APPROACHES TO INFER WIND FIELD FROM AIRBORNE DOPPLER RADAR DATA

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RESUMO
Este trabalho apresenta uma revisão sobre alguns métodos de restituição das componentes do vento a partir de dados coletados por radares Doppler embarcados em aviões. Os métodos discutidos são COPLAN, MANDOP e CARTESIANO (ODD, quad-Doppler e EODD). As bases essenciais dos métodos, suas vantagens e limitações são apresentadas.
Palavras-chave: campo de vento, radar Doppler embarcado, métodos de restituição do campo de vento

SUMMARY
This paper presents a review of some methods to infer wind components from airborne radar Doppler data. The methods presented are COPLAN, MANDOP and CARTESIANO (ODD, quad-Doppler and EODD).
Here are presented the essential bases of the methods and its advantages and limitations.

Keywords: wind field, airborne Doppler radar, methods to infer wind field

1 INTRODUCTION

Doppler weather radars have been used in a variety of field research programs, since the early 1970's. However, these studies have as experimental support Doppler radars ground based, what presents certain limitations, that don't allow to follow the complete evolution of a convective system.

Since the 1980's, airborne Doppler radars have been increasingly used to investigate meteorological phenomena, and many efforts have been deployed to evaluate their potential in convective storms or mesoscale convective systems, as a tool to obtain accurate description of the associated airflow (JORGENSEN et al., 1983; HILDEBRAND & MUELLER, 1985; RAY, et al. 1985). These studies have combined two or three quasi-orthogonal flight legs forming an 'L' or 'U' shaped flight pattern, which provide pseudo-dual or pseudo-triple Doppler data over domains of about 80 x 80 km². Although the analysis methods can provide reliable wind fields, one major problem is the relatively large data collection times reaching > 10 min., which is inadequate for highly evolving convective storms (RAY & STEPHENSON, 1990).

To overcome the temporal sampling problem caused by storm evolution, a Fore/Aft Scanning Technique (FAST) was proposed by FRUSH et al. (1986) in order to collect dual-Doppler data from a single straight flight path, by switching mechanically, at each sweep, the antenna towards the fore or aft direction at about 20-25° from the plane normal to the flight track (JORGENSEN & DUGRANRUT, 1991). This reduces the observation time by
a factor of two compared to the time of an 'L-shaped' pattern. However, the most significant advantage of FAST is in allowing dual-Doppler sampling while flying a simple straight-line track. This also allows sampling under conditions when executing 'L-shaped' patterns is impractical, e.g., in the presence of long and impenetrable lines of deep convection. At the same time, and in the context of the French-American ELDORA (ELectra DOppler RAdar)/ASTRAIA (Analyse STéreooscopique par Radar A Impulsions Aéroporté) airborne Doppler radar project (HILDEBRAND et al., 1994), a dual-beam system has been developed, consisting of a pair of antennae mounted back to back, pointing at a tilt angle of ±20°. A version of this system was installed in July 1991 on one of the two NOAA WP-3D (N43RF) research aircraft, and was tested during the Convection and Precipitation/Electrification (CaPE) experiment carried out in central Florida in summer 1991, and the data analysis allowed to show the equipment capacity (DOU, 1993; CHONG & TESTUD, 1996). Figure 1 gives a schematic view of the dual-beam radar system installed on the tail of the Electra and WP-3D aircraft, each beam prescribing an helical scan as the aircraft moves forward. Due to the rotation of the antennae about the aircraft's longitudinal axis, the dual observations from the fore and aft beam can be readily organized into tilted half-planes (or coplanes) as shown in Fig. 2. The operational use of the EUDORA/ASTRAIA took place during TOGA-COARE (Tropical Ocean Global Atmosphere - Coupled Ocean Atmosphere Response Experiment; WEBSTER & LUKAS, 1992) field project, that involved three airborne Doppler radar equipped to make FAST observations, during the Intensive Observing Period from November 1992 to February 1993 in the Western Pacific warm pool.

The Doppler radars are able to measure just the parallel component to the pointing beam direction (radial velocity), therefore to obtain the three wind components it is necessary to use one of the various approaches available to infer wind field. So, the aim of this paper is to present
a review of some methods to infer wind components from airborne Doppler radar data.

2 COPLAN METHOD

The COPLAN (COordinate coPLANar) method was conceived initially for the case of two Doppler radars ground based (ARMijo, 1969; LHERMITTE & MILLER, 1970; MILLER & STRAUCH, 1974) and in the sequence improvements were made (TESTUD & CHONG, 1983; CHONG et al., 1983; CHONG & TESTUD, 1983). The application of this method to the case of airborne Doppler radar data was developed later by CHONG & TESTUD (1996).

