

Sediment transport for nonuniform sediment mixtures: the Egiazaroff equation and its application in a canyon river in Brazil

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ABSTRACT

Understanding the sediment yield and transport is one of the major topics of hydrosedimentology today. Although there are several methods to evaluate the incipient movement, the Shields model is mostly-used. However, the model proposed by Shields assumes that the sediments are uniform in an homogeneous and non-cohesive mixture. As such conditions are not easily found in nature, Egiazaroff proposed a sheltering coefficient, which considers the sediments in a non-homogeneous mixture in a wide range of granulometry. This coefficient allows correcting the Shields critical shear stress for nonuniform mixtures and can be used in mountain rivers, where the sediment size varies in several orders of magnitude. The equation proposed by Egiazaroff has been neglected for so many years in Brazil, and it is not mentioned in hydraulics or hydrosedimentology books. Thus, the present paper aimed to introduce the model proposed by Egiazaroff. Applying this equation to one natural river in a canyon, southern Brazil, the paper shows the good performance of this equation and recommends its use in mountain rivers in Brazil.

Keywords: Sediment transport; Incipient motion; Egiazaroff equation

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RESUMO

A compreensão da produção e do transporte de sedimentos é um dos principais tópicos da hidrossedimentologia atualmente. Embora existam diversos métodos para avaliação de início de transporte de sedimentos, o modelo de Shields é largamente utilizado. No entanto, o modelo proposto por Shields pressupõe que os sedimentos apresentam tamanho uniforme em uma mistura homogênea e não-coesa. Como tais condições não são facilmente encontradas na natureza, Egiazaroff propôs um coeficiente de abrigo de sedimentos que considera os sedimentos em uma mistura não-homogênea em uma ampla faixa granulométrica. Esse coeficiente permite corrigir a tensão de cisalhamento crítica de Shields para misturas não-uniformes e pode ser utilizado em rios montanhosos, onde o tamanho dos sedimentos varia em várias ordens de magnitude. A equação proposta por Egiazaroff vêm sendo negligenciada há anos no Brasil, não sendo mencionada nos livros de hidráulica ou hidrossedimentologia. Assim, o presente trabalho teve como objetivo introduzir o modelo proposto por Egiazaroff aplicando esta equação a um rio natural de um cânion no sul do Brasil e indicar o seu uso, visto que os resultados demonstram um bom desempenho da equação.

Palavras-chave: Transporte de sedimentos; Início de transporte de sedimentos; Equação de Egiazaroff

1 INTRODUCTION

The comprehension of sediment yield and consequently the sediment transport in rivers is one of the main subjects of hydrosedimentology nowadays. In the hydrosedimentological cycle the sediments are yielded, disaggregated, transported and then deposited, reaching rivers, lakes, reservoirs or seas. As any person can expect, the sediments are different in size and weight, depending on location where they are yielded. Hence, understanding the processes that define how the sediments are transported is a big theme in hydrosedimentology.

According to Carvalho (2008), as far the sediments are from the production zone, more uniform in size they tend to be. On the other hand, the closer to the production zone they are, the larger the size variation becomes. Thus, the process of sediment transport may differ in these cases. Sediments can be transported by washload, suspension, saltation, or as bedload, being finer sediments transported by suspension and larger

sediments transported as a bedload. Several studies (Hjülstrom, 1939; Shields, 1936; Einstein, 1950; Egiazaroff, 1965) tried to describe the incipient sediment movement in rivers. For achieving them, some considerations such as uniform grain size distribution, constant gradient, mean velocity and constant shear stress, have been usually done in order to simplify the sediment transport.

In Brazil, the mostly-used model for estimating the incipient bedload sediment transport is the Shields equation, followed by the Hjülstrom equation. Meanwhile Shields (1936) used a physically-based process, trying to identify the forces acting on a particle submerged in a channel, Hjülstrom (1936) used laboratory experiments relating mean velocity and sediment diameter. Both of them used some simplifications in order to estimate the incipient movement of sediments.

Although Shields equation is widely used for many purposes, Okazaki et al. (2001) demonstrated that Egiazaroff equation is more suitable for nonuniform sediment transport, presenting better results with comparison to other works. This equation is popularly used in Asia, especially in Japan (Nakagawa et al., 1982; Okazaki et al., 2001), where they dedicated themselves for the mountain-rivers management. In Brazil, the Egiazaroff equation has been neglected for many decades, not appearing in hydraulics handbooks as an applicable equation for estimating sediment transport.

