# DECOMPOSITION OF WIND VELOCITY AND TEMPERATURE FLUCTUATIONS INTO WAVE AND TURBULENT SIGNALS, USING WAVELET TRANSFORM

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**RESUMO**

Na camada limite planetária estável, o espectro de energia pode estar contaminado por vários tipos de ondas. Uma vez que o problema de difusão turbulenta relaciona-se fortemente com o espectro das flutuações de velocidade do vento, é importante separar as contribuições devido às ondas daquelas originadas pelo escoamento turbulento. Transformada de ondeleta, discreta e de pacotes, foi aplicada a flutuações de velocidade vertical e temperatura medidas em Candiota, RS (31°28´ S, 53°40´ W), amostradas com freqüências de 1 e 10 Hz. Ondas foram detectadas nestas séries temporais através de técnicas de espectro cruzado. Transformada de ondeleta foi aplicada para decompor as séries, permitindo a identificação daquelas contribuições relacionadas com as ondas detectadas. Estas foram então subtraídas

dos sinais originais. Portanto, esse método pode ser utilizado para decompor as séries em dois sinais, um contendo ondas e o outro correspondendo ao escoamento turbulento.


## ABSTRACT

In the stable stratified atmospheric boundary layer, spectra might be affected by several kinds of waves. Since the problem of turbulent diffusion is closely related to the spectra of wind velocity fluctuations, it is necessary to separate the contributions of those waves from that of turbulence. Discrete wavelet packets transform was applied to temperature and vertical velocity fluctuations measured in Candiota, RS (31°28´ S, 53°40´ W), sampled with a frequency of 10 Hz and 1 Hz. In these datasets, waves have been detected through cospectral techniques. Wavelet transform was used to decompose the series. It allowed identification of those contributions that are related to the detected waves, which were then removed from the original signals. Therefore, this method may be used to decompose the series in two signals, one containing waves and other corresponding to random turbulence of the flow.


## INTRODUCTION

Parameterization of the Stable Boundary Layer (SBL) has been a central issue in atmospheric turbulence research for awhile (Wyngaard, 1975; Yamada, 1979; Nieuwstadt and Tennekes, 1981). Wetzel (1982), for instance, discuss three different formulations for the theoretical SBL height.

Solitary waves have been observed, under certain circumstances, in a strongly stably stratified SBL, during, for instance, the winter in Antarctica (Rees et al., 1998). Those authors report identification of several wave-like phenomena, such as a) small-scale, horizontally propagating waves, generated by orography and associated with broad-band frequency spectra; b) shear driven turbulent eddies, which occur when the local Richardson number falls below 0.25; c) Kelvin-Helmholtz instabilities, with large amplitude and generated in the layers of sub-critical Richardson number near de surface; d) solitary waves, which maintain their form over large distances due to a balance between nonlinear effects and dispersion. They were able to determine that those later waves have a duration of the order of one minute and wavelength of 200 m. They probably have a topographic effect as a controlling factor.

During a stable stratification period, turbulence is largely due to mechanical effects and maybe strongly contaminated by buoyancy waves, which transport energy and momentum. Most parameterizations are closely related to some form of the energy equation. They might eventually overestimate the total turbulent energy due to the contribution of the waves. Therefore, it is necessary to identify those waves in order to study their relation to the total energy of the system.

In this paper, the main goal is to apply discrete wavelet packets transform as a method to identify the components due to waves. Then, they can be subtracted from the original measured signal and the resultant will be the turbulent component of the signal.

In the following section, a brief description of the experiment site and of the measurements is presented. Then, the method of analysis is outlined, followed by a description of the main results.

## DATA ANALYSIS

The experiment site, which is near a thermoelectric power plant, is located in a very flat terrain, covered mostly with short grass, but presenting some scattered trees. The measurements were made in a 12 m tower with the instruments placed on the 10 m level. The temperature sensor was a thin wire thermometer, the vertical velocity was measured with a sonic anemometer, and the horizontal components of velocity with Gill anemometers, all of which could be sampled at 1 or 10 Hz. The data analyzed was taken from two experiment periods, in May, 27th, and November, 15th, 1995.

