Smooth transition regression models: theory and applications in jmulti

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RESUMO

O objetivo deste tutorial é analisar modelos não lineares de modelos de Regressão de Transição Suave com o software JMulTi e contribuir para o entendimento da especificação STR, desde a estimativa até o ciclo de avaliação desses modelos. Fornece explicações pedagógicas, combinando conceitos teóricos e resultados empíricos de forma coerente. Especialmente nas relações económicas, onde é frequentemente encontrado um comportamento assimétrico com efeitos distintos nas contracções e expansões. Como as séries econômicas geralmente apresentam comportamento assimétrico/não-linear, os modelos de Regressão de Transição Suave (STR) fornecem uma estratégia empírica flexível que permite capturar os impactos de possíveis tipos de assimetria nos dados, Souza (2016). Esses modelos não lineares descrevem os movimentos dentro da amostra da série de retornos de ações melhor que o modelo linear correspondente. Os dados foram recolhidos da base de dados DataStream, para o período entre 02 de janeiro de 1995 e 29 de março de 2013, totalizando 4761 observações. As séries recolhidas consideram cinco dias por semana, tendo todas as séries diárias de índices bolsistas sido convertidas para a base 100, calculando-se em seguida as rendibilidades com base nas primeiras diferenças das séries de preço logaritmizadas.

Palavras-chave: Transição suave, Ruptura estrutural, Não linearidade, Séries temporais

ABSTRACT

This tutorial aims to analyze nonlinear models of Smooth Transition Regression with JMulTi and contribute to the understanding of STR specification, from the estimation until the evaluation cycle of these models. It provides pedagogical explanations, combining theoretical concepts and empirical results coherently. Especially in economic relationships, where an asymmetric behaviour with distinct effects is often found on contractions and expansions. As economic series generally present asymmetric/nonlinear behaviour, Smooth Transition Regression (STR) models provide a flexible empirical strategy that allows capturing the impacts of possible types of asymmetry in the data, Souza (2016). An overview of theory and applications in software is described. These nonlinear models describe in-sample movements of the stock returns series better than the corresponding linear model. The data used in this study consist of daily prices index from January 02, 1995 to March 29, 2013, a total of 4761 observations, from Germany (DAX30). The data was collected from the DataStream database considering 5 days a week. The data (price index) is converted to base 100 and the yields are then calculated based on the first differences in the log price series. 10-year interest rates treasury bond regarding the same markets identified has also been collected for the same period.

Keywords: Smooth transition, Structural Break, Nonlinearity, Time Series

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1 INTRODUCTION

JMulti is a free and interactive econometric software designed for univariate and multivariate time series analysis. Key features are: a) standard interface to communicate to a number of external engines to reuse existing math kernels and libraries, such as GAUSS, Ox, Matlab and R; b) it can read and write datasets in various formats (ASCII, all Gauss binary formats!, Matlab, Excel), in fact it could be used as a file converter; c) it comes with a rich set of GUI components to gather user input and to present results (time series selector, editable data tables, number selectors with interval based validation, etc.); d) has a powerful event-driven, thread-save, XML serializable, extendable internal data model (data frames, dates, date ranges, matrices, string arrays, ...); e) it makes easy to use integrated project management for settings and data, only some specific application handlers need to be written; f) all subsystems can be extended and customized for special purposes (http://www.jstatcom.com/, accessed 11/07/2019). Considering all the aspects described above, we use this software to implement the STR model.

STR models are used when the time series under study shows different behaviours along time so that the series can be divided in regime, Teräsvirta (1994), each one with different behaviours. STR models provide a method to test the existence of nonlinearities of the "smooth transition" type, which belongs to the range of nonlinear models for time series known as regime-switching (change in variables regime) (Teräsvirta and Anderson (1992) and Skalin and Teräsvirta (1999)). [1]STR models include both linear as autoregressive with TAR threshold, and particular cases (Tsay, 1989), therefore flexibility is a key advantage before other nonlinear models.

Smooth transition (S_t,) models allows to avoid the search for a rigid threshold between regimes, specifying the transition variable. In short, in STR models, the regime occurs in a given period and is determined by the value of the transition variable (S_t) and by the value associated with it, Souza (2016).

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[1] Switching Models are nonlinear models that allow analyzing different types of schemes. In these models, the transition between regimes is carried out instantaneously. STR models are a generalization of switching models, allowing a smooth transition between schemes.

The aim is to produce a tutorial that allows analyzing nonlinear models of Smooth Transition Regression with JMulTi software, developing theory and applications and contributing to the understanding of STR specification, from the estimation until the evaluation cycle of these models.

The estimation procedure developed in this tutorial is based on Teräsvirta (1994) methodology and its co-authors mentioned previously.

This tutorial develops the modeling cycle of smooth transition Regression models in JMulTi, based on three stages: specification, estimation and evaluation. The analysis of nonlinear models of STR type using JMulTi described it practically, but recommended that the user have a priori theoretical knowledge about these models.

The remainder of the paper is as follows. Section 2 presents an overview of unit root test including the concept and application in JMulti. Section 3 presents the implementation of STR model, results and a discussion of them. Finally, section 4 notes some conclusions.

2 UNIT ROOT TEST

In a non-stationary process, shocks amplitudes are persistent, not allowing a return to the initial equilibrium situation — this phenomenon of persistence results from profound shocks. With deep shocks, the process goes through irreversible changes over time so they can significantly influence the series level and the subsequent statistical analysis. Thus, the violation of stationarity assumption may lead to important limitations, such as spurious regressions resulting from common deterministic trends or tests of inefficient OLS (Ordinary Least Squares) estimates. In addition to investigating the stationarity of the series under study, it is crucial to verify the existence of structural breaks to separate the series from other types of deviations, such as unit-roots.

The structural break has a potentially similar effect on second-order properties of a historic series, and are a feature usually found in economic data. Found structural breaks, conventional unit root tests normally used (aiming to determine the integration order of a given time series), lose their test power face to different regimes conditions like a deterministic trend. In the presence of a change in the level data generation

process, unit root tests with a structural break are used to analyze series instability. These tests allow the identification of possible structural breaks as well as the year of their occurrence. In effect, structural break implies a significant change in the level and tendency in the time series, and a change may have a permanent or temporary character. If the series is stationary, shocks should have temporary effects. Otherwise, they will have permanent effects, that is, they will not recover to the initial level. The investigation of possible structural break and the date of its occurrence will be presented by test of Lanne et al. (2001, 2002). The evaluation of structural breaks is essential to determine the use of STR models.