In this method, the restitution of the 3D wind field is made considering a cylindrical frame. Figure 2 shows that the organization of the dual-Doppler radar data is described entirely in a cylindrical coordinate system, considering the flight track as the axis of the cylinder. So, we have the coordinate system \((x, \ell, \alpha)\), where \(x\) represents the flight track, \(\ell\) the perpendicular direction and \(\alpha\) the elevation angle from the horizontal plane. At a specific point \((x, \ell)\) within an \(\alpha\) plane where are observed both radial wind velocities \(V_1\) and \(V_2\) from the fore and aft antennae (negative velocities are receding from the radar), the coplanar components that results from geometric combination of the measured wind vectors are given by:

\[
\Gamma = (-V_1 \cos \theta_2 + V_2 \cos \theta_1) / \sin(\theta_1 - \theta_2)
\]

\[
\psi = (V_1 \sin \theta_2 - V_1 \sin \theta_1) / \sin(\theta_1 - \theta_2) + v_T \sin \alpha
\]

where \(v_T\) is the terminal fallspeed (positive downward) of precipitating particles accounts for their contribution on the measured velocity and can be estimated from an empirical relationship with the observed radar reflectivity (Z). Here \(\theta_1\) and \(\theta_2\) are the tilt angles relative to the \(\ell\) axis (positive
clockwise), that is, positive and negative in Fig. 1, respectively. Equation (1) shows that only $\psi$ is contaminated by precipitation fallspeed and that uncertainty in the estimation of $v_T$ may have substantial effect, in particular at high elevation angles.

The third component $\Phi$ is then estimated by using the anelastic continuity equation expressed in a cylindrical frame as

$$\frac{\partial \Phi}{\partial \alpha} + l(\frac{\partial \Gamma}{\partial x} + \frac{\partial \Psi}{\partial l}) + \Psi(l - kl \sin \alpha) - kl \cos \alpha \Phi = 0 \quad (2)$$

where $k = -\frac{\partial \ln \rho}{\partial z}$ accounts for air density decrease.

Finally, these cylindrical components are readily related to the horizontal wind components $u$ and $v$ ($u$ being along x axis) and the vertical wind component $w$ as

$$\Gamma = u$$
$$\Psi = v \cos \alpha + w \sin \alpha$$
$$\Phi = -v \sin \alpha + w \cos \alpha \quad (3)$$

It can be observed that the equations (1) and (2) leads to a complete mathematical solution of the three wind components using two radar measures without any specific hypothesis, except the boundary condition for the integration of the continuity equation (2). This is the advantage of the COPLAN method that results of a very well placed problem (ARMIJO, 1969). Another interesting point is that this method takes into account the advection speed that is made in a very simple way, considering the cylindrical frame associated to the advected flight track, as showed CHONG & TESTUD (1996).

In spite of that, one of the greatest problems of the radar data analysis is the integration of the continuity equation. It is the physical condition, $w=0$ at the surface in the downward integration. This integration causes however errors that tend to amplify with altitude, while an upward
integration stabilizes these errors and also the deviations in the estimate of the vertical speed, due to arbitrary choice of the boundary condition in the top. To solve these problems, different methods are proposed. The first, largely used, is the O'BRIEN (1970) method, that consists to modify uniformly the divergence profile in order to annul the vertical velocity in the surface and in the top of the observed zone, that is not really the top of the cloud. Another variational method proposed by CHONG & TESTUD (1983, 1996) consists in searching the conditions in the surface, such that the vertical wind field be regular horizontally (derived minimum) in the whole domain, these vertical velocities in the surface verify in the statistical sense the physical condition.

The performance of the COPLAN method was analyzed by these authors by using data collected by airborne Doppler radar during CaPE.

The application of this method is, however, limited in the space, on the two sides of the straight-line flight track. The non use of above and below airborne Doppler radar data is associated to the absence of continuity in the measurements along the patterns of integration, due to vertical extension of the phenomenon, very inferior than the horizontal extension. Typically the restitution zone corresponds to the measurements placed in the inclination planes between -45 and 45 degrees.

The use of this method needs a previous data interpolation in a cylindrical mesh that is accomplished using the Cressman (1959) function.

3 MANDOP METHOD

The MANDOP (Multiple Analytical Doppler) method developed by SCIALOM & LEMÎTRE (1990) is an extension of the method proposed by MATEJKA & SRIVASTAVA (1982) to determine the horizontal wind using
data from any radar number. This method is totally different from the methods COPLAN and CARTESIAN.