## METHODOLOGY

### Phase Spectrum

It is known that the cross spectrum phases can be used to identify the presence of waves (Stull, 1988; Rees, 1991). The cross spectrum $\leftrightarrow_{AB}$, between A and B signals, is defined as

$$\leftrightarrow_{AB}= \leftrightarrow_{A}^{*}.\leftrightarrow_{B} \tag{1}$$

where $\leftrightarrow_{A}^{*}$ is the complex conjugate of the Fourier transform of signal A, and $\leftrightarrow_{B}$ is the Fourier transform of signal B. The real part of $\leftrightarrow_{AB}$ is defined as the cospectrum $Co_{AB}$, and the imaginary part is called the quadrature spectrum $Q_{AB}$. Then, the phase spectrum $\Phi$ can be defined as

$$\Phi_{AB}= \arctan(Q_{AB}/Co_{AB}) \tag{2}$$

It represents the phase difference between the two signals A and B that yields the greatest correlation for any frequency (Stull, 1988).

Since waves do not transport heat, their vertical velocity (denoted by $w$) and temperature (denoted by $T$) fluctuations must be $\pi/2$ out of phase. Therefore, when the phase of the cross spectrum between vertical velocity and temperature fluctuations, denoted $\Phi_{wT}$, is equal to $\pm\ \pi/2$ for a certain frequency, that frequency corresponds to waves. The selection of two datasets, one sampled at 1 Hz and the other at 10Hz, was performed through the application of this propriety. Both sets have $\Phi_{wT} \cong \pm\ \pi/2$ at least in two frequencies.

### Wavelet Transform

The wavelet transform of a sign is a decomposition using a finite function, which is translated and expanded, as the kernel of the transform. The wavelet transform is local, because of its compact support and its definition in a region of space. It allows the determination of singularities and partial reconstruction of the original sign.

Therefore, the discrete form of the wavelet transform, denoted WD, applied to a function f(x), may be defined as:

$$WDf(m,n) = \int_{-\infty}^{+\infty} f(x)\psi_{mn}^{*}(x)dx \tag{3}$$

where the kernel of the transform $\psi_{mn}$* is the complex conjugate of

$$\psi_{\mathrm{mn}} = \ell_0^{-m/2}\psi\left(\ell_0^{-m}\mathrm{x}-\mathrm{n}b_0\right) \tag{4}$$

and $\psi$ is the mother (or generating) wavelet, $\ell_0^{-m}$ is the scale dilation parameter, and n$b_o$ is the translation parameter, while $\ell_0^{-m/2}$ is a normalization factor (Farge,1992). A set of complete orthogonal wavelets is obtained when $\ell_0$=2 and $b_0$=1. The mother wavelet has to be carefully chosen, since there is a contribution of the analyzing function in the transformed field which may led to misinterpretations (Qiu et al., 1995).

Daubechies (1988) analyzed the proprieties of orthogonal wavelets and developed a family of orthogonal mother wavelets which were later named *daublets.* Orthogonal mother wavelets yield a unique and complete reconstruction of the signal (Farge, 1992).

Wavelet packet decomposition was employed to detect the various contributions, because this technique provides a more precise estimation of the wave's characteristic. A wavelet packet family is generated by translation and dilation of the mother wavelet, as one would do to apply the discrete wavelet transform, but also by modulation of a mother wavelet (Farge,1992). This technique was developed in analogy to Gabor’s Windowed Fourier Transform. The wavelet packets transform provides a more flexible analysis than that the wavelet transform, because of its larger basis of analyzing functions.

Turner and Leclerc (1994) were some of the first researchers to apply wavelet transform to separate a turbulent sign. Their technique consists in separating the original sign in two, one recomposed using the higher coefficients and the other, the remaining ones. Then, they could analyze, in the first, the coherent structures that are a characteristic of sheared turbulent flows. That is possible, since “the wavelet transform conserves energy not only globally but also locally” (Farge,1992) and the total energy can be split among the different scales.

Hence, in analogy to Turner and Leclerc’s work, the various components of a measured signal will be decompose using wavelet packets transform and then recompose using adequate coefficients to separate the waves from the turbulent signal.

## RESULTS

The cross spectrum technique detected two possible waves in the 1 Hz series and three in the 10 Hz one. Table 1 shows the critical frequencies, as number of cycles per time period, and their associated cross spectra phase values. Those are also registered in Figure 3 (a) and 3 (b). An error of ~6% was allowed, and phases between ±90° and ±85° where considered as possible wave components.

**Table 1**- Critical Frequencies and Cross Spectra Phase

| Sampled at | Critical Frequencies | $\Phi_{wT}$ (º) |
|---|---|---|
| 1 Hz | 3 | -85,51 |
| | 19 | -87,85 |
| 10 Hz | 11 | 87,32 |
| | 25 | 89,66 |
| | 29 | 85,43 |

For the 1Hz series, the two critical frequencies were associated with negative phase values, while for the 10 Hz series, the phase values were all positives. Nevertheless, the critical frequencies' range were similar.