According to Ferrer (2012), financial data rarely follow the Normal distribution and many studies are developed in this analysis focus. Nelson and Plosser (1982), for example, analyse by unit root test if shocks on actual production had permanent effects on the system. Thus, if the series in analyze has unit roots, the impact of long-term structural reforms will be balanced by other shocks. Li (2000), on the other hand, points out that if the real product has a stationary tendency, it implies that only big shocks are intended to change the fundamentals and will have at least semi-permanent effects on growth trajectory. Lee and Tsong (2012) apply a set of unit root tests to verify the existence of true long-term interest rate parity among G10 countries from 1971 to 2007. Chang et al. (2012) apply a threshold unit root test to test the long-term validity of purchasing power parity in a sample of nine East Asian countries.

Empirical studies provide evidence on the importance and necessity of unit root test, helping to determine the models types and their estimation, avoiding spurious results. The concept of stationarity says that a time series is stationary when it evolves randomly in time, revealing some sort of stable equilibrium, around the average. The basic idea of stationarity is that the laws of probability that act in the process do not change over time. In fact, and especially in economic data, stationarity is an unusual characteristic and usually, these series present some linear tendency, be it positive or negative.

A set of random variables $Z = \{Z_t : t \in T\}$ defined in a probability space (Ω, A, P) is said to be stationary if the data set statistics do not vary at instants t and $t+k$, $k = 0, 1,$

2,…, *n*. That is, the $E[Z_t] = E[Z_{t+k}]$ and $Var[Z_t] = Var[Z_{t+k}]$. The process can be strictly stationary (or strong) if all finite dimensional information remains the same under time translation, that is

$$F_{Z_{t1}, \cdots, Z_{tT}}(z_1, \ldots, z_T) = F_{Z_{t1}+k, \cdots, Z_{tT}+k}(z_1, \ldots, z_T), \tag{1}$$

where F represents the joint distribution function of random variables Z_t, *t* =1, 2, …,*T*. On the other hand, the process may contain second order (or weak) stationarity, so the process is considered weakly stationary if its averages and variances remain constant over time and the autocovariance function depends only on the lag among the instants of time. The stationary condition implies that the average and the variance of the process are constant and the covariance between Z_t e Z_{t-1} depends only on the "*lag k*".

A stochastic process $Z = \{Z_t : t \in T\}$ will be weakly stationary if and only if:

$$E(Z_t) = \mu \; \forall t = 1, \ldots, T \tag{2}$$
$$Cov(Z_t, Z_{t-k}) = \gamma_k \; \forall t = 1, \ldots, T, k = 0,1,2\ldots{}'$$

where μ is a real constant and γ_k has a constant structure for each *k*. By doing *k* = 0 in equation (3), it has the variance of Z_t, given by Var(Z_t) = γ_0. The term γ_k is a *k*-function called autocovariance function in the literature. On the other hand, the autocorrelation function (ACF) consists of the *k*-series autocorrelation coefficient or autocorrelation, which performs, together with the average and variance, a pre-requisite in stationarity study of stochastic process. ACF allows capturing the temporal extension and memory of the process by measuring the correlation of the current process values with their past values. Note that:

$$\rho_k = \frac{\operatorname{cov}(Z_t, Z_{t-k})}{Var(Z_t)} = \frac{\gamma_k}{\gamma_0}, \tag{3}$$

where $\gamma_0 = \sigma^2$ is a constant, on the assumption that the process Z_t, *t*=1,2, …,*T*, is homoscedastic. An important property of autocorrelation function is that it is a semi-definite positive form, that means:

$$\tag{4}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} \upsilon_i \upsilon_j \rho_{|t_i - t_j|} \geq 0,$$

for any set of time moments t_1, t_2, …, t_n and any real numbers $\upsilon_1, \upsilon_2, \dots$.

The autocorrelation function has the property of attenuating itself as k increases, exhibiting similar behavior in many cases, making it difficult to distinguish between processes of different order. Normally, Partial Autocorrelation Function (PACF) is used to aid in this distinction which is obtained from the following process AR (k):

(5)

$$Z_t = \phi_{k1} Z_{t-1} + \dots + \phi_{kk} Z_{t-k} + e_t,$$

The k-th partial autocorrelation is given by the coefficient ϕ_{kk} of the model (5). An important feature of the partial autocorrelation function is that it takes into account the entire process memory up to the correspondent k-order lag, by measuring the intensity of the relationship between two observations of the series, keeping the effect of the others constant (for more information on the subject see Enders, 2004). Thus, the characteristics of ACF and PACF are important to signal the possible generator process and its order and also the study of stationarity.

An example of a stationary process is white noise disturbance. The term applies itself to a sequence of random errors (or shocks) with a series of random variables independent and identically distributed. This is the particular case of a weakly stationary process. Then, the white noise is temporally homogeneous, stationary and with no time dependence. In a strong white noise process, the sequence of random variables Z_t, are uncorrelated and identically distributed, with zero mean, constant variance and Normal distribution.

In a non-stationary process, the ranges of shocks are persistent, not allowing return to initial balance situation. This phenomenon of persistence results from uncommon deep shocks such as the 2008 crisis[2]. On the other hand, frequent and small shocks tend to have a quick return to the deterministic trend.

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[2] In the United States, the financialization of the economy has resulted in the subprime crisis 2008. The subprime crisis has developed into the worst globally financial crisis for decades, leading to a severe real-economy recession.

With deep shocks, the process goes through irreversible mutations over time in order to significantly influence subsequent statistical analyzes. The violation of the stationarity assumption may lead to important limitations, such as: spurious regressions resulting from common deterministic trends or tests of inefficient OLS (Ordinary Least Squares) estimates. Besides investigating stationarity, it is crucial to explore the existence of structural breaks to separate the series from other types of deviations, such as unit roots, since the structural break has a potentially similar effect on second order statistics properties (variance) of a historic series.