The principle of the MANDOP method is based on the representation of the Cartesian wind components by the product of three space functions, each function being a development in series of orthogonal functions of the considered coordinate. The analytic form for each component is obtained by the adjustment of the measurements, made in a considered volume, using the least squares method to minimizing the following function:

$$ F = \sum (V_{th} - V_{obs})^2 $$

where $V_{obs}$ is the measurement, $V_{th}$ is the radial velocity analytical form in (x,y,z) and for a pointing angle defined by the azimuth AZ relative to y direction and elevation relative to horizontal (see Fig. 1 where y would be the north direction). $V_{th}$ is given by:

$$ V_{th} = u \sin AZ \cos EL + v \cos AZ \cos EL + (w + v_T) \sin EL $$

with

$$ u = f_1(x).g_1(y).h_1(z) $$
$$ v = f_2(x).g_2(y).h_2(z) $$
$$ w = f_3(x).g_3(y).h_3(z) $$

$f_k$, $g_k$ and $h_k$ are the developments in orthogonal polynomials (polynomial of Legendre, Fourier...); $v_T$ is the terminal fallspeed, obtained by an empirical relationship with the observed radar reflectivity. The development order, combined to the dimension of the analysis domain, defines the scale of the phenomenon that we want to restore.

In the dual-Doppler case, additional boundary conditions are needed and are introduced in the variational formalism. These are the
following physics boundary conditions: (i) \( w = 0 \) in the ground and (ii) mass conservation (continuity equation).

Like for the COPLAN method, MANDOP approach adapted to airborne Doppler radar data was tested with dual-Doppler data from CaPE (DOU, 1993). This method is adapted to infer wind fields in stratiform regions, but it can be valid to convective systems as showed DOU (1993) using an elevated order of development functions and a small domain. An advantage of this method is its capacity to use the whole measurements without geometric limitations, contrarily the COPLAN analysis. In order to facilitate the calculations, a previous interpolation in grid points is made.

4 CARTESIAN METHOD

The Cartesian method for the analysis of dual-Doppler data was initially proposed by HEYMSFIELD (1978) and used by several researchers, for the ground based radar case (Ray et al., 1980) and for airborne Doppler radars (HILDEBRAND & MUELLER, 1985; JORGENSEN & DUGRANRUT, 1991). In this method a cartesian frame \((x,y,z)\) is used to infer wind field. As in Eq. (5), the beam orientation is defined by its azimuth \( AZ \) relative to the \( y \) axis, and by its elevation \( EL \) relative to the horizontal. At points where radial velocities from the fore \((V_1)\) and aft \((V_2)\) beams intersect, the components are related to the cartesian wind components \((u,v,w)\) in a cartesian frame Oxyz as

\[
\begin{align*}
  u \sin AZ_1 \cos EL_1 + v \cos AZ_1 \cos EL_1 &= V_1 - (w + v_T) \sin EL_1 \\
  u \sin AZ_2 \cos EL_2 + v \cos AZ_2 \cos EL_2 &= V_2 - (w + v_T) \sin EL_2
\end{align*}
\]

where \( AZ \) and \( EL \) are the azimuth (clockwise direction from \( y \)-axis) and elevation (from horizontal) angles of the fore (subscript 1) and aft (subscript 2) beam direction reported in Figure 1, and \( v_T \) is the terminal fallspeed of
precipitating particles which contributes to the measured radial velocity and can be estimated from an empirical relationship with the observed radar reflectivity $Z$.

The above underdetermined two-equations system for $u$, $v$ and $w$ requires the additional mass continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} - kw = 0 \quad (7)$$

where $k = -\frac{\partial \ln \rho}{\partial z}$ accounts for air density decrease.

The Cartesian method uses an iterative procedure to solve equations (6) and (7):

1. assuming $w=0$ in eq. (6) and solving $u$ and $v$;
2. calculating $w$ by (7);
3. injecting this estimate of $w$ in the eq. (6) to obtain new values for $u$ and $v$.

These two last iterative process steps are repeated until the solution convergence. It is a relatively simple method, what explains its use by most of the researchers. Some limitations of this method led CHONG & CAMPOS (1996) to propose an extension of this method.