The mother wavelet chosen for this analysis was a daublet of order 10, since it has the form of a modulated wavelike sign, but it is not symmetrical, which may allow for those waves that are growing or have begun during the measurements. The application of the wavelet packets transform generates a 12 level decomposition tree. Each $m^{th}$ level has m associated bands, where m=1,2,..12 and $\ell_0^{-m}$, with $\ell_0$=2, is the dilation parameter. The part of the signal related to each band is then reconstructed using the inverse wavelet packets transform. For instance, reconstruction of band 1 of level 1 yields the original signal, while the sum of the recomposed 12 bands of level 12 reproduces the original signal.

The 11th level of the decomposition tree provide a detailed decomposition of both series (temperature and vertical velocity). The 11 bands of this level were then recomposed and visually inspected to detect the bands where the components were out of phase during most of the time.

Through visual inspection, it was determined that the bands 4 and 8 of the $11^{th}$ level of the decomposition tree, for the sample taken at 1 Hz, where the ones in which the components, associated to the vertical velocity and temperature series, where out of phase during most of the time, denoted as d(11,4) and d(11,8). For the sample taken at 10 Hz, the bands where d(11,1), d(11,8), d(11,9), and d(11,11). These can be observed in figures 1 (1 Hz) and 2 (10 Hz). The original temperature series are presented in figure 1(a) and 2(a) , the reconstruction signals are represented in figures 1(b) and 2(b), along with their sum, and the final series, which is the result of the subtraction of the sum of the reconstruction signal from the original signals, are shown in figures 1(c) and 2(c). Similar results where obtained to the vertical velocity series, not shown here.

Notice that the 10 Hz series needs a larger number of bands to reconstruct its associated waves, which might indicate that at least one of their critical frequencies are due to a superposition of two waves.

Finally, the cross spectra phase was recalculated, using the resulting signals, to verify if the waves where removed. Figures 3(c), for the 1 Hz series, and 3(d), for the 10 Hz series, show those results. It can be noticed that, although there are no longer phases larger than |85º|, most of the phases were shifted, specially the ones associated with smaller frequencies.

## CONCLUSION

The wavelet transform was successfully applied to decompose a turbulent sign into its wavelike and turbulent components. The wavelike signal can be detected by comparison between the wavelet packets decomposition of vertical velocity and temperature, which are out of phase for those frequencies associated with waves.

Therefore, this technique can be used to remove the contribution of waves from a measured signal whenever there is a need to evaluate its turbulent component and the presence of wave(s) is detected using the cross spectrum phase. Moreover, it can be applied regardless of the frequency of sampling, since the results are valid both for 1 Hz and 10 Hz data.

Nevertheless, further research should be done to verify how the shift of phases observed after the removal of the wavelike contributions may affect other results. There is also need to classify the waves identified.


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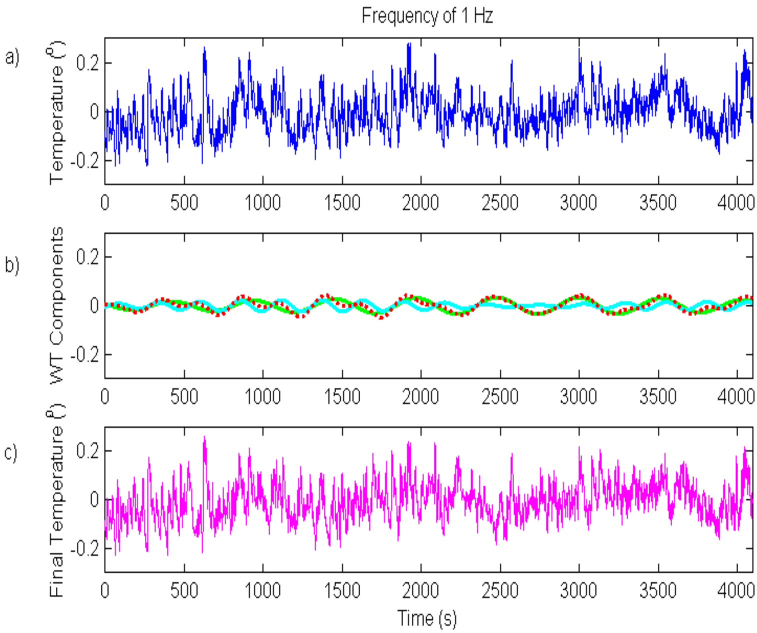


**Figure 1:** (a) Original temperature series, sampled at 1 Hz, (b) wavelet tranform components, where the green line is the d(11,4) component and the blue line is the d(11,8) one, while the dotted red line is their sum, (c) final temperature series, which is the result of the wavelet components subtracted from the original series.