In case of a structural break in deterministic trend, conventional unit root tests, such as Augmented Dickey-Fuller (ADF) (Said and Dickey, 1984) and Kwiatkowski-Phillips-Schmidt-Shin (1992) (KPSS) lose their power and lead to biased conclusions that tend to reject null hypothesis incorrectly (see Perron, 1989, Perron and Vogelsang 1992, Lee and Tsong, 2012). There are many tests that examine the existence of unit roots in the presence of structural breaks, (Banerjee et al., 1992, Zivot and Andrews 1992, Amsler and Lee 1995, Lumsdaine and Papell 1997, Perron 1990, 1994, 1997, (2005), Sikkonen and Lütkepohl, 2001, 2002, Lütkepohl et al., 2001, Lanne et al., 2002, Lee and Strazicich, 2003, 2004, Cavaliere and Georgiev, 2005a, b, Glynn et al., 2007, Vogelsang and Perron 1998, Rossi and Sekhposyan, 2014). These tests are usually based on endogenous determination of the break date, which reduces the bias of the statistics of test.

-   *Unit root test with a structural break*

Lanne et al. (2001, 2002) allow the identification of possible structural breaks as well as the year of their occurrence. The structural break implies a significant change in the level and trend of a time series, which can have a permanent or temporary character. If the series is stationary, shocks should have temporary effects. Otherwise they will have permanent effects, that is, they will not recover the initial level.

The evaluation of structural breaks in this study is fundamental to make inferences about their effects and their implications, such as the period in which they occur. Therefore, spurious results are avoided, such as rejecting the null hypothesis of a unit root, when in fact the series is under the effect of structural break, being able to

detect if structural ruptures are associated or not to a certain crisis. This is supported by several empirical studies (see: Berks et al., 2006, Perron, 2006, Andreou and Ghysels, 2009, Lee et al., 2003, Lee et al., 2004).

-   *Lanne et al. (2002) test – Unit root test with one structural break*

Lanne et al. (2002) proposed a change in the model developed by Perron (1989), in such a way that it became similar to the original model of Augmented Dickey-Fuller (ADF) to test the presence of a unit root in time series. The model with a linear trend $\mu_1 t$ and a term of change in differences $f_1(\theta)'\gamma$ is represented as follows, plus an error term υ_t:

$$\Delta Y_t = \mu_1 t + \Delta f_1(\theta)'\gamma + \upsilon_t \qquad (6)$$

Equation (6) denotes the formulation that can be used to test the presence of a unit root with a structural break in the series Y_t depending on the form assumed by the function $f_1(\theta)$. A statistical test can be performed on the estimated parameter γ. The modified null hypothesis is that the stochastic process has a unit root with a structural break, and the critical values of the modified statistic are presented in Lanne *et al*. (2002). The Lanne et al. (2002) belongs to the family of unit root tests for processes with level change/structural break with a regime change function. The model (7) is the basis for the test estimate:

$$X_t = \mu_0 + \mu_1 t + f_t(\theta)'\gamma + e_t, \qquad (7)$$

where $\mu_0 + \mu_1 t$ represents a linear deterministic trend, $f_t(\theta)'\gamma$ indicates the level change function and e_t is a residue generated by an AR (p) process with a possible unit root. θ and γ are parameters or vectors of unknown parameters.

Lanne et al. (2002) proposed three different cases of level change functions. The first case considers only a dummy of level change with a T_B data. The function does not incorporate other parameters in the vector θ, and γ is a scalar:

$$f_t^{(1)} = \begin{cases} 0, & t < T_B \\ 1, & t \geq T_B \end{cases} \qquad (8)$$

The second case considers a nonlinear gradual change (or smooth transition) based on exponential distribution function:

$$f_t^{(2)}(\theta) = \begin{cases} 0, & t < T_B \\ 1 - \exp\left[-\theta\left(t - T_B + 1\right)\right], & t \geq T_B \end{cases} \tag{9}$$

In this case, θ and γ are scalar or scalar vectors and $\theta > 0$. Finally, the third case considers a rational function of the lag operator L applied to a dummy of level change given by:

$$f_t^{(3)}(\theta) = \left[\left(1 - \theta L\right)^{-1} f_t^{(1)} \quad \left(1 - \theta L\right)^{-1} f_{t-1}^{(1)}\right] \tag{10}$$

where $\theta \in [0,1]$ e $\gamma = [\gamma_1\ \gamma_2]'$. For certain values of θ, the last two cases generate abrupt changes in a single T_B moment, constituting more general cases than the first.

In order to avoid problems with spurious rejections, and uncertainty regarding the correct point of the structure break, it is advisable to use an alternative unit root test. Tests with endogenous structural breaks significantly reduce the bias of test statistics, endogenous unit root tests with a structural break may exhibit distortions such as the null hypothesis of unit roots can often reject. When using such tests, it can incorrectly conclude that a time series is stationary with a structural break, when in fact, the series is non-stationary with structural break. Therefore, spurious rejections may occur to increase the break magnitude (Lee and Strazicich, 2004). These distortions have previously been observed by Nunes *et al*. (1997), Vogelsang and Perron (1998) and Lee and Strazicich (2001).

We recommend the joint use of a unit root test with two structural breaks alternative to the unit root test with a structural break, avoiding the problems mentioned above.

- Implemented features in JMulti software

JMulTi was originally designed as a tool for certain econometric procedures in time series analysis that are especially difficult to use and that are not available in other free packages of the same type. Now, many other features have been integrated as well

to make it possible to convey a comprehensive analysis. The software is divided in 7 section as following:

Section 1. Initial Analysis allows many tools for creating, transforming, editing time series.