4.1 ODD METHOD

If more than two measures (more than two radars) are available the dual-Doppler Cartesian method still can be used, but in an overdetermined version, called "Overdetermined Dual-Doppler Analysis " (ODD). This method consists of rewriting (6) in a least square sense, as a function to minimize:

$$F = \sum_{i}[u \text{sen}AZ_i \cos EL_i + v \cos AZ_i \cos EL_i + (w+\gamma \text{sen}EL_i - V_i)^2]$$

where $i(=1,2,3,...)$ define the considered radar number. Deriving $F$ relative to $u$ and $v$ leads to
where \( V_i = V_i - (w + v_T) \sin EL_i \). The eq. 9 is solved considering the continuity equation (7), using the same iterative process described previously. It can be shown that the solution of (9) is exactly that for (6) when only two measurements are considered. The ODD method is therefore a general formalism that allows the treatment of two or more simultaneous measurements, following the iterative process described above. As for the precedent methods, the data interpolation in grid points by the pondered average is made previously. A variant of the ODD formalism was proposed by ROUX & SUN (1990), that integrates the ponderation process, avoiding the previous interpolation.

### 4.2 QUAD-DOPPLER METHOD

This method was proposed by JORGENSEN et al. (1995) to take advantage of the additional contributions of the measurements when two airborne double-beam Doppler radar flying parallel tracks to observe a precipitation system. Thus, each point in the domain of interest can be viewed from four different orientations, doubling like this the system of equations (6). The three components of the wind \( u, v \) and \( w + v_t = W \), are then solutions of an overdetermined system which can be readily solved in a least square sense as

\[
F = \sum_i \left( \alpha_i u + \beta_i v + \gamma_i W - V_i \right)^2 \text{ minimum}
\]

where \( i = 1 \) to 4 and
\[ \alpha_i = \sin AZ_i \cos EL_i \]
\[ \beta_i = \cos AZ_i \cos EL_i \]
\[ \gamma_i = \sin EL_i \]

The solution of (10), obtained by deriving \( F \) relative to \( u, v \) and \( W \), is given by

\[
\begin{bmatrix}
\sum_i \alpha_i^2 & \sum_i \alpha_i \beta_i & \sum_i \alpha_i \gamma_i \\
\sum_i \alpha_i \beta_i & \sum_i \beta_i^2 & \sum_i \beta_i \gamma_i \\
\sum_i \alpha_i \gamma_i & \sum_i \beta_i \gamma_i & \sum_i \gamma_i^2
\end{bmatrix}
\begin{bmatrix}
u \\ v \\ W
\end{bmatrix}
= \begin{bmatrix}
\sum_i \alpha_i V_i \\
\sum_i \beta_i V_i \\
\sum_i \gamma_i V_i
\end{bmatrix}
\] (11)

The inversion of (11) gives a direct solution to the 3 wind components, that differs from dual-Doppler solution (9), where only two wind components are defined. The vertical wind component \( w \) being the residual between the obtained \( W \) and the terminal fallspeed of particle deduced from a suitable \( v_T - z \) relationship. However such a wind field is not ensured to verify the mass continuity equation. RAY et al. (1985) and JORGENSEN et al. (1995) proposed the use of equation (7) to compute the vertical velocity since the horizontal components are not significantly contaminated by uncertainties in \( v_T \). Also JORGENSEN et al. (1995) suggested the use of the vertical velocity at top levels, obtained from the minimization problem (10), as an improved upper boundary condition for a downward integration.

It is important to point out that the (11) formalism was already used by RAY et al. (1985) for multiple Doppler radar data.

### 4.3 EODD METHOD

The EODD (Extended Overdetermined Dual-Doppler analysis) method was proposed by CHONG & CAMPOS (1996). It is an extension of the ODD method (Dual Overdetermined Doppler), to which two additional boundary conditions were added, with the aim to controlling: (a) the variations
of $u$ and $v$ in each step of the iterative process used to solve the system of
equations for $u$, $v$ and $w$; (b) the horizontal variations of the horizontal wind
field when it is not well determined in the regions, above and below the
aircraft in the dual-Doppler method.

The mathematical formalism used in the EODD method is
written considering the function $F$ minimum, where $F$ is given by:

$$ F = \int \left\{ \sum_{j} \left[ \alpha_j u + \beta_j v + \gamma (w^0 + v_T) - V \right]^2 + \mu_1 \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w^0}{\partial z} - k w^0 \right]^2 + \mu_2 \left[ J_2(u) + J_2(v) \right] \right\} dx \, dy \tag{12} $$

where $\alpha$, $\beta$ and $\gamma$ are functions of the elevation angles and azimuth of the
radar antenna, $u$ and $v$ are the horizontal velocities, $w^0$ is the vertical velocity
(specified in the beginning of the iterative process and obtained from the
continuity equation after), $v_T$ is the terminal fallspeed, $V$ is the radial velocity,
$\mu_1$ is a normalization factor, $\mu_2$ is a filter proportional to the cutoff wavelength,
k=$\partial \ln p/\partial z$, $J_2 = \left( \frac{\partial^2}{\partial x^2} \right)^2 + 2 \left( \frac{\partial^2}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2}{\partial y^2} \right)^2$ is a term that
minimizes the second derivative of the $u$ and $v$ field and $i \geq 2$.

