A routine calculating the time dependent mixing height in coastal sites

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ABSTRACT

The paper describes an algorithm for the calculation of the time dependent mixing height h(t) in coastal sites, using momentum and heat flux and wind and temperature time series as input data.

A stationary expression for the coastal mixing height is used to avoid the solution of a partial differential equation for the evolution of h as function of time and fetch, thus reducing the problem to a direct integration in time. The algorithm has been implemented in a previously developed scheme that allows a continuous calculation over the diurnal cycle. The calculated evolution of h(t) during a few days is compared with Sodar estimates in a coastal site.

Topic: Measurements of micrometeorological parameters in the planetary boundary layer
Key words: mixing height, turbulent fluxes, semianalytical modelling, Sodar.

RESUMO

Descreve-se um algoritmo de cálculo da altura h da camada de mistura atmosférica a partir de dados pontuais de fluxo de calor sensível e momento, vento e temperatura , em função do tempo.

O algoritmo utiliza uma expressão estacionária para a altura da camada de mistura costeira que permite evitar a solução de uma equação diferencial em derivadas parciais para a evolução de h em função do tempo t e da distância a sotavento da linha da costa, reduzindo o problema a uma integração direta no tempo. O algoritmo foi introduzido dentro de um esquema, previamente desenvolvido, que permite a execução contínua do cálculo no ciclo diurno (24 horas). Compara-se a evolução de h(t) calculada por alguns dias num sítio costeiro com estimativas de h obtidas por medidas de Sodar.

Tópico: Medidas de parâmetros micrometeorólogicos na camada limite planetária.

Palavras chave: altura de mistura, fluxos turbulentos, modelagem semianalítica, Sodar.

1. INTRODUCTION

The height h of the turbulent atmospheric boundary layer is characterised by a typical diurnal cycle, growing during daytime due to the heat flux from the surface (mixing height) and usually strongly decreasing during the night. The determination of this height is important in applications that go from meteorological modelling and forecasting through dispersion problems for atmospheric pollutants. Simple one-dimensional models have been developed for the mixing height growth, basically modification and refinings of the Tennekes-Carson model (Tennekes, 1973), in which the mixing height is linked to the time integral of the surface heat flux, and limited by the temperature gradient and the inversion strength above the mixing height In coastal sites the mixing height can be also strongly influenced by advection. In fact, depending on the over-land fetch x, function of the wind direction, the actual time to cover the distance from the coast to the site, over which the heat flux is effective, can limit the effective time integral of the flux. In this case, at least a two-dimensional modelling is necessary, requiring a partial differential equation to be solved for h(x,t). If the travel time over x is small compared with the flux/wind variation time scale, stationary models demonstrated to be effective in describing h(x), but perform less well over long fetches, were the stationarity of the near surface condition can hardly hold during the travel time. Considering that also the fetch depends on the wind direction (that usually has a high degree of variability in coastal sites) it follows that the performance of a stationary model can vary from time to time, and does not depend only on the site topography.

For this reason, a practical algorithm is presented to cope with the problem of the time dependent growth of h(x,t) in a fixed position, avoiding a numerical solution of a partial differential equation.

The algorithm uses a recently proposed stationary solution that allows the reduction of the problem to a direct integration in time only. The algorithm needs heat and momentum fluxes and wind and temperature as input data. It has been implemented within a previously developed scheme (Martano-Romanelli, 1997), that allows continuous computation during the diurnal cycle. Results for a few days application in a coastal site are compared with Sodar estimations of the mixing height.

2. THE MODEL

The model equations in the Tennekes-Carson scheme can be written taking into account the advection, using the total (material) derivatives $D/Dt \equiv \partial/\partial t + U\partial/\partial x$, where U is the average wind speed in the mixing layer, (e.g.: Steyn and Oke, 1982):

$$h(D\theta_m /Dt) = Q_s - Q_i \qquad (1)$$

$$\Delta(Dh/Dt) = -Q_i \qquad (2)$$

$$D\Delta/Dt = y(Dh/Dt) - D\theta_m /Dt \qquad (3)$$

Here, Q is the potential temperature flux at the surface (subscript s) and at the upper temperature inversion (subscript i), Δ is the temperature gap above the mixing height (inversion strength), θ_m is the avearge potential temperature in the mixing layer and

$y=d\theta/dz|_{z=h}$ is the lapse rate above the thermal gap. Known the surface fluxes, the model can be mathematically closed through Zilitinkevich (1975) expression for the heat flux Q_i in the upper inversion:

$$\frac{g}{T}Q_i=-C_K\frac{w_m^3}{h}+C_T\frac{w_m^2}{h}\frac{Dh}{Dt} \qquad (4)$$

where $w_m^3 = w_*^3 + C_N u_*^3$, u_* is the friction velocity (square root of the module of the velocity flux), $w_*^3=ghQ_s/T$ is the convective velocity scale, g is the gravity acceleration, T the absolute temperature and C_N, C_K, C_T, are constants (Luhar, 1998) .