- Unit Root tests: ADF, HEGY (quarterly, monthly), Schmidt-Phillips, KPSS, Unit Root test with structural break
- Co-integration tests: Johansen Co-integration test with response surfaces, Saikkonen & Lütkepohl test
- kernel density estimation
- spectral density plots
- cross-plots
- autocorrelation analysis

   Section 2. VAR (can be used for univariate modelling as well)

- VAR modelling (with arbitrary deterministic/exogenous variables)
- subset model estimation
- output in matrix form
- automatic model selection (various strategies based on information criteria)
- residual analysis with tests for non-normality, autocorrelation, ARCH, spectrum, kernel density, autocorrelation plots, cross correlation
- GARCH analysis for residuals
- Impulse Responses with bootstrapped confidence intervals also for accumulated responses, orthogonal and forecast error versions
- Forecast Error Variance Decomposition
- forecasting, also levels from 1st differences, asymptotic confidence intervals for levels
- causality tests
- stability analysis: bootstrapped Chow tests, recursive parameters, recursive residuals, CUSUM test
- SVAR modelling: AB model, Blanchard-Qua Model with bootstrapped standard errors

- SVAR Forecast Error Variance Decomposition
- SVAR Impulse Responses with bootstrapped confidence intervals

Section 3. VECM

- VECM modelling (with arbitrary deterministic/exogenous variables)
- restrictions on cointegration space, Wald test for beta restrictions
- Johansen, Two Stage, S2S estimation procedures
- EC term can be fully or partly predetermined
- subset model estimation
- output in matrix form
- automatic model selection (various strategies based on information criteria)
- residual analysis with tests for non-normality, autocorrelation, ARCH, spectrum, kernel density, autocorrelation plots, cross-correlation
- Impulse Responses with bootstrapped confidence intervals also for accumulated responses, orthogonal and forecast error versions
- Forecast Error Variance Decomposition
- forecasting, also levels from 1st differences, asymptotic confidence intervals for levels
- causality tests
- stability analysis: bootstrapped Chow tests, recursive parameters, recursive eigenvalues
- SVEC modelling with bootstrapped standard errors
- SVEC Forecast Error Variance Decomposition
- SVEC Impulse Responses with bootstrapped confidence intervals

Section 4. GARCH Analysis

- univariate ARCH, GARCH, T-GARCH estimation with different error distributions
- residual analysis for ARCH residuals with <u>robustified</u> test for no remaining ARCH (S. Lundbergh, T. Teraesvirta), plotting of variance process, kernel density for residuals
- multivariate GARCH(1,1) estimation, residual analysis, plotting of variance process together with univariate estimates, kernel density for residuals

Section 5. Smooth Transition Regression

- STR model specification with exogenous/deterministic variables

- linearity tests

- STR estimation

- various specification tests for no remaining nonlinearity, non-normality, no remaining serial dependency, parameter constancy

- various plots to check estimated model

Section 6. Nonparametric Analysis

- lag selection for univariate models based on linear and nonlinear selection criteria

- nonlinear estimation with configurable 3D plots

- residual analysis

- model selection for volatility process

- estimation of volatility process

- residual analysis for volatility estimation residuals

Section 7. ARIMA Analysis with fixed regressors (univariate)

- lag selection for AR and MA parameters with Hannan-Rissanen procedure

- estimation with fixed regressors

- residual analysis

- ARCH modelling of residuals

- forecasting with fixed regressors

In this tutorial we work with the UR test in section 1 and the estimation of STR model in section 5.

- Application in JMulti

We describe a way to perform the unit root test with one structural break (UR test). Figure 1 shows how we should go about unit root test with one structural break. On the output history on, select initial analysis. On the window initial analysis, select testing procedure/UR with a structural break, choose one option of residual analysis,

automatic search, choose one of the variables in the right window/confirm selection and execute.

Figure 1: Unit Root test with one structural break
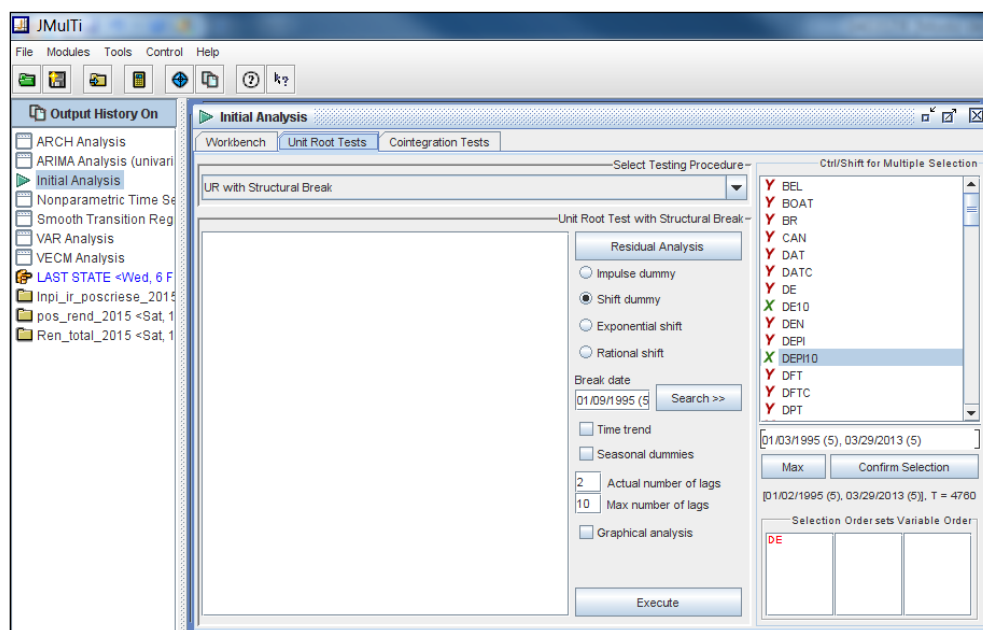
Note: Data is processed by authors (software: JMulti)

3 STR MODEL

STR model allows evaluating the effect of the variation of series under study besides the regime change. Linearity test is the first step of modelling procedure. It is used to verify if there is linearity or not, to determine which transition variable *(St)* is fundamental for modelling since it assumes the reference moment for regime change and to suggest which logistic models LSTR1 or LSTR2 should be used. In this context, we investigate the logistics models (k = 1) or LSTR1 and (k = 2) or LSTR2 in what the null hypothesis of linearity is $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$. So, the transition can be modelled by LSTR1 or LSTR2. For this selection, the variable that has the strongest rejection of the test (the lowest *p-value*) is used as a decision rule, especially if the differences are huge (Tsay, 1989).

