operators are known to be non-local and, therefore, have the property of adding memory to the competition (see Observation 2.4 in Diethelm (2010)).

2 MULTI-FRACTIONAL COURNOT MODEL

In this section, we will present the multi-fractional Cournot model that we propose in this work. This model assumes that the adjustment process of the firm's quantities $qi(t)$ for i = 1,2 and for $t \geq 0$, follows the partially rational dynamics given by

$$\begin{cases} q_1(t+1) = & q_1(t) + \rho_1 D^{1-\alpha_1}\Pi_1(q_1(t),q_2(t)) \\ q_2(t+1) = & q_2(t) + \rho_2 D^{1-\alpha_2}\Pi_2(q_1(t),q_2(t)) \end{cases}, \tag{1}$$

where ρ_i represents the firm's adjustment speed, for $i = 1,2$, respectively. Also, $D^{1-\alpha_i}$ represents Caputo's fractional differential operator of order $1 - \alpha_i$, for $\alpha_i \in [0,1]$, partial with respect to the variable $q_i(t)$, for $i = 1,2$. The definition of the operator $D^{1-\alpha_i}$ can be obtained in Diethelm (2010). In particular, for $\alpha_i = 0$, the fit model (1) corresponds to the Cournot myopic fit model, as in Martin (1993). Furthermore, we assume that, at $t \leq 0$, firms act as monopolies, which results in the initial conditions for the model (1), given by.

$$q_i(t=0) = q_i(0)\,, \quad i = 1{,}2\,. \tag{2}$$

2.1 Multi-fractional Cournot model for a linear inverse demand

For the sake of simplicity, we will assume that the adjustment (1) are such that the affine function gives the inverse demand function of the model by the quantities produced by firms 1 and 2 as

$$P(q_1(t), q_2(t)) = a - b(q_1(t) + q_2(t))\,, \tag{3}$$

where a and b are positive constants and $a > b$. Furthermore, we will assume that the costs of firms are given by

$$C_i(q_i) = cq_i \tag{4}$$

where c is a positive constant.

As a result, the revenue (total collected) by the firm $i,j = 1{,}2$ with ineqj is given by

$$R_i(q_i(t)) = P(q_i(t), q_j(t))q_i(t) \tag{5}$$

From this, it follows that the profit function of each firm is given by

$$\pi_i(q_i(t), q_j(t)) = P(q_i(t), q_j(t))q_i(t) - cq_i(t)\,, \tag{6}$$

for $i,j = 1{,}2$ and $i \neq j$.

Therefore the profit function $\Pi(\cdot,\cdot)$ is a polynomial function.

Since, the fractional derivatives according to Caputo Diethelm (2010), given by

$$D_x^{n-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \frac{d^n f(t)}{d^x} dt\,, \tag{7}$$

where n is an integer, α a real number belonging to the interval (0,1) and $n-\alpha$ the order of the derivative.Applying the derivative (7) to the function $\Pi_i(\cdot,\cdot)$, we find the following expression

$$D^{1-\alpha_1}\Pi_1(q_1(t),q_2(t)) = (a-c-bq_2(t))\,\frac{q_1(t)^{\alpha_1}\Gamma(1)}{\Gamma(1+\alpha_1)} - 2b\frac{q_1(t)^{1+\alpha_1}\Gamma(2)}{\Gamma(\alpha_1+2)}\,,$$
$$D^{1-\alpha_2}\Pi_2(q_1(t),q_2(t)) = (a-c-bq_1(t))\,\frac{q_2(t)^{\alpha_2}\Gamma(1)}{\Gamma(1+\alpha_2)} - 2b\frac{q_2(t)^{1+\alpha_2}\Gamma(2)}{\Gamma(\alpha_2+2)}\,, \tag{8}$$

where $\Gamma(Z)$ is the Gamma function of $Z \in R$. It follows from (8) that the model (1) is rewritten as

$$\begin{cases} q_1(t+1) & = q_1(t) + \rho_1\left((a-c) - bq_2(t)\frac{q_1^{\alpha_1}(t)\Gamma(1)}{\Gamma(1+\alpha_1)} - 2b\frac{q_1^{1+\alpha_1}(t)\Gamma(2)}{\Gamma(\alpha_1+2)}\right) \\ q_2(t+1) & = q_2(t) + \rho_2\left((a-c) - bq_1(t)\frac{q_2^{\alpha_2}(t)\Gamma(1)}{\Gamma(1+\alpha_2)} - 2b\frac{q_2^{1+\alpha_2}(t)\Gamma(2)}{\Gamma(\alpha_2+2)}\right). \end{cases} \tag{9}$$