Inserting equation (4) in equation (2) an expression for Dh/Dt as function of h and Δ can be obtained:

$$\frac{Dh}{Dt}=\frac{\partial h}{\partial t}+U\frac{\partial h}{\partial x}=\frac{C_K w_m^3}{C_T w_m^2+gh\Delta/T} \qquad (5)$$

3. STATIONARY FORMULATION

A variable transformation t'=t-x/U y x'=x gives D/Dt=U∂/∂x' and the whole above formulation can be transformed in the correspondent stationary form in x' (Steyn y Oke, 1982). An efficient stationary solution have been proposed by Gryning-Batchvarova, (1990), and by Luhar (1998).

Luhar's solution reads:

$$\frac{Q_s\gamma(3+2C_K)}{3C_K\delta}\left[-\frac{\beta}{\alpha^2}(h-h_0)+\frac{\beta^2}{\alpha^3}\ln\left(\frac{\alpha h+\beta}{\alpha h_0+\beta}\right)+\frac{1}{2\alpha}\left(h^2-h_0^2\right)\right]$$
$$+\left[(h+\eta)^{\frac{2}{3}}-\frac{(h_0+\eta)^{\frac{2}{3}+\frac{1}{C_K}}}{(h+\eta)^{\frac{1}{C_K}}}\right]=\left(\frac{3+2C_K}{3C_T}\right)\left(\frac{g}{T}Q_s\right)^{\frac{1}{3}}\frac{x'}{U} \qquad (6)$$

where $\alpha=(1+2C_K)Q_s$, $\beta=2C_KC_N^3u_*^3T/g$, $\delta=(Qs^2\ T/g)^{1/3}$, $\eta=C_N^3u_*^3T/(gQ_s)$ and h_0 is the initial mixing height.

This expression uses an approximation for the temperature gap Δ, formerly proposed by Gryning and Batchvarova (1990) and then generalized by Luhar (1998) as:

$$\Delta = \Delta_{GB} + \Delta_{Y=0} \tag{7}$$

The Gryning-Batchvarova expression Δ_{GB} is given as a function of the Monin Obhukhov length $L = -u_*^3T/(kgQ_s)$, where $k=0.4$ is the Von Karman constant and reads:

$$\Delta_{BG} = \frac{\gamma h\left(C_K h - C_K C_N^3 kL\right)}{\left(1+2C_K\right)h - 2C_K C_N^3 kL} \tag{8}$$

Luhar's (1998) generalisation allows the model to be non-singular also for Y→0, adding the term:

$$\Delta_{\gamma=0} = \frac{3\delta C_K}{\left(3+2C_K\right)}\left[-\frac{\left(h+\eta\right)^{2/3}}{h} + \frac{\left(h_0+\eta\right)^{\frac{2}{3}+\frac{1}{C_K}}}{h\left(h+\eta\right)^{\frac{1}{C_K}}}\right] \tag{9}$$

4. THE PROPOSED ALGORITHM

Following Steyn and Oke (1982), the term $U\partial h/\partial x$ in the time-dependent problem can be explicitly calculated as derivative of a stationary expression of h(x') in x'=x, multiplied by U.

In the present case, a direct derivation is not possible, as h(x') has the implicit form of equation (6), but the problem can be overcome by first calculating h numerically by (6) in x'=x and then using equation (5) (with $D/Dt \equiv U\partial/\partial x'$) to obtain the numerical value of $U\partial h/\partial x'$ in x'=x.

After finding the term $U\partial h/\partial x$ for the given x (fetch between the site of interest and the coastline), as explained, the algorithm numerically integrates in time equation (5) , using (7), (8) and (9) to calculate Δ. The integration is made in the time interval between two steps in the data input time series (typically one hour), after which new input data are introduced and the whole calculation is repeated from the new values of x and $U\partial h/\partial x$ (depending on wind direction).

The proposed algorithm calculates the mixing height from time series of wind, temperature and momentum and temperature fluxes as input data. All data can be acquired at the same time by the use of a sonic anemometer (Cassardo et al., 1995), or calculated from local meteorological data by numerical algorithms (e.g.: Beljaars-Hostlag, 1990). An estimate of the temperature lapse rate γ is also required (see next section).