The Linearity test suggests an adequate transition variable for each period as well as the best model specification, which in this case was a logistic model (LSTR1 or LSTR2).

Once the linearity is rejected and the transition variable is selected, the initial values of γ, C e S_t to estimate the STR model are determined by the minimum sum squared of residual (SSR) value.

This procedure is adopted to ensure that the values of the transition function have enough sample variation for each choice of γ, C and S_t. It creates a linear grid in C and a log-linear grid in γ.

For each value of γ and C, the sum squared of residual is calculated and the smaller values are taken as initial values. It should be also noted that γ is divide by $\hat{\sigma}_s^K$ so as not to reduce the test power of the Kth sample standard deviation of the transition variable.

Then, you should choose between LSTR1 or LSTR2 specifications. If LSTR2 is selected, the grid consists of C_1, C_2 and γ, and for LSTR1 it consists of C_1 and γ. In case of the LSTR2 specification, C_1, C_2 can only be estimated together.

The second-order logistic model is adequate in cases where the regimes have similar dynamic behavior and the intermediate regime has different behaviors. And LSTR1 allows characterizing dynamic behaviors of different variables of the two regimes (Teravirta, 2004).

The SSR graphs of sum squares of residuals function of γ and C are important for observing the contour surface since the maximum results are usually more visible in such graphs.

It is possible to perform the linearity test under additional constraints on θ. A variable can be excluded from the nonlinear part if $\theta_i = 0$. In linearity test, this can be considered by configuring elements of $\beta_j = 0$. In JMulTi this can be done by selecting elements from the respective Table 2.

If all variables are excluded from the nonlinear part, then the test cannot be calculated for s_t that are part of z_t. However, it still works for transition variables not contained in z_t, for example, t.
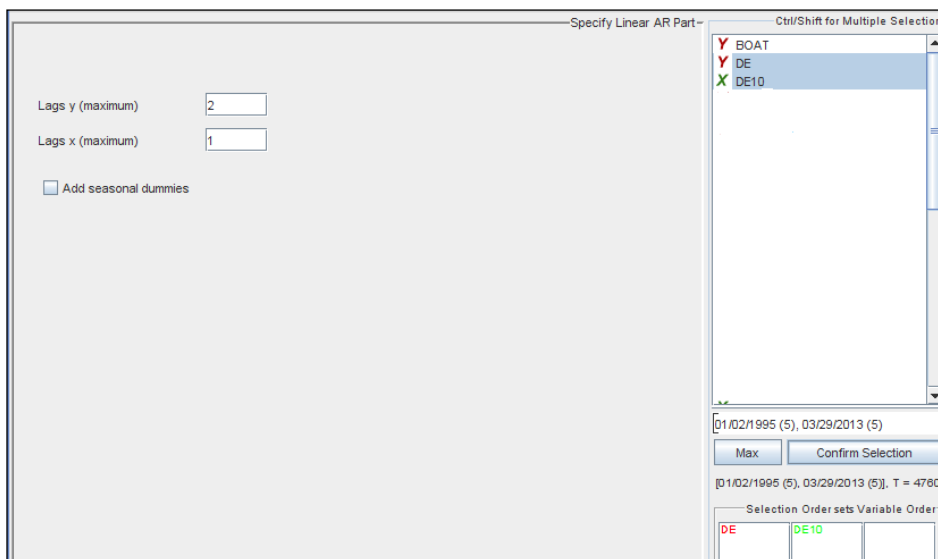
- Application in JMulti
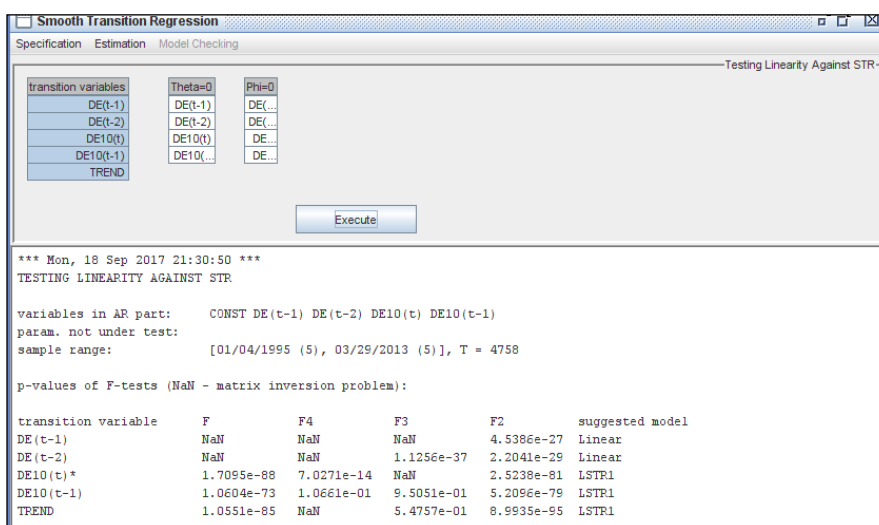
Figure 2: STR
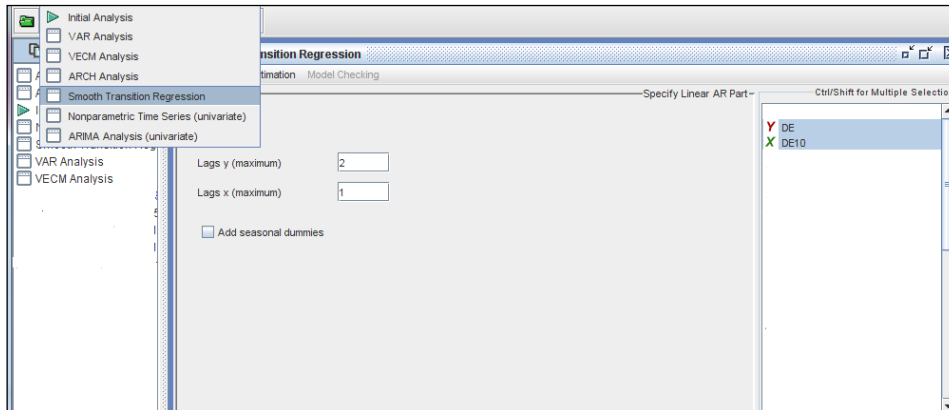
Note: Data is processed by authors (software: JMulti)

Figure 3: Choose of variables

Note: Data is processed by authors (software: JMulti) - Crtl/Shift for multiple selections

Figure 4: Test linear[3]

Note: Data is processed by authors (software: JMulti)

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[3] If the elements of S_t are close to zero or one, they can lead to invertibility problems. This is because in test regression the powers of S_t transition variable are included in the regressor. In this case, the output is NaN.