3 INFLUENCES OF THE MEMORY IN THE COURNOT MODEL: SIMULATED SCENARIO

In this section, we will present numerical simulations of the influence of memory (choices of α_i in the model (9)) on the existence of equilibrium points and stability of the proposed model, as well as on the profit of firms, for a choice parameter specific to $a = 3$, $b = c = 1$. More general results will be investigated in future contributions.

3.1 Existence of equilibrium points and stability

This section will begin by investigating the existence of equilibrium points for the model (9). The stability of the equilibrium points for different choices of the order of derivatives $\alpha_j \in [0, 1]$ will then be numerically formulated.

Claim 1: The only equilibrium point of the model (9) is given by

$$(q_1^*, q_2^*) = \left(\frac{2(1-\alpha_2)(1+\alpha_1)}{[4-(1+\alpha_2)(1+\alpha_1)]}, \frac{2(1-\alpha_1)(1+\alpha_2)}{[4-(1+\alpha_2)(1+\alpha_1)]}\right). \tag{10}$$

Indeed, it follows from the definition of equilibrium of discrete models (see Zibiani (2015)) that (q_1^*, q_2^*) must satisfy

$$\begin{aligned}(2-q_2^*)\,\frac{(q_1^*)^{\alpha_1}(\Gamma(1)}{\Gamma(1+\alpha_1)}-2\frac{(q_1^*)^{1+\alpha_1}\Gamma(2)}{\Gamma(\alpha_1+2)} &= 0\\ (2-q_1^*)\,\frac{(q_2^*)^{\alpha_2}\Gamma(1)}{\Gamma(1+\alpha_2)}-2\frac{(q_2^*)^{1+\alpha_2}\Gamma(2)}{\Gamma(\alpha_2+2)} &= 0\,.\end{aligned} \tag{11}$$

Using the fact that $\Gamma(2+\alpha_j) = \alpha_j(1+\alpha_j)\Gamma(\alpha_j)$ and that $q_j^* \neq 0$, for $j = 1, 2$, in (11), we obtain that the reaction function of the firm i must satisfy

$$q_1^* = (1+\alpha_1) - q_2^*\frac{(1+\alpha_1)}{2} \tag{12}$$

$$q_2^* = (1+\alpha_2) - q_1^*\frac{(1+\alpha_2)}{2} \tag{13}$$

Substituting (13) into (12), we can conclude that

$$q_1^* =(1+\alpha_1) - \left[(1+\alpha_2) - q_1^*\frac{(1+\alpha_2)}{2}\right]\frac{(1+\alpha_1)}{2}\,. \tag{14}$$

Isolating q_1^* above, we conclude that the first identity in (10) is satisfied. By substituting q_1^* into the equation in (13), we obtain the exact result stated in claim 1.

It is worth noting that when $\alpha_1 = \alpha_2 = 0$, we obtain the classic Cournot model results presented in Cournot (1897).

In the following, we will present numerical simulations for various ρ_1, ρ_2, α_1, and α_2 values. The goal is to determine the effect of parameters on the asymptotical stablability of stationary points for the Cournot model (9). For that fate, we will use the theory of linearized stability presented in Luìs and Rodrigues (2017) which corresponds to determining the eigenvalues of the Jacobian matrix of the model (9) which is given by

$$J(q_1(t),q_2(t)) = \begin{bmatrix} m_1(q_1(t),q_2(t)) & m_2(q_1(t),q_2(t)) \\ m_3(q_1(t),q_2(t)) & m_4(q_1(t),q_2(t)) \end{bmatrix} \tag{15}$$