Therefore, the objective of the present paper was to introduce the Egiazaroff's theory and equation, recommending its use in Brazil, especially in mountain regions where the sediment's size varies largely. Furthermore, its good performance will be demonstrated with one case study..

2 METHODOLOGY

2.1 Sediment Transport

Sediment transport is the movement of particulate material due to the water action (Chanson, 2004). In hydrosedimentology, sediment transport is one of the processes that have the most uncertainty in their values. In mountain rivers where the sediments are usually in nonuniform mixtures, Brardinoni et al. (2015) commented that their transport depends on a series of complex interactions among river discharge, activation of sediment sources from different types, and river morphodynamics. The authors furthermore commented that there is a lack of field observations with appropriate quantity and quality capable to allow the development of physically-based models.

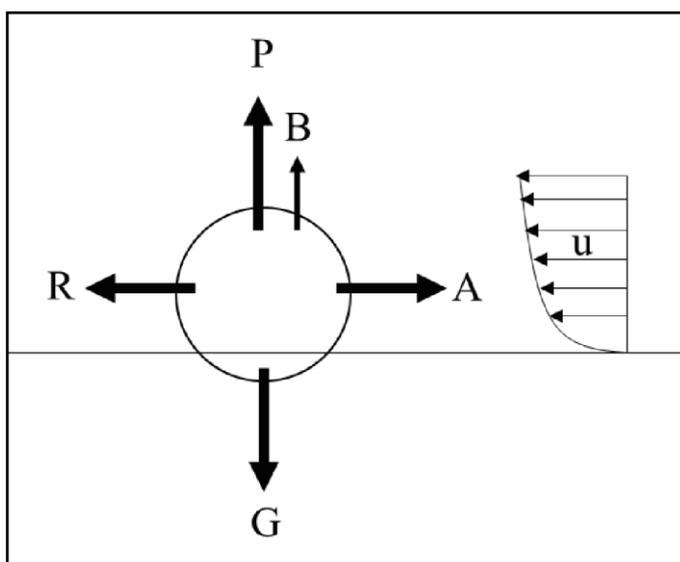
One of the main physically-based models actually in use for the incipient movement of sediments is that developed by Shields (1936). In his theory, it is considered the equilibrium between gravity and fluid forces, where the gravity resists to flow meanwhile fluid forces try to suspend and drag the sediments. Also, the Shield's theory considers the hypothesis of a homogeneous, non-cohesive particle, and constant channel gradient. These conditions are easily encountered in laboratory experiments but hardly observed in natural rivers.

Egiazaroff (1965) did some considerations on transport of nonuniform sediment mixtures. Thus, it is important to verify the Shields' theory and then the Egiazaroff's considerations.

2.2 Shields Theory

The classical approach for the incipient sediment movement proposed by Shields (1936) considers the forces and the processes involved on determination of critical condition in which the sediments will move due to the shear stress and the Reynolds number for a grain during the critical velocity (Figure 1). The critical condition is that situation for which the threshold of shear stress (considering velocity, discharge or deep) is achieved, and from this condition the sediments will move on.

Figure 1 – Scheme of forces considered in Shields theory: P is lift force; R is hydrodynamic force; G is the weight force; B is the floating pulse; A is the friction force; u is the shear velocity



The experiments conducted by Shields (1936) allowed identifying the combination of these forces, which were compiled on Shields' diagram (Figure 2). The Shields diagram's parameters are:

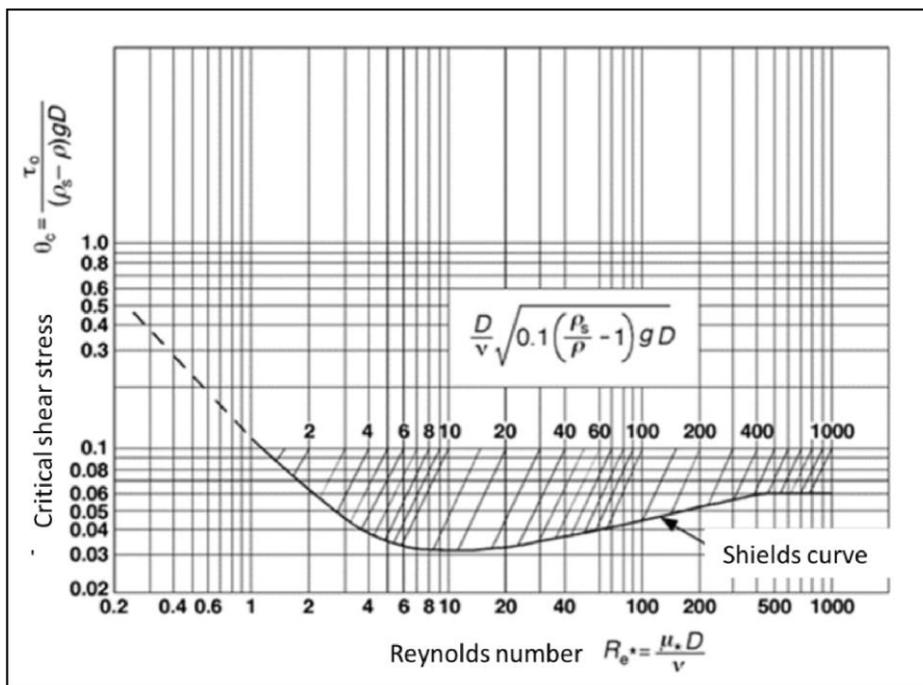
$$\tau = \frac{\rho_w \cdot u_*^2}{g \cdot (\rho_s - \rho_w) \cdot D} \quad (1)$$

$$R_e = \frac{u_* \cdot D}{\nu} \quad (2)$$

$$u_* = \sqrt{g \cdot R_h \cdot S} \quad (3)$$

where τ is the shear stress; ρ_w is the mass density of water; u_* is the shear velocity of flow; g is gravitational acceleration; ρ_s is the mass density of the sediments; D is the sediment diameter; R_e is the Reynolds number; R_H is the hydraulic radius; ν is the kinematic viscosity of water; and S is the river gradient.

Figure 2 – Shield's diagram



When the combination between critical shear stress and Reynolds number is found above the Shields' curve, the sediment will be transported. If the combination of values is located below the Shields' curve, the sediment will not be transported. However, the Shields theory has many limitations: for example, the sediments are considered as

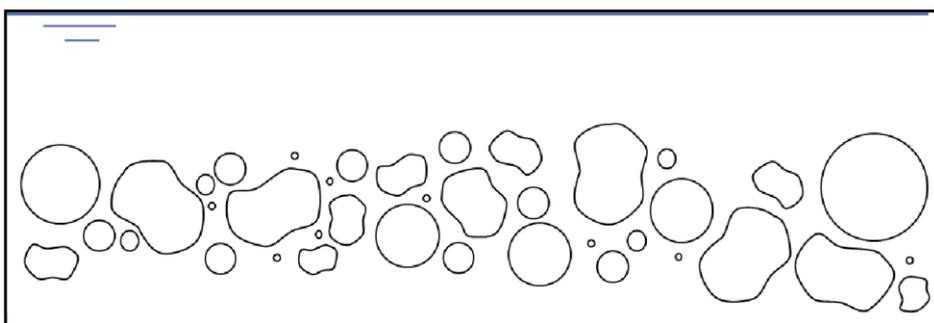
uniform, homogeneous, non-cohesive, and the channel has a constant gradient. Also, the critical shear stress is considered constant at a cross section. According to Vollmer and Kleinhans (2007), although the Shields' theory presents good approximations on shear stress, it is necessary to consider fluctuations on velocity flow due to turbulences. Besides, Byrd and Furbish (2000) commented that it is especially needed in streams which have high gradients, in which the velocity profiles are not logarithmic.

Although the Shields curve is easily applicable, their unrestricted use could super- or sub-estimate the necessary discharges for transporting sediments. Thereby, some investigations have been done in order to understand and minimize these errors. In case to evaluate the effect of non-homogeneous mixtures, Egiazaroff did some assumptions for correcting the critical shear stress in such condition.

2.3 Egiazaroff Assumption

As commented by Armanini (2005), it is very hard to find sediments composed by homogeneous mixtures in natural rivers, especially in mountain rivers where the sediment sizes vary in a large order of magnitude (Paixão and Kobiyama, 2019). Thus, Egiazaroff (1965) commented the incipient condition of motion is highly sensitive to the material's dimensions. Therefore, in mountain ones, small particles are protected by larger particles in a phenomena called sheltering or hiding or armouring (Figure 3).