In this step, choose all or the transition variable of interest, and click Run, Table 1 shows the linearity test values.

Table 1: Linear Test

$(DE)_{(t-1)}$	NaN	NaN	NaN	4.5386E-27	Linear
$(DE)_{(t-2)}$	NaN	NaN	1.1256E-37	2.2041E-29	Linear
$(DE10)_{(t)*}$	1.7095E-88	7.0271E-14	NaN	2.5338E-81	LSTR1*
$(DE10)_{(t-1)}$	1.0604E-73	1.0661E-01	9.5051E-01	5.2096E-79	LSTR1
Tendência	1.0551E-85	NaN	5.4757E-01	8.9935E-95	LSTR1

Note: H_0: $\beta_3 = 0$; H_{03}:$\beta_2 = 0$ | β_3=0; H_{02}:β_1=0 | β_2=β_3=0
* show the model selected by linearity test.

The linearity test in Table 1 suggests the most adequate transition variable, which in this case is $(DE10)_{(t)}$ as well as the best specification of the model, which was a logistic model (LSTR1). The best choice suggested by Linear test is signalled by a STR. Once you rejected the linearity hypothesis and selected the transition variable, the start values γ, C e S_t to estimate STR model are determined by minimum value sum squared of residual (SSR).

This procedure is adopted to ensure that the values of transition function contain enough variation of sample for each choice of γ, C e S_t. A linear grid in C and a log-linear grid in γ were created. For each value of γ e C the sum square of residuals is calculated and the minimum value is considered with start values.[4]

Following, two smooth transitions equations are estimated, the specifications of LSTR1 or LSTR2 are choose. If LSTR2 is selected, the grid of parameters C_1, C_2 and γ, LSTR1 is constructed of C_1 e γ. In case of specifications of LSTR2, C_1, C_2 they can just be estimated together. The second-order logistic model will be adequate in cases of extreme regimes with similar dynamic behavior and the intermediate regime with distinct behavior. The LSTR1 allows characterizing dynamic behavior from different variables in two regimes (Teravirta, 2004).

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[4] It must also note that, γ it is divided for $\hat{\sigma}_s^K$ in way not to reduce the power of the test of the K-th. A standard deviation of the sample of the variable of transition.

The SSR charts from sum square of residuals function of γ and C are important to observe the contour and the surfaces, because the results are in general more visible in charts.

- SSR (sum squared of residual) plot

The SSR graphs of the sum squares of residuals function of γ and C are important for observing the surface and the contour, since the maximum results are usually more visible in such graphs.

Figure 5: Grid Search
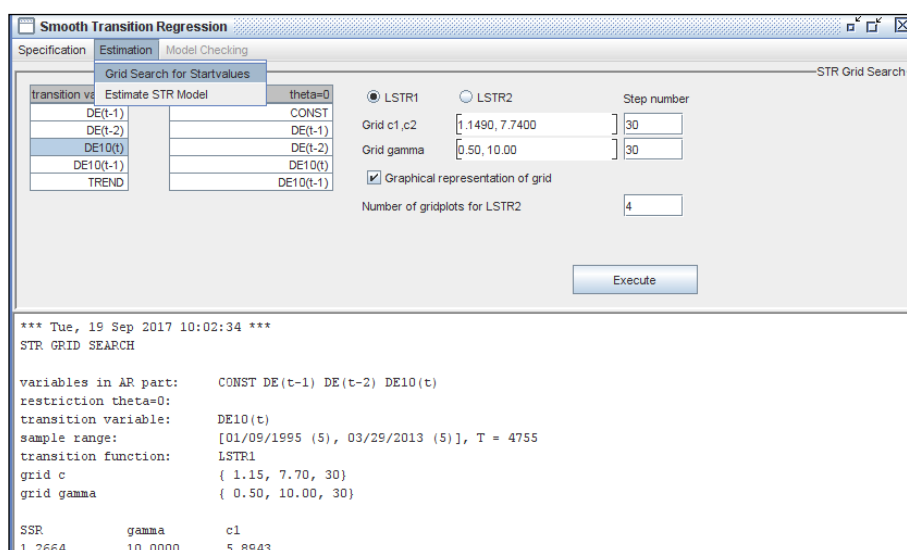
Note: Data is processed by authors (software: JMulti)

Table 2: Result of start values of SSR, γ and C

Mercado	S_t	Período	Modelo	SSR	γ	C_1
DE	DE10	1995-2013	LSTR1	1.2664	10.0000	5.8943

In addition to the constant (Const), the variables in the linear part AR are: $(DE)_{(t-1)}$, $(DE)_{(t-2)}$ and $(DE10)_{(t)}$, the constraint used is $\theta=0$. The analysis period is 03/01/1995 to 03/29/2013 (analysis performed 5 days a week). These results are obtained from estimation of grid search for start values Figure 5.

Figure 6: Graphical representation of the grid
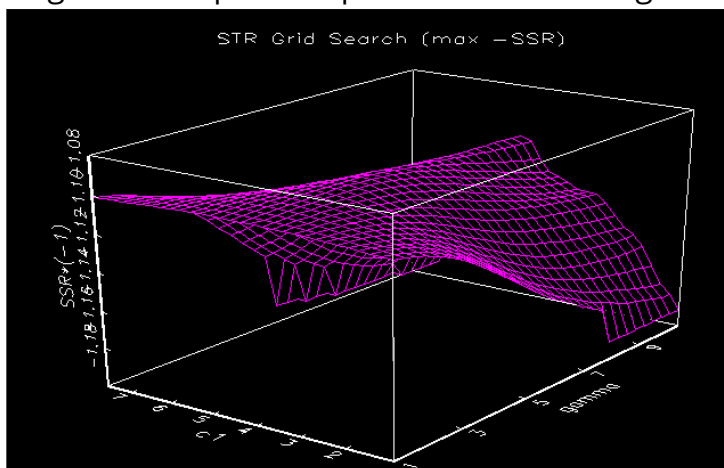
Note: Data is processed by authors (software: JMulti)

From the above results (Linear test), it is concluded that linear model is not the most adequate to explain the joint behavior of the variable under study. Therefore, next step is choosing the appropriate STR model type.