where

$$
\begin{aligned}
m_1(q_1(t), q_2(t)) &= 1 + \rho_1 \left(-q_2(t) \frac{q_1^{\alpha_1 - 1}(t))}{\Gamma(\alpha_1)} - 2 \frac{q_1^{\alpha_1}(t))}{\Gamma(\alpha_1 + 1)} \right) , \\
m_2(q_1(t), q_2(t)) &= -\rho_1 \frac{q_1^{\alpha_1}(t))}{\Gamma(1 + \alpha_1)} , \\
m_3(q_1(t), q_2(t)) &= -\rho_2 \frac{q_2^{\alpha_2}(t))}{\Gamma(1 + \alpha_2)} \quad e \\
m_4(q_1(t), q_2(t)) &= 1 + \rho_2 \left(-q_1(t) \frac{q_2^{\alpha_2 - 1}(t))}{\Gamma(\alpha_2)} - 2 \frac{q_2^{\alpha_2}(t))}{\Gamma(\alpha_2 + 1)} \right) .
\end{aligned} \tag{16}
$$

It follows from replacing the equilibrium points (q_1^*, q_2^*) given by (10) in the Jacobian matrix, that the characteristic polynomial is given by

$$
p(\lambda) = \lambda^2 - (m_1 + m_4)\lambda + m_1 m_4 - m_2 m_3 . \tag{17}
$$

Hence, the eigenvalues are

$$
\begin{aligned}
\lambda_1 =& \frac{m_1 + m_4 + \sqrt{(m_1 + m_4)^2 - 4(m_1 m_4 - m_2 m_3)}}{2} \\
\lambda_2 =& \frac{m_1 + m_4 - \sqrt{(m_1 + m_4)^2 - 4(m_1 m_4 - m_2 m_3)}}{2} ,
\end{aligned} \tag{18}
$$

where $m_1 = m_1(q_1^*, q_2^*)$, $m_2 = m_2(q_1^*, q_2^*)$, $m_3 = m_3(q_1^*, q_2^*)$, $m_4 = m_4(q_1^*, q_2^*)$.

Next, we will analyze different conditions in α_i and ρ_i, for $i = 1,2$ in order to verify the stability of the equilibrium points. That is, check for which values of ρ_i, ρ_2, α_1 and α_2 the eigenvalues of the Jacobian matrix are less than one according to Luìs and Rodrigues (2017).

Case 1: We will assume that $\alpha_1 = \alpha_2 = \theta \in [0,1]$ and $\rho_1 = \rho_2 = \rho$. The results of the the eigenvalues of the Jacobian matrix are displayed in Table 1.

Table 1 – Values of λ_1 and λ_2 for $\alpha_1 = \alpha_2 = \theta$ and $\rho_1 = \rho_2 = \rho$

	ρ = 0,1		ρ = 0,3		ρ = 0,5		ρ = 0,7		ρ = 0,9	
θ	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$
0,1	0,89	0,69	0,66	0,055	0,44	0,57	0,22	1,20	5,54e-03	1,83
0,2	0,88	0,67	0,63	0,01	0,38	0,65	0,14	1,30	1.10e-01	1.96
0,3	0,87	0,66	0,6	0,03	0,33	0,71	0.06	1,4	2,14e-01	2,08
0,4	0,85	0,65	0,56	0,06	0,27	0,77	0,02	1,48	3,14e-01	2,19
0,5	0,84	0,63	0,53	0,1	0,22	0,83	0,1	1,56	4.10e-01	2.29
0,6	0,83	0,62	0,5	0,13	0,17	0,88	0,18	1,63	5,02e-01	2,38
0,7	0,82	0,62	0,47	0,15	0,12	0,92	0,23	1,69	5,87e-01	2,45
0,8	0,81	0,61	0,44	0,17	0,07	0,95	0,3	1,74	6,66e-01	2,52
0,9	0,81	0,6	0,42	0,19	0,04	0,98	0,35	1,77	7,37e-01	2,57

Source: The authors

Case 2: In this scenario, we will assume that $\rho_1 = \rho_2 = \rho$ and $\alpha_1 \neq \alpha_2 \in [0,1]$. The values for the eigenvalues $|\lambda_i|$ of the Jacobian matrix for the simulated scenarios and for $i = 1,2$ are presented in Table 2 3 and 4, respectively.