Figure 3 – Sheltering effect in natural river



According to Komar and Li (1988), Egiazaroff (1965) assumed that a grain is entrained when the grain velocity equals to the settling velocity in posed water. Then, he proposed a hiding coefficient, ζ_j , in order to compensate the critical shear stress in such condition:

$$\theta_{cj} = \theta_{cu} \times \zeta_j \quad (4)$$

where θ_{cj} is the critical shear stress for the mixture; θ_{cu} is the critical shear stress for d_{50} ; and ζ_j is the sheltering coefficient.

Then, Egiazaroff (1965) proposed that the sheltering coefficient may be calculated by:

$$\zeta_j = \left(\frac{\log_{10} 19 \frac{d_j}{\bar{d}}}{\log_{10} 19 - \frac{d_j}{\bar{d}}} \right)^2 \quad (5)$$

where ζ_j is the sheltering coefficient; d_j is the representative diameter of a considered fraction in the sediment distribution; \bar{d} is the median diameter of the sediment mixture.

Later, Ashida and Michiue (1978) observed that the equation (5) sub-estimated the mobility of small particles ($d_j < 0.4\bar{d}$) although it worked well for larger particles. So, they proposed a conditional function, as described below:

$$\zeta_j = 0.85 \times \left(\frac{\bar{d}}{d_j} \right)^2, \text{ if } \left(\frac{\bar{d}}{d_j} \right) < 0.4 \quad (6)$$

$$\zeta_j = \left(\frac{\log_{10} 19 \frac{d_j}{\bar{d}}}{\log_{10} 19 - \frac{d_j}{\bar{d}}} \right)^2, \text{ if } \left(\frac{\bar{d}}{d_j} \right) \geq 0.4 \quad (7)$$

The Egiazaroff's assumption was considerably neglected for some years, especially because they thought that it was not considering lift forces. However, Okazaki et al. (2001) demonstrated that lift forces were implicitly considered. Also, Okazaki et al. (2001) reported that the Egiazaroff assumption presented good results for nonuniform mixtures. Therefore, the present study recommends the Egiazaroff equation use in mountain rivers in Brazil, because sediment diameter's size varies very largely.

2.4 Case study

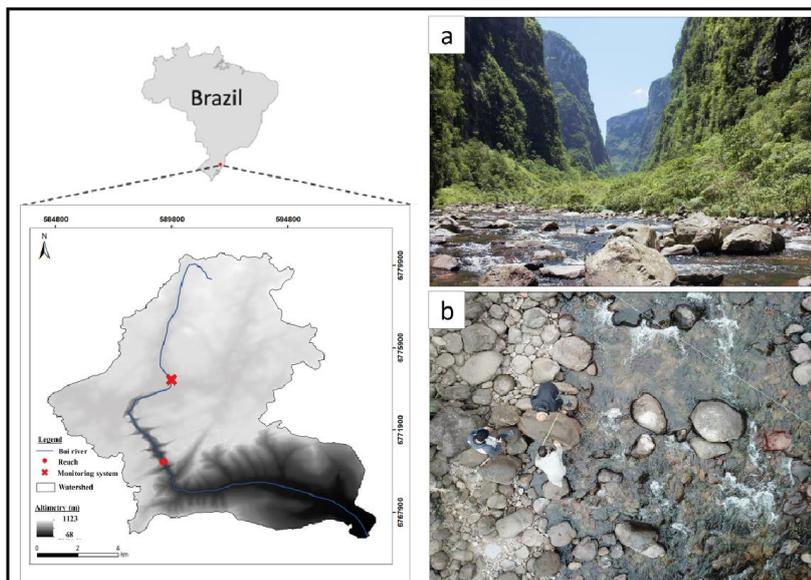
2.4.1. Study area

The study area is the Boi River basin (128 km²) (Figure 4), located in the South of Brazil, inside the Aparados da Serra National Park at the border between Rio Grande do Sul and Santa Catarina states. The basin's altitudes vary between 1012 and 85 m.

According to Köppen-Geiger classification, the climate is classified as Cfb upstream the canyon, and Cfa in the region downstream the canyon. The mean precipitation in the study area is 1800 mm/year.