A proper correspondence table between wind direction sectors and fetches x for the site of interest must be included, that being the only needed topographical information.

Note that the algorithm can be also used in topographically homogeneous sites, simply putting $U\partial h/\partial x=0$ in equation (5).

It is also straightforward to take into account a subsidence vertical velocity w_s, if known, in the time integration, simply adding the term w_s/Δ in equation (2).

5. RESULTS

The algorithm results have been compared with mixing height estimations by Sodar measurements, in a coastal plain site at about 5 Km SW from the town of Lecce (Italy), in a central position on the Salento peninsula, the furthest SE region of Italy. The site is surrounded by the Mediterranean sea, with a minimum distance of about 15 Km from the coast. The input data have been measured by a sonic anemometer on top of a 15 m mast, and consist of hourly continuous (day and night) time series, for a few days in June 1997.

The present algorithm have been used within a previously developed scheme (Martano-Romanelli, 1997), that adds an exponential decay towards an equilibrium value for the night-time boundary layer height, allowing a continuous calculation during the 24 hours diurnal cycle. Two ways of estimating γ have been compared. First a daily constant value has been used, estimated above the mixing height from routine midday sounding at Brindisi, 40 km NE from the measurement site. Secondly, the simple calculation of the nocturnal inversion and its inner lapse rate in Martano-Romanelli (1997) algorithm (MR) has been used, associated to the average value of the mentioned daily soundings for the outer lapse rate, kept fixed as 0.005 K m^{-1} for all the days of test. In this last form, the algorithm needs just surface data to work (Martano-Romanelli, 1997).

The results are shown in fig. 1. The routine gives reasonable results for the daytime mixing height (the present algorithm), compared with the Sodar estimates in both cases, with an overall better performance when the daily values the lapse rate are used. In the night, the height appears to be underestimate, but, as noted elsewhere (Martano-Romanelli, 1997), it is possible that Sodar estimates be not reliable when the boundary layer height can be of the some order or even less than the Sodar minimum vertical range (50 m). Also, it must be pointed out that advection effects, that could potentially increase the nocturnal boundary layer height, are not explicitly taken into account in night-time.

The results can be compared with those obtained in the absence of the advection term ($U\partial h/\partial x=0$), These strongly overestimate the Sodar mixing height. It is apparent that, in spite of its simplicity, the proposed directional algorithm appears to catch the relevant mechanism that controls the daytime mixing height in the site.

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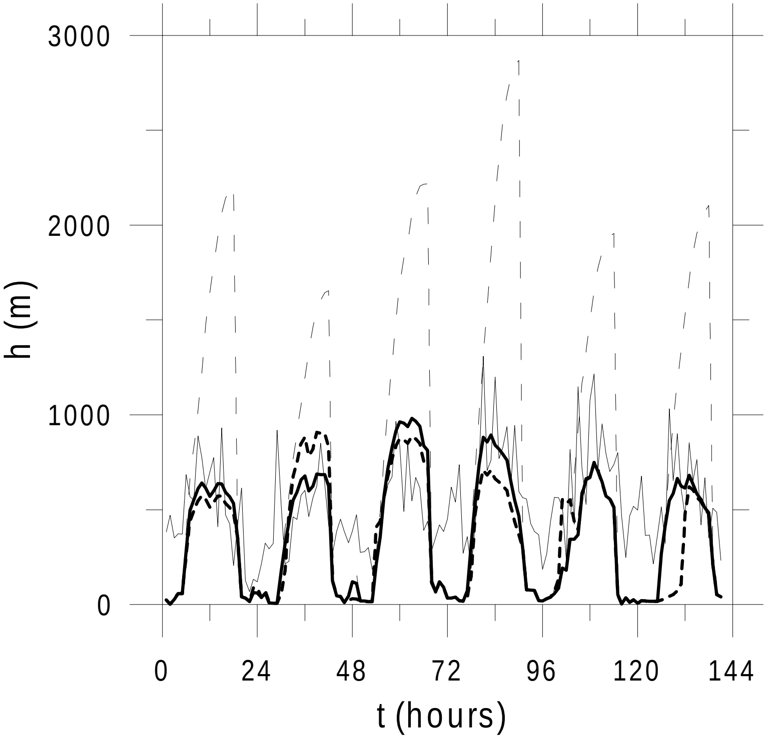

Fig. 1. Thin line: Sodar estimates. Thick line: model results with measured lapse rate. Thick dashed line: model results with estimated lapse rate. Thin dashed line: model without advective term (U∂h/∂x=0). Starting time: 09/07/1997, at 00h.

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