Table 2 – Values of λ_1 and λ_2 for $\alpha_2 = 0.1$ and $\rho_1 = \rho_2 = \rho$

	ρ = 0,1		ρ = 0,3		ρ = 0,5		ρ = 0,7		ρ = 0,9	
α_1	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$
0,1	0,79	1,14	0,66	0,06	0,44	0,6	0,22	1,2	0.01	1.83
0,2	0,78	1,06	0,66	0,05	0,44	0,58	0,18	1,26	0,05	1,9
0,3	0,78	0,96	0,67	0,05	0,45	0,58	0,15	1,32	0.1	2
0,4	0,77	0,83	0,69	0,06	0,48	0,56	0,11	1,39	0,14	2,07
0,5	0,77	0,68	0,72	0,08	0,53	0,53	0,08	1,48	0.18	2.18
0,6	0,78	0,52	0,75	0,1	0,58	0,5	0,04	1,58	0,24	2,32
0,7	0,78	0,37	0,78	0,13	0,63	0,46	0,03	1,72	0,32	2,5
0,8	0,79	0,29	0,79	0,15	0,66	0,41	0,14	1,9	0.46	2.73
0,9	0,79	0,62	0,73	0,15	0,54	0,42	0,44	2,2	0,85	3,11

Source: The authors

Table 3 – Values of λ_1 and λ_2 for $\alpha_2 = 0.3$ and $\rho_1 = \rho_2 = \rho$

	$\rho = 0{,}1$		$\rho = 0{,}3$		$\rho = 0{,}5$		$\rho = 0{,}7$		$\rho = 0{,}9$	
α_1	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$
0,1	0,88	0,67	0,63	0	0,39	0,66	0,15	1,32	0,1	1,98
0,2	0,87	0,66	0,61	0	0,36	0,68	0,1	1,36	0,16	2,08
0,3	0,87	0,66	0,6	0,02	0,33	0,71	0,06	1,4	0.21	2.08
0,4	0,86	0,65	0,58	0,04	0,3	0,75	0,02	1,45	0,26	2,15
0,5	0,85	0,64	0,56	0,07	0,27	0,79	0,03	1,51	0,32	2,23
0,6	0,85	0,63	0,54	0,11	0,23	0,85	0,08	1,6	0,39	2,34
0,7	0,83	0,61	0,50	0.16	0,17	0,93	0,16	1,7	0,49	2,48
0,8	0,81	0,59	0,44	0,23	0,07	1,04	0,3	1,87	0,68	2,7
0,9	0,76	0,55	0,28	0,35	0,2	1,26	0,67	2,16	1,14	3,06

Source: The authors

Table 4 – Values of λ_1 and λ_2 for $\alpha_2 = 0.5$ and $\rho_1 = \rho_2 = \rho$

	$\rho = 0{,}1$		$\rho = 0{,}3$		$\rho = 0{,}5$		$\rho = 0{,}7$		$\rho = 0{,}9$	
α_1	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$
0,1	0,87	0,65	0,62	0,04	0,36	0,74	0,11	1,43	0,15	2,13
0,2	0,86	0,65	0,59	0,04	0,32	0,75	0,05	1,45	0,22	2,15
0,3	0,86	0,65	0,58	0,05	0,3	0,75	0,02	1,46	0,26	2,16
0,4	0,86	0,65	0,57	0,06	0,28	0,76	0	1,47	0,29	2,17
0,5	0,85	0,64	0,56	0,07	0,27	0,78	0,02	1,49	0,31	2,2
0,6	0,85	0,64	0,56	0,08	0,26	0,8	0,03	1,52	0,33	2,25
0,7	0,85	0,63	0,55	0,11	0,25	0,84	0,05	1,58	0,35	2,32
0,8	0,85	0,62	0,54	0,15	0,23	0,91	0,09	1,68	0,4	2,44
0,9	0,83	0,6	0,48	0,21	0,15	1,01	0,2	1,82	0,54	2,62

Source: The authors

Based on the values presented in the Tables 1 - 6 and, considering Luìs & Rodrigues, 2017 Theorem 5, we can draw some conclusions about the stability of the model (9).