The studied reach is inserted in the Itaimbezinho Canyon, a typical mountain environment with canyon landscape (Figure 4a). The reach is composed by bars, step-pool and cascade sequences, and its mean gradient is 0.03 m/m.

Figure 4 – Study area: a) canyon landscape at considered reach in the Boi River; and b) sediment measurements of nonuniform mixture during field survey



Source: elaborated by the authors.

2.4.2 Hydrology, topobatismetry and sediment data

This basin has a meteorological and fluviometric station which is recording data every 10 minutes. Rating curve was well established for this station, which allows estimating discharge at considered reach. The present study considered an event with 7-days duration, between 24th July and 30th July 2018. This event presents the highest discharges since the monitoring system was installed in the basin, in October 2017. During the event, an elevation of approximately 1 m in the Boi river was registered. Topobatismetry was performed with field survey in order to obtain detailed cross-sections in the considered reach. Sediments were collected and measured in field by zig-zag pebble-count procedure proposed by Bevenger and King (1995).

3 RESULTS AND DISCUSSION

Calculations were performed with Shields and Egiazaroff equations for sediment incipient movement. The summary of results is shown in Table 1. The values of the critical shear stress were calculated by considering the typical diameters obtained during field survey for both Shields and Egiazaroff equations.

Table 1 – Summary of results

Diameter	$\theta_{C_Shields}$	$h_{c_Shields}$ (m)	$\theta_{C_Egiazaroff}$	$h_{c_Egiazaroff}$	h_{m1} (m)	h_{m2} (m)
d_{16} (mm)	19	18.45	0.08	7.79	0.03	
d_{50} (mm)	45	43.69	0.18	43.69	0.18	
d_{62} (mm)	60	58.25	0.24	77.66	0.32	
d_{70} (mm)	80	77.66	0.31	138.10	0.56	0.20 0.56
d_{83} (mm)	163	158.24	0.65	573.18	2.34	
d_{90} (mm)	320	310.65	1.27	2209.10	9.01	
d_{100} (mm)	936	908.66	3.71	18900.19	77.10	

Obs.: d_N is the diameter which N% of the sediment sample is smaller than this diameter; $\theta_{C_Shields}$ is the Shields critical shear stress; $\theta_{C_Egiazaroff}$ is the Egiazaroff critical shear stress; $h_{c_Shields}$ is the Shields critical depth; $h_{c_Egiazaroff}$ is the Egiazaroff critical depth; h_{m1} is the mean water depth condition during the field survey; and h_{m2} is the mean water depth condition during the considered event.

It is noted that for particles smaller than d_{50} the critical shear stress considering the Egiazaroff equation present lower values than Shields equation. Also, for particles larger than d_{50} , the values of the critical shear stress calculated by considering the Egiazaroff equation are higher than those with Shields equation. It can result from the sheltering effect. As the larger particles are sheltered in a sediment mixture, they need higher shear stress to be moved out. Otherwise the shear stress acts predominantly on small particles, because lower values are necessary for moving smaller particles. In such condition, however, Armanini (2005) commented that Egiazaroff equation may subestimate flow conditions for incipient movement.

Critical mean water-depth for Shields and Egiazaroff were compared with the mean water-depth in two conditions: i) mean water-depth during field survey (h_{m1}); and ii) mean water-depth during the event of July 2018 (h_{m2}). Table 1 shows that sediments up to d_{62} could be continuously transported in both conditions. This situation, however, was not observed during field survey, which suggests the critical conditions for sediment

transport for smaller particles than d_{50} are not describing appropriately the field conditions.

For larger particles, Shields equation shows it is possible to move sediments up to d_{83} , approximately. However, such condition was not observed in field, being the Egiazaroff equation more realistic, keeping immovable the sediments larger than d_{83} .

4 CONCLUSIONS

The Egiazaroff equation was neglected for so many years, especially because it was thought that the equation did not consider lift forces. However, Okazaki et al. (2001) demonstrated the lift forces were implicitly included in Egiazaroff's assumption.

The Egiazaroff equation is largely used in Asia, especially in Japan, where the most part of rivers belong to mountain environments presenting nonuniform mixtures. In Brazil this equation is not considered neither in hydraulics or hydrosedimentology for incipient sediment movement. Thus, the present paper showed the good performance of the Egiazaroff equation and strongly recommends its use for evaluating the sediment transport, especially in mountain rivers.

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