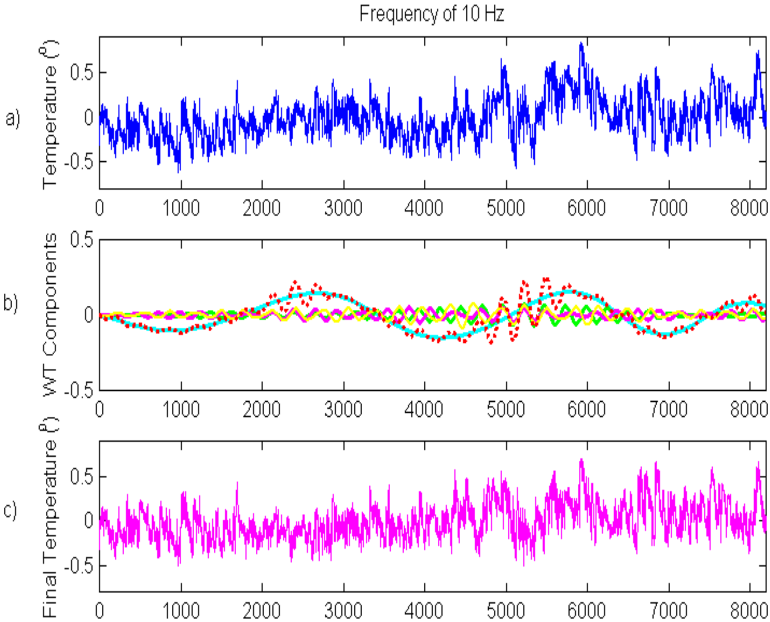


**Figure 2: (a)** Original temperature series, sampled at 10 Hz, **(b)** wavelet transform components, where the blue line is the d(11,1) component, the green line is d(11,8), the magenta line represents the d(11,9), and the yellow line is the d(11,11) one, while the red dotted line is their sum, **(c)** final temperature series, which in the results of the sum of the wavelet components subtracted from the original series.

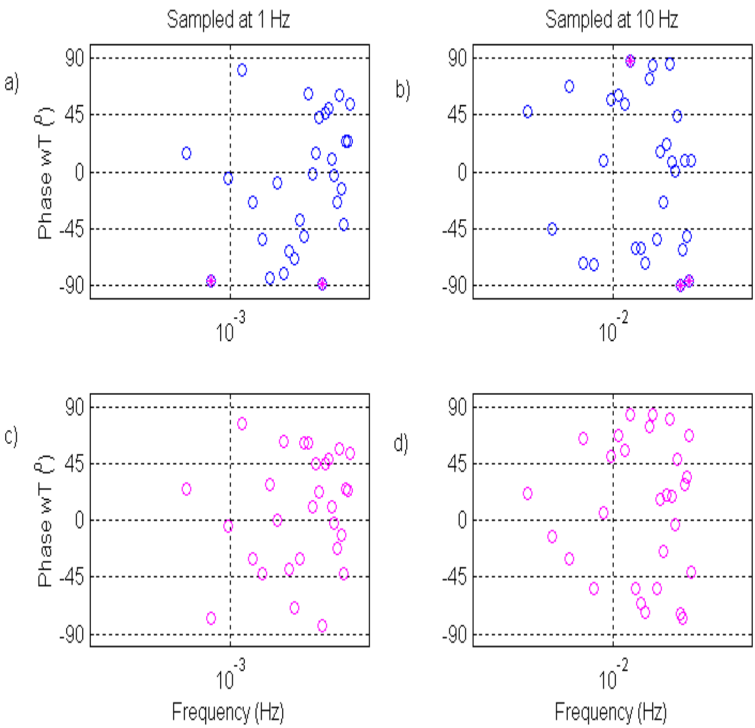


**Figure 3:** Cospectra of original (**(a)** and **(b)**) and final (**(c)** and **(d)**) temperature and vertical velocity series, sampled at 1 Hz (**(a)** and **(c)**) and 10 Hz (**(b)** and **(d)**). Magenta stars in original cospectra denote phases with absolute value larger than 85º.