The order of the derivative influences the model (9) convergence to the equilibrium point. Indeed:

Both the values of ρ_j and α_j, for $j = 1, 2$, determine the stability of the studied equilibrium points. In particular, regardless of the α_i and α_j values used, the model (9) does not converge to the equilibrium point for $\rho_1 = \rho_2 = \rho \geq 0.7$.

When $\rho \leq 0.5$, the eigenvalues have moduli less than one in the case where $\alpha_1 = \alpha_2 = \theta$, as shown in Table 1, and thus the model converges to the equilibrium point thanks to Luìs & Rodrigues, 2017, Theorem 5.

For any simulated case with $\alpha_1 \neq \alpha_2$, the eigenvalues have moduli less than one, but only if $\rho \leq 0.5$, indicating that the stationary point is asymptotically stable. Otherwise, the moduli values of the eigenvalues attain values greater than 1, in some scenarios of choosing α_1 and α_2 see the lasted lines of Tables 2-6.

A detailed analysis regarding the stability of the equilibrium points in terms of the parameters α_i and ρ_i for the model (9) can be done using a bifurcation strategy and will be done in a future contribution.

Table 5 – Values of λ_1 and λ_2 for $\alpha_2 = 0.7$ and $\rho_1 = \rho_2 = \rho$

	ρ = 0,1		ρ = 0,3		ρ = 0,5		ρ = 0,7		ρ = 0,9	
α_1	$\lvert\lambda_1\rvert$	$\lvert\lambda_2\rvert$	$\lvert\lambda_1\rvert$	$\lvert\lambda_2\rvert$	$\lvert\lambda_1\rvert$	$\lvert\lambda_2\rvert$	$\lvert\lambda_1\rvert$	$\lvert\lambda_2\rvert$	$\lvert\lambda_1\rvert$	$\lvert\lambda_2\rvert$
0,1	0,85	0,61	0,56	0,17	0,27	0,94	0,03	1,72	0,32	2,5
0,2	0,84	0,61	0,52	0,17	0,2	0,94	0,12	1,72	0,43	2,5
0,3	0,83	0,61	0,50	0,16	0,17	0,93	0,16	1,71	0,49	2,48
0,4	0,83	0,62	0,49	0,15	0,15	0,92	0,18	1,69	0,52	2,46
0,5	0,83	0,62	0,49	0,15	0,14	0,91	0,2	1,68	0,54	2,44
0,6	0,83	0,62	0,48	0,15	0,13	0,91	0,21	1,67	0,56	2,43
0,7	0,82	0,62	0,47	0,15	0,12	0,91	0,23	1,69	0,59	2,45
0,8	0,81	0,61	0,45	0,17	0,84	0,95	0,28	1,73	0,65	2,51
0,9	0,8	0,6	0,4	0,21	0,75	1,02	0,41	1,83	0,81	2,64

Source: The authors

Table 6 – Values of λ_1 and λ_2 for $\alpha_2 = 0.9$ and $\rho_1 = \rho_2 = \rho$

	$\rho = 0,1$		$\rho = 0,3$		$\rho = 0,5$		$\rho = 0,7$		$\rho = 0,9$	
α_1	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$	$\|\lambda_1\|$	$\|\lambda_2\|$
0,1	0.79	0,54	0,38	0,37	0,03	1,28	0,44	2,2	0,85	3,11
0,2	0,77	0,54	0,30	0,38	0,16	1,3	0,63	2,22	1,09	3,14
0,3	0,76	0,55	0,29	0,35	0,19	1,26	0,67	2,16	1,14	3,06
0,4	0,77	0,56	0,30	0,31	0,17	1,18	0,63	2,06	1,1	2,94
0,5	0,78	0,58	0,33	0,27	0,12	1,12	0,56	1,97	1,01	2,82
0,6	0,79	0,59	0,36	0,24	0,06	1,07	0,48	1,89	0,91	2,72
0,7	0,8	0,6	0,4	0,21	0	1,02	0,41	1,83	0,81	2,64
0,8	0,81	0,6	0,42	0,19	0,03	0,99	0,36	1,79	0,75	2,58
0,9	0,81	0,6	0,42	0,19	0,04	0,98	0,36	1,77	0,74	2,57