Term A is the classical least-squares minimization formalism of
the ODD method, it makes the adjustment of $u$ and $v$ to the radial velocity
measurements.

Term B express that the mass continuity equation be verified in
the least-squares sense. Because $w^0$ is an input for (12), it is equivalent to
consider terms involving $w^0$ in B as a specified divergence at a previous step
and to search $u$ and $v$ such that the associated divergence is bounded (at the
initial step, $w^0{=}0$ is generally assumed everywhere and the first horizontal
wind field should be nearly nondivergent). This implies that contamination of \( u \) and \( v \) by errors in estimating \( w \) through integration of the continuity equation (7) should be moderate. The weight, \( \mu_1 \) in B, also accounts for units consistency between A, B and C.

Finally, term C is a low-pass filter that is controlled by the weight \( \mu_2 \) (see TESTUD & CHONG, 1983). The main property of applying this second derivative constraint is to provide regular fields by filtering out small-scale variations such as those involved in regions of increased errors, for instance above and below aircraft when only two measurements are available. In essence, C realizes a regular extrapolation in these regions from surrounding 'correctly conditioned' areas.

The discretization of the function \( F \) is made in each grid point \( k= (j-i)n_x+i \), where \( i \) varies from 1 to \( n_x \) along the x-axis and \( j \) varies from 1 to \( n_y \) along the y-axis. The minimization of \( F \) is obtained by deriving \( F \) relative to \( u_k \) and \( v_k \), e.g.: \( \partial F / \partial u_k = 0 \) and \( \partial F / \partial v_k = 0 \) where \( k= 1, n_x n_y \) leading to a linear equation system \( 2n_x n_y \), which can be written under the following matricial form: \( \mathbf{M} \mathbf{v} = \mathbf{p} \), where \( \mathbf{M} \) is the matrix of minimization, function of the coordinates of the observations, \( \mathbf{v} \) is the searching vector coefficients and \( \mathbf{p} \) is the minimization vector that depends on the measurements.

The inversion of this system is made in a iterative way, using the conjugated gradients method. The component \( w \) is obtained then by integration of the continuity equation, using the variational method proposed by CHONG & TESTUD (1983).

To validate the EODD method, airborne Doppler radar data collected on February 22, 1993, during TOGA-COARE (CHONG & CAMPOS, 1996) were used. Three comparisons were made: 1) EODD method - dual-Doppler Cartesian method; 2) EODD-2 method (combining two
measurements)- EODD-4 method (combining four measurements) and 3) EODD method - quad-Doppler method.

The results showed that the EODD method is able to minimize the problems of geometric order that obstruct the use of dual-Doppler Cartesian method and the quad-Doppler method. The EODD can be applied so well to one pair of observations (dual-Doppler) as for two pairs (quad-Doppler). This is an evident advantage of the EODD relative to quad-Doppler method, which is restricted to areas where 4 measurements are available. An advantage of the EODD-4 analysis over the EODD-2 analysis, is the significant increase of the wind field restitution domain, what allows a more global analysis of the systems.

5 CONCLUSIONS

In this article some methods to infer wind field from airborne Doppler radar data were presented. All discussed methods, except MANDOP, using a direct formalism to obtain the 3D wind field.

The analysis of the advantages and limitations of these methods allow the user to make its choice in function of the intended objective.
6 REFERENCES


Figure 1: Airborne dual-beam radar system. $\theta$ is the tilt angle of the antenna fore or aft from a plane normal to the aircraft axis. $\phi$ is the rotation angle of the projection of the beam onto this plane, EL and AZ are respectively the azimuth and elevation angles relative to the earth surface. ROLL is the angle that the wings make with the horizontal plane. PITCH is the angle that the aircraft longitudinal axis makes with the horizontal plane. TRK (TRACK) is the angle measured clockwise from the north to the aircraft ground trajectory, on the horizontal plane. HDG (HEADING) is the angle measured clockwise from the north to the projection of the aircraft's longitudinal axis on the horizontal plane. (Chong & Testud, 1996).
Figure 2: Spatial distribution of the dual-beam radar observations. Subscripts 1 and 2 refer to as fore and aft radar observations of radial velocity. Half-planes with elevation $\alpha$ from the horizontal depict the coplane organization (CHONG & TESTUD, 1996).