- STR Model

When initial values are found, the STR model is estimated and displayed at Table 3. The first step to specify a STR model is to select the linear part in the linear start model (AR linear). The selection mechanism allows choosing an endogenous variable y_t and an arbitrary number of exogenous factors X_t and deterministic variables. The maximum lag order is determined by the number of lags to include. In Table 3, the results are presented for all markets analyzed in the present study, and STR model found for the total period, pre and post 2008 crisis are shown.

Figure 7: Estimating the model
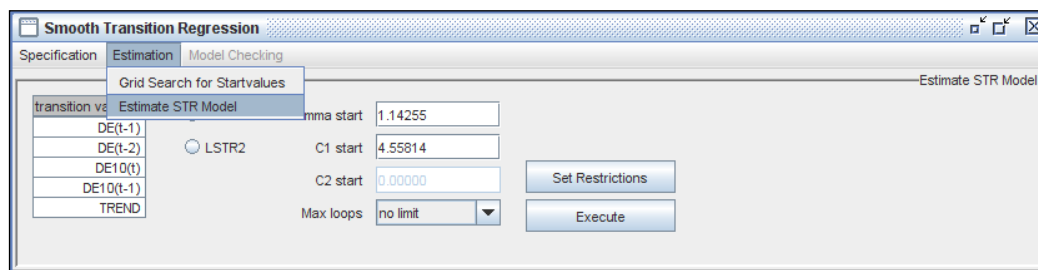
Note: Data is processed by authors (software: JMulti)

Table 3 shows the results of STR model for Germany.

Table 3: Estimation for STR model

|	Start	Estimate	Standard Error	p-value
Linear Part				
CONST	0.00980	0.01553	0.0144	0.2806
DE(t-1)	0.99502	0.96788	0.0235	0.0000
DE(t-2)	0.00311	0.02907	0.0233	0.2133
DE10(t)	0.11926	0.29564	0.0409	0.0000
DE10(t-1)	-0.11971	-0.29597	0.0408	0.0000
Nonlinear Part				
CONST	0.03991	-0.01896	0.0375	0.6131
DE(t-1)	-0.00718	0.03509	0.031	0.2572
DE(t-2)	-0.00076	-0.03216	0.0303	0.2888
DE10(t)	-0.22953	0.45191	0.0807	0.0000
DE10(t-1)	0.22758	0.45191	0.0807	0.0000
γ	10.0000	1.09674	0.2817	0.0001
C1	5.89430	4.43667	17.1075	0.0000

The model specification procedure (Table 3) for DE suggests a logistic transition function model LSTR1 by linear test, with a transition variable that is given by DE10. It is important to analyze the threshold parameter C1. In this case, it presents a positive value indicating a behavior similar to large shocks and falls of smaller proportions and positive shocks. Therefore, in the crisis regime, i.e., when there are large negative returns, long-term interest rates usually have a large impact on stock market.

- Validation of STR models

The evaluation of STR models consists, of tests based on the residuals of the estimated model and of considering the long-term properties of the model (Eitrheim and Teraisvirta, 1996). The validity of the hypotheses underlying estimation should be investigated since the parameters of STR models were estimated. We used the Lagrange (LM) and Eitrheim Teräsvirta (1996) multiplier tests built for this purpose. The assumption of non-autocorrelation error should be tested. In addition, it is useful to know if there are non-linearities left in the process after a STR model adjustment. This possibility is investigated by testing the hypothesis of any additive non-linearity against the alternative hypothesis that there is an additional STR component. Finally, the constancy of parameters is tested against the hypothesis that parameters change monotonically and without problems over time.

Model evaluation also includes verifying if the estimates seem reasonable and of course checking the residues for ARCH and normality effects. For more details see Eitrheim and Teräsvirta (1996). Diagnostic tests for autocorrelation, constancy of parameter, residual non-linearity test and tests for ARCH-LM and normality effects are applied to residuals of STR models shown in figures 8 and 9.
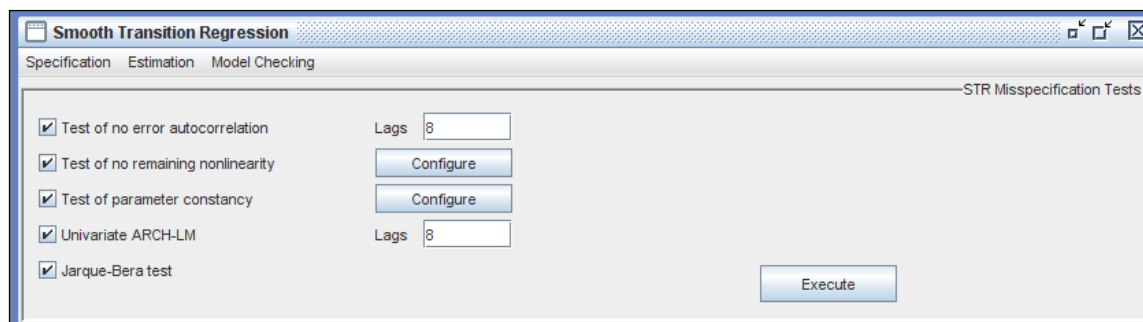
Figure 8: STR Model evaluation
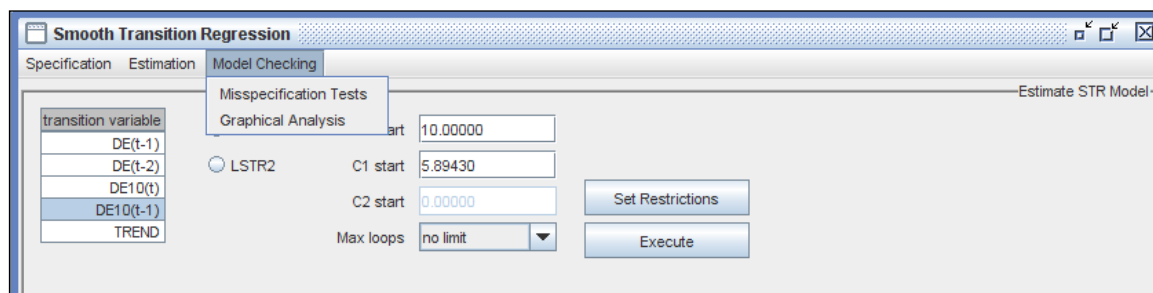
Note: Data is processed by authors (software: JMulti)