Source: The authors

3.2 Analysis of the impact of the adjustment strategy with memory on the firm's profit

One of the main assumptions of the Cournot model is that the firms involved in the oligopolies aim to maximize their profit. However, it is worth noting that in an oligopoly competition, one of the strategies may be to reduce the profit of the competing firm, and, therefore, reduce its competitors' space in the market. In what follows, we will make a preliminary analysis of the effects of the fractional derivative order (from memory) on the profit of the firms involved, if they follow the adjustment dynamics given by the model (9).

Theorem 1. Assume that both firms follow the fractional Cournot adjustment model (9), with $a > c > 0$ and $b > 0$. If the firm i strategy in (9) is such that $\alpha_1 > \alpha_2$, then firm one's profit will be greater than firm two's profit at equilibrium.

Demonstração. It follows from (6), that the firm's i profit, for $i = 1,2$, calculated at the equilibrium points (10) are such that

$$\begin{aligned}\Pi_2((q_1^*,q_2^*)) - \Pi_1((q_1^*,q_2^*)) &= (a-c-b(q_1^*+q_2^*))\,(q_2^*-q_1^*)\\ &= \frac{((3+\alpha_1)(1+\alpha_2)-(3+\alpha_2)(1+\alpha_1))\,(a-c)}{[4-(1+\alpha_1)(1+\alpha_2)]\,b}\\ &= \frac{2(\alpha_2-\alpha_1)\,(a-c)}{[4-(1+\alpha_1)(1+\alpha_2)]\,b} < 0\,,\end{aligned} \tag{19}$$

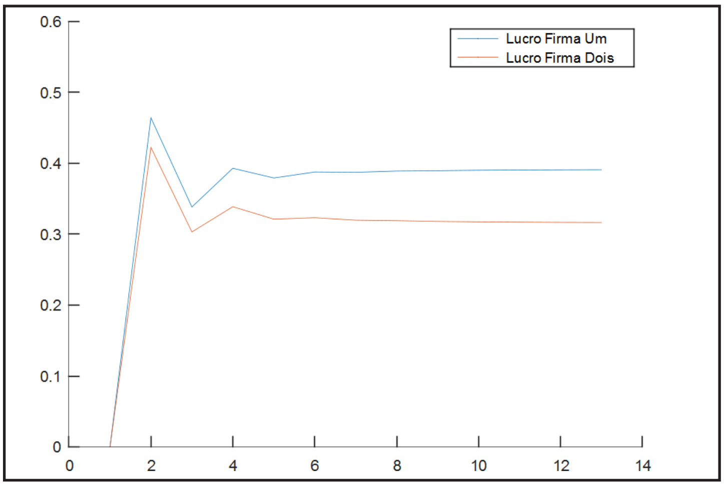
because the denominator will always be positive whenever $\alpha_1 > \alpha_2$, for $\alpha_i \in [0,1]$.

We show a numerical simulation of the Theorem 1 result in Figure 1. To create this simulation, the problem constants were set to $c = 1$, $b = 1$, and $a = 3$, and the initial conditions of $q_1 = q_2 = 1$ were used.

4 CONCLUSIONS AND FUTURE DIRECTIONS

In this article, we propose a generalization for a Cournot-type adjustment process in a duopoly market, incorporating memory into the model arising from the nonlocality of fractional differential operators. Assuming that the inverse demand and firm costs are affine functions of the produced quantities, we numerically show some conditions on the memory (order of the fractional derivative) and on the speed of the adjustment process so that the proposed model converges to the generalized Cournot stationary points.

Figure 1 – Profit of firms 1 and 2 for α_1 = 0.3 and α_2 = 0.2, c = 1, b = 1, and a = 3, showing that Theorem 1 assertions follows



Source: The authors

The approach's main result is demonstrated in Theorem 1, which guarantees that the firm with more memory profits more.

This is just a preliminary work, whose proposed generalizations to more complex adjustment models will be the result of future contributions.

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