Model Checking ⇨ Misspecification Test ⇨ Execute

Figure 9: STR Model evaluation

Note: Data is processed by authors (software: JMulti)

Sellect all test ⇨ Execute

Table 4: Test of no error autocorrelation

Mercado DE		
Modelo LSTR1		
S_t: DE10		
Lag	F-value	p-value
1	4.9806	0.026
2	4.0691	0.017
3	3.5742	0.013
4	3.4172	0.009
5	3.6851	0.003
6	3.3453	0.003
7	3.0736	0.003
8	2.9901	0.002

Nota: H_0: ausência de autocorrelação.

Table 5: Test of parameter constancy

H	F-value	p-value
H1	8.6059	0.000
H2	5.1520	0.000
H3	4.1961	0.000

Note: H_0: constant parameters.

Testing parameter constancy is an important way of checking the adequacy of a linear model and retains its importance even in the present framework because nonlinear STR models are also estimated assuming constant parameters. Testing Ho derives with H3 alternative.

Table 6: Test of no remaining nonlinearity

	DE			
	F-value			
St	F	F4	F3	F2
DE10	2.38E-03	1.88E-02	1.13E-02	2.09E-01

In the test of no remaining nonlinearity, the alternative is assumed to be an additive STR component just as before (Eitrheim and Teräsvirta, 1996). Therefore, after the STR has been fitted, it should be checked whether there is remaining nonlinearity in the model. The test assumes that the type of the remaining nonlinearity is again of the STR type. The null hypothesis of no remaining nonlinearity is that $H_0^* : \beta_1 = \beta_2 = \beta_3 = 0; H_{04}^* : \beta_3 = 0; H_{03}^* : \beta_2 = 0 | \beta_3 = 0; H_{02}^* : \beta_1 = 0 | \beta_2 = \beta_3 = 0.$ The resulting $F_{statistics}$ are given in the same way as for the test on linearity.

Table 7: Jarque-Bera test

Jarque Bera	Skewness	kurtosis
3681.4298(0.000)	-0.0215	7.309

Note: In parentheses, p-value of the test
H_0: $E(u_t^5)^3 = 0$ and H_0: $E(u_t^5)^4 = 3$

Skewness and kurtosis are measured by conventional test statistics; normality refers to the test of Jarque and Bera (1980) for linear models, and to that of Lomnicki (1961) and Jarque and Bera (1980) for non-linear models. Looking at Table 4, it is analyzed the results regarding model specification (if well specified).

Table 5 tests the null hypothesis of constant parameters against continuous parameter change. Residual nonlinearity tests (Table 6) are important in STR models, since they consist of testing null hypothesis that parameters are constant against continuous smooth change in parameters, that is, models are sufficient to completely characterize non-linearity. The test in Table 8 measures the existence of ARCH effects or cyclic heteroskedasticity at a significance level of (0.05).

The STR models are a feasible alternative for a behavioral adjustment, for example between interest rates and stock indexes. The data used are only informative, because in this tutorial the aim is to demystify STR modeling using Jmulti software.

4 CONCLUSION

The STR models provide a method for modeling the existence of nonlinearity of smooth transition type, this model belongs to the models scale not linear for time series or regime-switching. The function of transition between regimes presupposes a different type from expected dynamic behavior. In other words, the historical economic data may depend differently on the state of the economy and may have different behavior in each state. The statistical properties and dynamic behaviours can be different in each one of the regime or state, where does an equation locally varies from a regime to other in function of a variable of transition (S_t).

Therefore, the STR models are a generalization of the nonlinear models that belong to the range nonlinear to time series or regime-switching and which allows to analyze different regime. In these models, the transition is made instantaneous but in the STR models, nonlinearity is of smooth transition type. What constitutes one of the advantages of this model. In addition, this advantages highlight other important points: (a) the model cannot simultaneously exist with actions of the economic agents, as it is locally linearly, allowing a more simplified interpretation; (b) the model can be interpreted as a linear model with stochastic coefficients.

Therefore, the STR models allow the transition between two different regimes associating a variable S_t. The variable S_t, therefore, is called "transition variable", and may be an exogenous variable, endogenous or even a linear trend, which would give rise to a model with variable parameters in a smooth way. Parameter c, in turn, can be

interpreted as a "location parameter" of the transition, that is, as the threshold between one regime and another, since the logistic function grows monotonically from 0 to 1 as the value of the transition variable S_t. γ determines the smoothness in changing the value of the logistic function, it has the interpretation of "degree of smoothness" of the transition between regimes.

Therefore, the STR model tells us that the regime that occurs in a given period is determined by the value of the transition variable and by the value associated with the transition function. It remains to be seen, however, whether the most suitable model to describe the data is in fact nonlinear. For this, STR modeling provides us with a linearity test, which has power against any kind of nonlinearity in the relationship in question. The results to Germany (DE) suggest that in the crisis regime, i.e., when there are large negative returns, long-term interest rates usually have a large impact on the stock market. The estimation for Germany suggests that the total effect is dominated by the pre-crisis effect and not so much by the post-crisis. Thus, the returns of stock market indices affect the previous values of the market itself.

The most recent information has more weight in the overall effect than the previous ones, that is, the memory relative to more distant moments is not transmitted with the same intensity to the current moment. The STR models applied in this research are a feasible alternative to a behavioural adjustment between interest rates and stock market indices of Germany. It Highlight the importance of modelling the cyclical behaviour of stock markets, identifying the influence of interest rates.

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